Adaptive MMSE-based Precoding in Multiuser MIMO Broadcasting with Application to Cognitive Radio

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***Abstract*—Since cognitive radio (CR) in conjunction with non-orthogonal multiple access is a candidate techniques for future mobile radio networks, realizing underlay spectrum sharing with high capacity gain is an important issue. In this paper, we propose an adaptive linear precoder design for a CR based multiuser MIMO broadcasting system. The proposed adaptive precoder employs regularized channel inversion based on the minimum mean square error criterion (MMSE). Unlike the conventional MMSE based precoder, the proposed adaptive MMSE based precoding approach develops a new kind of diversity, so-called "precoding diversity", which considerably improves the signal-to-interference-plus-noise (SINR) ratio at the receiver of each cognitive user (CU) while fulfilling the interference constraint at the primary user (PU). Simulation results illustrate that the proposed adaptive MMSE based precoder can achieve good SNR and spectral efficiency gains outperforming the state-of-the-art.**

***Index Terms*—Adaptive MMSE precoding, Cognitive radio, MIMO broadcasting, Precoding diversity.**

I. Introduction

Recently, cognitive radio (CR) has extensively been investigated as a promising solution for utilizing the spectrum more efficiently [1]. In the underlay spectrum sharing strategy, CR transmits concurrently with primary radio (PR), thus CR should constrain its transmit power to manage the interference at PR users (PUs) to be below a predefined threshold [2] aiming to protect the PR performance from degradation. Meanwhile, CR should provide a qualitative service for CUs fulfilling their minimum SNR. With such a challenge in the underlay CR network, CR can hardly provide a good quality of service for CUs. Motivated by this, some methods have been developed to build reliable platforms for multiuser CR downlink systems employing multiple antennas [3], [4], [5]. Typically, since CR systems are unlicensed and have limitations with respect to transmission, they should utilize the data-carrying time slots spectrally and computationally efficient. On one hand, non-linear processing like [3] performs serial-based computations and therefore is inefficient and can not be handled by parallel computing facilities despite its spectral efficiency. On the other hand, although linear precoding techniques developed in [5], [6], [7], [8], [9] (even in non-CR context) have lower sum-rate, they are highly preferred in CR networks as they require less computations.

In [5], [6], [7], MMSE based preprocessing has demonstrated better performance versus zero-forcing (ZF) based preprocessing for multiantenna multiuser downlink systems since it addresses the transmit power boost issue, which is equivalent to the noise enhancement issue in ZF linear receivers. The MMSE precoder regularizes channel inversion using the entire transmit power and noise variance. Recently, an MMSE CR based block diagonalization (CR-MMSE-BD) scheme [5] has been studied that extends the MMSE based channel inversion scheme [7] to meet CR requirements. However, we claim that the multiple antenna structure of the MIMO channels in the conventional MMSE based precoding approach has not been fully utilized.

In this paper, we propose two adaptive linear precoders based on ZF and MMSE criteria, respectively, both employing a precoding diversity concept for a multiuser MIMO CR based downlink. Unlike the conventional ZF-BD CR based (CR-ZF-BD) precoder, the proposed scheme utilizes the MIMO channel paths and spaces to create precoding diversity in order to excite a multiuser interference (MUI) diversity. Such a diversity can achieve considerable SINR and spectral efficiency gains in the low SNR region. Although the adaptive ZF-BD precoding (AZB) method improves the SINR, it still suffers from the transmit power boost issue. Therefore, we propose an adaptive MMSE-BD (AMB) scheme for the CR downlink. The proposed AMB employs a non-iterative solution and overcomes the transmit power issue inherent in AZB by means of engaging a regularization factor. We mitigate the interference produced by the PR system at the CUs' receivers by means of a whitening process. Although the proposed adaptive precoding requires higher complexity than the conventional CR-ZF-BD and CR-MMSE-BD precoders, it can be handled by parallel computing facilities unlike the non-linear precoder developed in [3]. As will be seen in the simulation results, the proposed AMB precoder considerably improves the SINR as well as the spectral efficiency and outperforms the conventional CR-MMSE-BD precoder.

