

Supercloseness of a Least-Squares Finite Element Method for Elasticity

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Least-squares FEM are an attractive class of methods for the numerical solution of partial differential equations, as the idea minimizing the L^2 residuals in the partial differential equations combines the advantages of the mixed FEM with the production of symmetric and positive definite discrete systems and an inherent a posteriori error indicator. The related physical equations in the context of linear elasticity are the equilibrium equation and the constitutive equation. To preserve the symmetry of the stress, a modified weak form obtained by introducing the vorticity in $L^2(\Omega)$. The new least-squares functional is shown to be elliptic and continuous in the $H(\text{div}, \Omega)^d \times H^1(\Omega)^d \times L^2(\Omega)$ norm, which leads to the optimal error estimates for its finite element subspaces. Due to the strong connection of the stress approximation to that obtained from a mixed formulation based on the Hellinger-Reissner principle, the error associated with momentum balance is proved (similarly to (2)) to be of higher order than the overall error for the least-squares approach. This implies that the favorable conservation properties of the dual-based mixed methods and the inherent error control of the least squares method can be combined.

(1) Z. Cai, G. Starke, *Least squares methods for linear elasticity*. SIAM J.Numer.Anal. (2004) 42:826-842

(2) J. Brandts, Y. Chen and J. Yang *A note on least-squares mixed finite elements in relation to standard and mixed finite elements*. IMA J.Numer.Anal. (2006) 26:779-789.