

Optimal Convergence Rates in dPG for Elasticity

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The discontinuous Petrov-Galerkin methodology enjoys a built-in a posteriori error control in some computable residual term plus data approximation terms. This talk establishes an alternative error estimator, which is globally equivalent, but allows for the proof of the axioms of adaptivity and so guarantees optimal convergence rates of the associated adaptive algorithm. The talk exemplifies the analysis for the Poisson model problem $-\Delta u = f$ with a right-hand side f in $L^2(\Omega)$ in the polyhedral domain Ω simultaneously for the four lowest-order discontinuous Petrov-Galerkin schemes. Those and a low-order ultraweak scheme for linear elasticity are rewritten in terms of the first-order nonconforming Crouzeix-Raviart functions $CR_0^1(\mathcal{T})$ and its conforming subspace $S_0^1(\mathcal{T})$, with respect to a shape-regular triangulation \mathcal{T} into simplices, some projection $Q : L^2(\Omega) \rightarrow L^2(\Omega)$ and a parameter α .

For solutions $(v_{CR}, u_C) \in CR_0^1(\mathcal{T}) \times S_0^1(\mathcal{T})$ to this reduced mixed system, the novel error estimator $\eta(T)$ consists of the expected volume contributions $|T|^{1/n} \|f - \alpha Q v_{CR}\|_{L^2(T)}$ and the jump terms of the piecewise gradient of v_{CR} across the sides of any simplex $T \in \mathcal{T}$. The estimator exclusively involves the variable v_{CR} and seemingly ignores the conforming contribution u_C , but surprisingly also controls the error term $u - u_C$. The optimal convergence rates rely on standard arguments for stability and reduction, while the discrete reliability involves an additional term $h_0 \eta(\hat{\mathcal{T}})$ for an admissible refinement $\hat{\mathcal{T}}$ of \mathcal{T} ; this eventually enforces the additional condition of a sufficiently small initial mesh-size h_0 for optimal convergence rates.