II. System Model and Problem Statement

A. System Model

Consider a cognitive base station (CBS) equipped with N_{CB} antennas that communicates with a set of K CUs denoted as $\mathcal{K} = \{1, \dots, K\}$ each having N_c antennas on a cognitive based multiuser MIMO broadcast channel as shown in Fig. 1. Denote the total number of CU receive antennas as $N_r = K\, N_c$. Let the MIMO channel between the CBS and the kth CU denoted as $\mathbf{H}_k \in \mathbb{C}^{N_c \times N_{BS}}$. A PR network (primary base station (PBS) has N_{PB} antennas that communicate with a single PU that has N_p antennas) coexists with the CR network. Denote the channel between the CBS and the PU as $\mathbf{G} \in \mathbb{C}^{N_p \times N_{CB}}$, the channel between the PBS and the PU as $\mathbf{T} \in \mathbb{C}^{N_p \times N_{PB}}$, and the channel between the PBS and the kth CU as $\mathbf{Z}_k \in \mathbb{C}^{N_c \times N_{PB}}$. The assumption is that all $\{\mathbf{H}_k\}$ and $\mathbf{G}$ are known at the CBS, while the kth CU only knows $\mathbf{H}_k$ and $\mathbf{Z}_k$ infers that the MUI should be managed at the transmitter side via preprocessing.

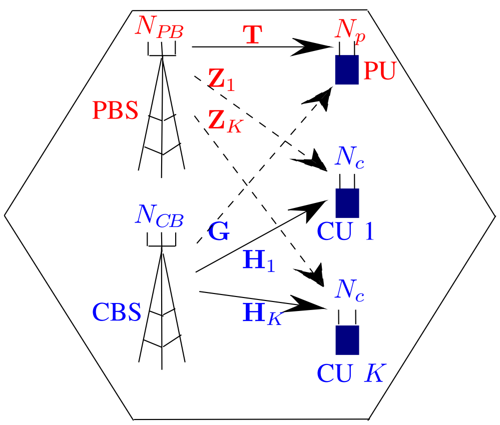

Fig. 1. System model of cognitive MU-MIMO broadcast channel.

The kth CR precoded transmit signal vector, data vector, noise vector, power matrix, preprocessing matrix, and postprocessing matrix are denoted as $\mathbf{x}_k \in \mathbb{C}^{N_c \times 1}$, $\mathbf{s}_k \in \mathbb{C}^{N_c \times 1}$, $\mathbf{n}_k \in \mathbb{C}^{N_c \times 1}$, $\mathbf{P}_k \in \mathbb{C}^{N_c \times N_c}$, $\mathbf{F}_k \in \mathbb{C}^{N_{CB} \times N_c}$, and $\mathbf{W}_k \in \mathbb{C}^{N_c \times N_c}$, respectively. Regarding the PR network, the precoded transmit vector, data vector, noise vector, power matrix, and precoding matrix are denoted as $\mathbf{x}_p \in \mathbb{C}^{N_p \times 1}$, $\mathbf{s}_p \in \mathbb{C}^{N_p \times 1}$, $\mathbf{n}_p \in \mathbb{C}^{N_p \times 1}$, $\mathbf{P}_p \in \mathbb{R}^{N_p \times N_p}$, and $\mathbf{F}_p \in \mathbb{C}^{N_{PB} \times N_p}$, respectively. The entries of noise vectors $\mathbf{n}_k$ and $\mathbf{n}_p$ are independent and identically distributed (i.i.d.) Gaussian random variables, i.e., $\mathbf{n}_k \sim \mathcal{CN}(\mathbf{0}, \sigma_n^2 \mathbf{I}_{N_c})$ and $\mathbf{n}_p \sim \mathcal{CN}(\mathbf{0}, \sigma_n^2 \mathbf{I}_{N_p})$. Therefore, the aggregate received signal of the CR system under investigation can be expressed as

$$\underline{\mathbf{y}} = \underline{\mathbf{H}}\ \underline{\mathbf{F}}\ \underline{\mathbf{s}} + \underline{\mathbf{Z}}\mathbf{F}_p\mathbf{s}_p + \underline{\mathbf{n}} \tag{1}$$

where the multiuser vectors and matrices of (1) are defined as follows: $\underline{\mathbf{y}} = [\mathbf{y}_1^T \ldots \mathbf{y}_K^T]^T$ is the $N_r \times 1$ receive vector, $\underline{\mathbf{s}} = [\mathbf{s}_1^T \ldots \mathbf{s}_K^T]^T$ is the $N_r \times 1$ data vector, $\underline{\mathbf{H}} = [\mathbf{H}_1^T \ldots \mathbf{H}_K^T]^T$ is the $N_r \times N_{CB}$ channel matrix, $\underline{\mathbf{F}} = [\mathbf{F}_1 \ldots \mathbf{F}_K]$ is the $N_{CB} \times N_r$ precoding matrix, and $\underline{\mathbf{n}} = [\mathbf{n}_1^T \ldots \mathbf{n}_K^T]^T$ is the $N_r \times 1$ noise vector. $\underline{\mathbf{Z}} = [\mathbf{Z}_1^T \ldots \mathbf{Z}_K^T]^T$ is the $N_r \times N_{PB}$ channel matrix of the PBS-CUs cross link. It is assumed that each data symbol has unit variance $\mathbb{E}\left\{\underline{\mathbf{s}}\ \underline{\mathbf{s}}^H\right\} = \mathbf{I}_{N_r}$, therefore the CBS transmit power fulfills $\mathbb{E}\left\{\|\underline{\mathbf{F}}\ \underline{\mathbf{s}}\|^2\right\} \leq P_T$.

In the PR network, the received signal at the PU, i.e. $\mathbf{y}_p \in \mathbb{C}^{N_p \times 1}$, can be written as

$$\mathbf{y}_p = \mathbf{T}\mathbf{F}_p\mathbf{s}_p + \mathbf{y}_{ni} \tag{2}$$

where $\mathbf{F}_p$ is a singular value decomposition (SVD) precoder, $\mathbf{P}_p$ is obtained by distributing the PBS power P_{PB} over the data streams (eigenmodes) of $\mathbf{T}$ via waterfilling such that $\mathrm{Tr}(\mathbf{P}_p) = \mathrm{Tr}(\mathbf{F}_p\mathbb{E}\left\{\mathbf{s}_p\mathbf{s}_p^H\right\}\mathbf{F}_p^H) \leq P_{PB}$ assuming that $\mathbb{E}\left\{\mathbf{s}_p\mathbf{s}_p^H\right\} = \mathbf{I}_{N_p}$. On one hand, the second term $\mathbf{y}_{ni} \in \mathbb{C}^{N_p \times 1}$ refers to the CBS interference plus noise induced at the PU, which is defined as $\mathbf{y}_{ni} = \mathbf{G}\underline{\mathbf{F}}\ \underline{\mathbf{s}} + \mathbf{n}_p$. On the other hand, the PBS interference plus the noise covariance matrix at the kth CU can be factorized as

$$(\mathbf{Z}_k\mathbf{P}_p\mathbf{Z}_k^H + \sigma_n^2\mathbf{I}_{N_c})^{-1} = \mathbf{\Gamma}_k\mathbf{\Gamma}_k^H \tag{3}$$

$\mathbf{\Gamma}_k \in \mathbb{C}^{N_c \times N_c}$ is defined as a receive whitening filter at the kth CU. The block of whitening filters can be written as $\underline{\tilde{\mathbf{W}}} = \mathrm{blockdiag}\,(\mathbf{\Gamma}_1 \ldots \mathbf{\Gamma}_K)$. Therefore, the whitened version of the entire received vector of all CUs defined in (1) can be written as

$$\underline{\tilde{\mathbf{y}}} = \underline{\tilde{\mathbf{H}}}\ \underline{\mathbf{F}}\ \underline{\mathbf{s}} + \underline{\tilde{\mathbf{n}}} \tag{4}$$

in the above equation $\underline{\tilde{\mathbf{H}}} = [\tilde{\mathbf{H}}_1^T\tilde{\mathbf{H}}_2^T \ldots \tilde{\mathbf{H}}_K^T]^T = \underline{\tilde{\mathbf{W}}}\ \underline{\mathbf{H}}$ and $\underline{\tilde{\mathbf{n}}} = [\tilde{\mathbf{n}}_1^T\tilde{\mathbf{n}}_2^T \ldots \tilde{\mathbf{n}}_K^T]^T = \underline{\tilde{\mathbf{W}}}(\underline{\mathbf{Z}}\mathbf{F}_p\mathbf{s}_p + \underline{\mathbf{n}})$. The whitened noise vector $\tilde{\mathbf{n}}$ is characterized as a zero-mean vector with identity covariance matrix. Thus, the whitened received vector of CU k is given by

$$\tilde{\mathbf{y}}_k = \tilde{\mathbf{H}}_k\mathbf{F}_k\mathbf{s}_k + \tilde{\mathbf{H}}_k\sum_{j=1, j\neq k}^{K}\mathbf{F}_j\mathbf{s}_j + \tilde{\mathbf{n}}_k \tag{5}$$

where $\tilde{\mathbf{H}}_k = \tilde{\mathbf{W}}_k\mathbf{H}_k$ and $\tilde{\mathbf{n}}_k = \tilde{\mathbf{W}}_k(\mathbf{Z}_k\mathbf{F}_p\mathbf{s}_p + \mathbf{n}_k)$. Therefore, the whitened noise vector $\tilde{\mathbf{n}}_k$ is characterized as a zero-mean vector with identity covariance matrix.

III. Proposed Adaptive Linear Preprocessing

Conventional linear preprocessing designs have substantially low complexity, therefore from the complexity perspective it is convenient to develop new linear preprocessing techniques in an adaptive manner utilizing the multiple antenna structure for enhancing spectrum efficiency. Particularly in the proposed adaptive linear preprocessing schemes, we map the antenna diversity to a precoding diversity with countable independent degrees of freedom (DoF). The precoding diversity produces a MUI diversity and thus improves the SNR of the CR system under investigation. The network adapts the precoding according to the maximum achievable SNR which fulfills the network constraints. Concerning the complexity, on one hand the comparable conventional precoding schemes, i.e. CR-ZF-BD and CR-MMSE-BD, only have one DoF compared to the proposed adaptive schemes which is an advantage in favor of computations reduction. On the other hand, advances of the parallel computing can handle the calculations of the independent precoding DoFs in the proposed adaptive methods efficiently. In the following, we present the details of the proposed adaptive linear precoders AZB and AMB. Without loss of generality, both MUI and PUI conditions in the adaptive linear preprocessing schemes are formed as: $\tilde{\mathbf{H}}_j\mathbf{F}_k \geq \mathbf{0}_{N_c \times N_c}$ and $\mathbf{G}\mathbf{F}_j \geq \mathbf{0}_{N_p \times N_c}, \forall j, j \neq k$, respectively. Therefore, the mechanism of the proposed approaches works on producing a precoding and a MUI diversity while satisfying the PUI condition $\sum_{k=1}^{K} \mathbb{E}\left\{\|\mathbf{G}\mathbf{F}_k\mathbf{s}_k\|^2\right\} \leq I_{th}$, and dissipating the entire transmit power P_T spectrally efficiently. Specifically, the key idea beyond the adaptive based preprocessing considers a part of complementary MIMO channel's paths (or subspaces) when computing a corresponding precoding DoF as will be described below. In other words, the design of the dth precoding DoF counts on one combination of complementary MIMO channel's paths (rows) or subspaces. Through scanning the entire combinations of paths (or subspaces), we can create a precoding set indexed for the kth CU as $\mathcal{D}_k = \{1, 2, \ldots, |\mathcal{D}_k|\}$. As we consider equal number of antennas per CU N_c, all CUs will have similar size precoding set, i.e. $|\mathcal{D}| = |\mathcal{D}_k|, \forall k$. From CUs' precoding sets, we combine the overall DoF set of network as K-tuples. To establish a low complexity solution, we combine the similar indices of the K precoding sets of the K CUs together, i.e. the K-tuple $\underline{\mathbf{F}}(d) \in \{\underline{\mathbf{F}}(1), \ldots, \underline{\mathbf{F}}(|\mathcal{D}|)\}$, where for instance the first K-tuple in the DoF set is defined as $\underline{\mathbf{F}}(1) = [\mathbf{F}_1(1)\mathbf{F}_2(1)\ldots\mathbf{F}_K(1)]$.

A. AZB Preprocessing

In this approach, we address the following sum-rate maximization problem

$$\begin{aligned} \max_{\{\underline{\mathbf{F}}(d), \forall d \in \mathcal{D}\}} \quad & \sum_{k=1}^{K} \log_2 |\mathbf{I}_{N_c} + \text{SINR}| \\ \text{subject to} \quad & \mathbb{E}\left\{\|\mathbf{G}\underline{\mathbf{F}}(d)\underline{\mathbf{s}}\|^2\right\} \leq I_{th} \\ & \mathbb{E}\left\{\|\underline{\mathbf{F}}(d)\underline{\mathbf{s}}\|^2\right\} \leq P_T \end{aligned} \tag{6}$$

where

$$\text{SINR} = \frac{\mathbf{W}_k(d)\mathbf{H}_k\mathbf{F}_k(d)\mathbf{F}_k^H(d)\mathbf{H}_k^H\mathbf{W}_k^H(d)}{\sigma_n^2\mathbf{I}_{N_c} + \mathbf{W}_k(d)\mathbf{H}_k\left(\sum\limits_{j\neq k}^{K}\mathbf{F}_j(d)\mathbf{F}_j^H(d)\right)\mathbf{H}_k^H\mathbf{W}_k^H(d)}$$

This problem is not convex as the MUI occurs in the denominator of each CU's SINR. To solve (6) in the context of the proposed AZB method, we should find the best K-tuple precoding $\underline{\mathbf{F}}(\hat{d})$ which maximizes the sum-rate.

To exploit the antenna diversity of CUs, we generate precoding diversity by two means: Subspace (orthogonal basis) combining and channel path combining. Note that the cardinality of $\mathcal{D}$ is a function of the number of rows of the complementary MIMO channel for the kth CU $\bar{\mathbf{H}}_k = [\mathbf{G}^T\tilde{\mathbf{H}}_1^T\ldots\tilde{\mathbf{H}}_{k-1}^T\tilde{\mathbf{H}}_{k+1}^T\ldots\tilde{\mathbf{H}}_K^T]^T$, i.e. $|\mathcal{D}| = f(\mathcal{R}_k)$, where $\mathcal{R}_k = N_p + N_r - N_c$. To this end, we consider the MUI seen by other $K-1$ CUs but not the intra-user interference seen by the kth CU itself. Note that the conventional CR-ZF-BD approach can be seen as one candidate DoF within the proposed AZB scheme and is the only case causing convexity. We will consider the waterfilling solution of this convex case $\mathbf{P}^{\text{ZB}}$ as a power allocation solution for the proposed AZB precoder. The details of both subspace and channel path combining methods follow.

1) *Subspace Combining (AZB-SC):*
In this approach, all paths of the complementary MIMO channel are considered in the preprocessing (rows of MIMO channel), however, the subspaces of the complementary MIMO channel are combined. Precisely, not only the null space of $\bar{\mathbf{H}}_k$ is taken into account, but also the spaces of the non-zero singular values. For the CU k, the cardinality of the precoding set in this method is $|\mathcal{D}| = \mathcal{R}_k + 1$ and the DoF set is indexed as $\mathcal{D} = \{0, 1, 2, \ldots, \mathcal{R}_k\}$. For instance, let the singular values of $\bar{\mathbf{H}}_k$ be ordered as $\bar{\lambda}_1 \geq \bar{\lambda}_2 \geq \cdots \geq \bar{\lambda}_{\mathcal{R}_k}$ and the null space is indexed as $d = 0$, therefore the index $d = 3$ refers to the spaces of the null as well as the smallest three non-zero singular values. For the CU k and the precoding index d, denote $\bar{\mathbf{H}}_k^{(d)} \in \mathbb{C}^{N_{CB}\times(N_c+d)}$ as the space of the null as well as the smallest d non-zero singular values in $\bar{\mathbf{H}}_k$. Then, apply the SVD as $\tilde{\mathbf{H}}_k\bar{\mathbf{H}}_k^{(d)} = \hat{\mathbf{U}}_k^{(d)}\hat{\mathbf{\Lambda}}_k^{(d)}\hat{\mathbf{V}}_k^{(d)H}$ to diagonalize the effective channel before applying power solution $\mathbf{P}_k^{\text{ZB}}$. The precoder and receive filter can be written as

$$\mathbf{F}_k^{\text{AZB1}}(d) = \bar{\mathbf{H}}_k^{(d)}\hat{\mathbf{V}}_k^{(d)}(\mathbf{P}_k^{\text{ZB}})^{\frac{1}{2}} \text{ and } \mathbf{W}_k^{\text{AZB1}}(d) = \hat{\mathbf{U}}_k^{(d)H}\mathbf{\Gamma}_k \tag{7}$$

Remark 1: For zero PUI constraint $I_{th} = 0$, we first define the null space of $\mathbf{G}$ denoted as $\mathbf{G}^{\perp}$ as follows:

$$\mathbf{G}^{\perp} = \mathbf{I}_{N_{CB}} - \mathbf{G}^H(\mathbf{G}\mathbf{G}^H)^{-1}\mathbf{G} \tag{8}$$

Then, we project the effective channel of the kth CU on $\mathbf{G}^{\perp}$ before we apply the SVD operation as $\tilde{\mathbf{H}}_k\mathbf{G}^{\perp}\bar{\mathbf{H}}_k^{(d)} = \hat{\mathbf{U}}_k^{(d)}\hat{\mathbf{\Lambda}}_k^{(d)}\hat{\mathbf{V}}_k^{(d)H}$ for the diagonalization and power allocation steps. The precoder becomes $\mathbf{F}_k^{\text{AZB1}}(d) = \mathbf{G}^{\perp}\bar{\mathbf{H}}_k^{(d)}\hat{\mathbf{V}}_k^{(d)}(\mathbf{P}_k^{\text{ZB}})^{\frac{1}{2}}$.

2) *Channel Path Combining (AZB-CPC):*
In this method, we define the complementary channel model of the CU k, excluding the kth CU's channel model $\tilde{\mathbf{y}}_k = \tilde{\mathbf{H}}_k\mathbf{s}_k + \tilde{\mathbf{n}}_k$ as

$$\underline{\tilde{\mathbf{y}}}_k^{\text{AZB2}}(d) = \underline{\tilde{\mathbf{H}}}_k(d)\ \underline{\mathbf{F}}_k(d)\ \underline{\mathbf{s}}_k + \underline{\tilde{\mathbf{n}}}_k(d) \tag{9}$$

where $\underline{\tilde{\mathbf{y}}}_k(d) = [\mathbf{y}_{ni}^T\tilde{\mathbf{y}}_1^T..\tilde{\mathbf{y}}_{k-1}^T\tilde{\mathbf{y}}_{k+1}^T..\tilde{\mathbf{y}}_K^T]^T$, $\underline{\mathbf{F}}_k(d) = [\mathbf{F}_1(d)..\mathbf{F}_{k-1}(d)\mathbf{F}_{k+1}(d)..\mathbf{F}_K(d)]$, $\underline{\mathbf{s}}_k = [\mathbf{s}_1^T..\mathbf{s}_{k-1}^T\mathbf{s}_{k+1}^T..\mathbf{s}_K^T]^T$, $\underline{\tilde{\mathbf{n}}}_k(d) = [\mathbf{n}_p^T\tilde{\mathbf{n}}_1^T..\tilde{\mathbf{n}}_{k-1}^T\tilde{\mathbf{n}}_{k+1}^T..\tilde{\mathbf{n}}_K^T]^T$, and

$$\begin{aligned} \underline{\tilde{\mathbf{H}}}_k(d) &= \mathbf{M}_k^{(d)} \circ \bar{\mathbf{H}}_k \\ &= \mathbf{M}_k^{(d)} \circ [\mathbf{G}^T\tilde{\mathbf{H}}_1^T..\tilde{\mathbf{H}}_{k-1}^T\tilde{\mathbf{H}}_{k+1}^T..\tilde{\mathbf{H}}_K^T]^T \end{aligned} \tag{10}$$

given that $\circ$ is the Hadamard "element-wise product" operator and $\mathbf{M}_k^{(d)} \in \mathbb{R}^{\mathcal{R}_k\times N_{CB}}$ is the so-called inclusion matrix which corresponds to the precoding index d of the CU k. The cardinality of the precoding set is $|\mathcal{D}| = \sum_{j=1}^{\mathcal{R}_k} {}^{\mathcal{R}_k}C_j = 2^{\mathcal{R}_k} - 1$ per CU, where ${}^{m}C_n$ refers to the number of possible n-tuples combined from m entities. The entries of each column in $\mathbf{M}_k^{(d)}$ are expressed as the binary conversion of the decimal index d as shown in TABLE I in which $\mathbf{0}^T$ and $\mathbf{1}^T$ are $1 \times N_{CB}$ all-zero and all-one row vectors, respectively. For instance, for $\mathcal{R}_k = 8$ and precoding index $d = 3$, the

TABLE I
ENTRIES OF $\mathbf{M}_k^{(d)}, \forall d \in \mathcal{D}$ OF AZB-CPC

Row index of $\mathbf{M}_k^{(d)}\downarrow$ $\mathbf{M}_k^{(d)}\rightarrow$	$\mathbf{M}_k^{(1)}$	$\mathbf{M}_k^{(2)}$	$\mathbf{M}_k^{(3)}$	..	$\mathbf{M}_k^{(\lvert\mathcal{D}\rvert)}$
1	$\mathbf{0}^T$	$\mathbf{0}^T$	$\mathbf{0}^T$		$\mathbf{1}^T$
$\vdots$	$\vdots$	$\vdots$	$\mathbf{0}^T$	..	$\vdots$
	$\mathbf{0}^T$	$\mathbf{1}^T$	$\mathbf{1}^T$		
$\mathcal{R}_k$	$\mathbf{1}^T$	$\mathbf{0}^T$	$\mathbf{1}^T$		$\mathbf{1}^T$

inclusion matrix $\mathbf{M}_k^{(3)}$ has N_{CB} equal column vectors each written as $[0\,0\,0\,0\,0\,0\,1\,1]^T$, where $\mathcal{R}_k$ determines the number of binary digits of the vector and d represents the decimal value of the binary column vector. For the CU k and index d, denote $\underline{\tilde{\mathbf{H}}}_k^{(0)}(d)$ as the null space of $\underline{\tilde{\mathbf{H}}}_k(d)$, then apply the SVD to the effective channel as $\tilde{\mathbf{H}}_k\underline{\tilde{\mathbf{H}}}_k^{(0)}(d) = \check{\mathbf{U}}_k^{(d)}\check{\mathbf{\Lambda}}_k^{(d)}\check{\mathbf{V}}_k^{(d)H}$ for the diagonalization purpose before the waterfilling power allocation solution $\mathbf{P}_k^{\text{ZB}}$. Thus, the ultimate form of the precoder and receive filter of the kth CU can be written as

$$\mathbf{F}_k^{\text{AZB2}}(d)=\underline{\tilde{\mathbf{H}}}_k^{(0)}(d)\hat{\mathbf{V}}_k^{(d)}(\mathbf{P}_k^{\text{ZB}})^{\frac{1}{2}} \text{ and } \mathbf{W}_k^{\text{AZB2}}(d)=\hat{\mathbf{U}}_k^{(d)H}\mathbf{\Gamma}_k \tag{11}$$

Remark 2: For zero PUI $I_{th} = 0$, similar to *Remark 1* the effective channel of the kth CU should be projected on $\mathbf{G}^\perp$ defined in (8) before applying the SVD operation as $\tilde{\mathbf{H}}_k\mathbf{G}^\perp\underline{\tilde{\mathbf{H}}}_k^{(0)}(d) = \hat{\mathbf{U}}_k^{(d)}\hat{\mathbf{\Lambda}}_k^{(d)}\hat{\mathbf{V}}_k^{(d)H}$. The precoder becomes $\mathbf{F}_k^{\text{AZB2}}(d)=\mathbf{G}^\perp\underline{\tilde{\mathbf{H}}}_k^{(0)}(d)\hat{\mathbf{V}}_k^{(d)}(\mathbf{P}_k^{\text{ZB}})^{\frac{1}{2}}$.

Therefore, the optimal K-tuple $\left\{\underline{\mathbf{F}}^{\text{AZB}}(\hat{d}), \underline{\mathbf{W}}^{\text{AZB}}(\hat{d})\right\}, \hat{d} \in \mathcal{D}$, should achieve the maximum sum-rate while satisfying the PUI condition.

B. AMB Preprocessing

In this section, we develop the conventional MMSE based precoding scheme into the proposed AMB precoding. As mentioned earlier, the precoding diversity enhances the network performance as it causes a notable improvement of the downlink SNR. Based on the MMSE criterion, the MMSE based precoder regularizes the channel inversion via a regularization factor containing the transmit power and noise covariance. We address the following MMSE optimization problem:

$$\begin{aligned} \min_{\gamma, \underline{\mathbf{F}}(d), d\in\mathcal{D}} \quad & \mathbb{E}\left\{\|\underline{\mathbf{s}} - \gamma^{-1}\underline{\tilde{\mathbf{y}}}\|^2\right\} \\ \text{subject to} \quad & \mathbb{E}\left\{\|\mathbf{G}\underline{\mathbf{F}}(d)\underline{\mathbf{s}}\|^2\right\} \leq I_{th} \\ & \mathbb{E}\left\{\|\underline{\mathbf{F}}(d)\underline{\mathbf{s}}\|^2\right\} \leq P_T \end{aligned} \tag{12}$$

where γ is a scaling factor for the received signal. In the proposed AMB approach, we develop a precoder to suppress the MUI and to fulfill both transmit power as well as PUI constraints. To exploit the multiple antenna structure of CUs, a DoF set with a cardinality $|\mathcal{D}|$ collecting all path combinations is computed as mentioned. Then, we find the optimal precoding K-tuple according to the following criterion:

$$\underline{\mathbf{F}}(\hat{d}) = \arg\max_{d\in\mathcal{D}} \sum_{k=1}^{K} r_k\left(\underline{\mathbf{F}}(d)\right) \tag{13}$$

where $r_k\left(\underline{\mathbf{F}}(d)\right)$ is the data rate of the kth CU given as a function of the dth K-tuple $\underline{\mathbf{F}}(d)$, $d \in \mathcal{D}$. Therefore, we first solve (12) for all DoFs, i.e. $\forall d \in \mathcal{D}$, then apply (13) to find the optimal network precoding $\underline{\mathbf{F}}(\hat{d})$. Towards this goal, we define the network channel model of the dth DoF as

$$\underline{\tilde{\mathbf{y}}}^{\text{AMB}}(d) = \underline{\tilde{\mathbf{H}}}(d)\ \underline{\mathbf{F}}(d)\ \underline{\mathbf{s}} + \underline{\tilde{\mathbf{n}}} \tag{14}$$

where $\underline{\tilde{\mathbf{y}}}^{\text{AMB}}(d) = [\tilde{\mathbf{y}}_1(d)^T\,\tilde{\mathbf{y}}_2(d)^T \ldots \tilde{\mathbf{y}}_K(d)^T]^T$, $\underline{\mathbf{F}}(d) = [\mathbf{F}_1(d)\,\mathbf{F}_2(d)\ldots\mathbf{F}_K(d)]$, and

$$\underline{\tilde{\mathbf{H}}}(d) = \mathbf{M}^{(d)} \circ \underline{\tilde{\mathbf{H}}} \tag{15}$$

where $\circ$ is the Hadamard operator and $\mathbf{M}^{(d)} \in \mathbb{R}^{N_r \times N_{CB}}$ is the inclusion matrix of the dth DoF for which the entries are tabulated in TABLE II. Here, we point out that the precoding set has a cardinality of $|\mathcal{D}| = \sum_{j=1}^{N_r} {}^{N_r}C_j = 2^{N_r} - 1$ DoFs. Each j-tuple corresponds to one path combination produced by (15). To this end, we apply the channel model defined in (15) to the MMSE network precoding resulted from solving (12) as described in [5]. The MMSE based precoding is given by

TABLE II
ENTRIES OF $\mathbf{M}_k^{(d)}$ OF AMB, $\forall d \in \mathcal{D}$

Row index of $\mathbf{M}^{(d)}\downarrow$ $\mathbf{M}^{(d)}\rightarrow$	$\mathbf{M}^{(1)}$	$\mathbf{M}^{(2)}$	$\mathbf{M}^{(3)}$	..	$\mathbf{M}^{(\lvert\mathcal{D}\rvert)}$
1	$\mathbf{0}^T$	$\mathbf{0}^T$	$\mathbf{0}^T$		$\mathbf{1}^T$
$\vdots$	$\vdots$	$\vdots$	$\mathbf{0}^T$	..	$\vdots$
	$\mathbf{0}^T$	$\mathbf{1}^T$	$\mathbf{1}^T$		
N_r	$\mathbf{1}^T$	$\mathbf{0}^T$	$\mathbf{1}^T$		$\mathbf{1}^T$

$$\begin{aligned} \underline{\hat{\mathbf{F}}}(d) &= \gamma\,\underline{\bar{\mathbf{F}}}(\mu, d) \\ &= \gamma\left(\underline{\tilde{\mathbf{H}}}(d)^H\underline{\tilde{\mathbf{H}}}(d) + \mu\mathbf{G}^H\mathbf{G} + \frac{N_r - \mu I_{th}}{P_T}\mathbf{I}_{N_{CB}}\right)^{-1}\underline{\tilde{\mathbf{H}}}(d)^H \end{aligned} \tag{16}$$

where $\gamma = \sqrt{P_T/\text{Tr}\left(\underline{\bar{\mathbf{F}}}(\mu,d)\underline{\bar{\mathbf{F}}}(\mu,d)^H\right)}$ and μ is a positive parameter that lies in the interval $0 \leq \mu \leq N_r/I_{th}$ in order to regulate the precoding matrix such that the entire transmit power P_T is dissipated while fulfilling the PUI constraint. μ can be found numerically by the bisection method [10].

Remark 3: For zero PUI constraint $I_{th} = 0$, the regularized channel inversion in (16) takes another form considering the null space of $\mathbf{G}$ defined in (8) as

$$\underline{\hat{\mathbf{F}}}(d) = \gamma\,\mathbf{G}^\perp\underline{\tilde{\mathbf{H}}}(d)^H\left(\underline{\tilde{\mathbf{H}}}(d)\mathbf{G}^\perp\underline{\tilde{\mathbf{H}}}(d)^H + \frac{N_r}{P_T}\mathbf{I}_{N_r}\right)^{-1} \tag{17}$$

Then, the space of the orthonormal bases of the kth CU which are spanned by the column vectors of the corresponding projection matrix $\hat{\mathbf{F}}_k(d)$ can be obtained by the QR decomposition as follows:

$$\hat{\mathbf{F}}_k(d) = \mathbf{Q}_k(d)\mathbf{R}_k(d), \forall k \tag{18}$$

where $\mathbf{Q}_k(d) \in \mathbb{C}^{N_{CB} \times N_c}$ contains N_c-dimensional columns of orthonormal bases. Then, we apply the power allocation counting on the MMSE combining matrix $\mathbf{P}_k^{\text{AMB}}$ for the kth CU which minimizes the sum MSE subject to a transmit power constraint as shown in [7]. Mathematically, $\mathbf{P}_k^{\text{AMB}}(d)$ is computed as $\mathbf{P}_k^{\text{AMB}}(d) = \beta(d)\bar{\mathbf{P}}_k^{\text{AMB}}(d)$, where β normalizes the sum power and defined as $\beta(d) = \sqrt{P_T / \Sigma_{k=1}^K \text{Tr}\left(\bar{\mathbf{P}}_k^{\text{AMB}}(d)^H \bar{\mathbf{P}}_k^{\text{AMB}}(d)\right)}$ and

$$\bar{\mathbf{P}}_k^{\text{AMB}}(d) = \left(\mathbf{Q}_k(d)^H \Sigma_{j=1}^K \tilde{\mathbf{H}}_j^H \tilde{\mathbf{H}}_j \mathbf{Q}_k(d) + \frac{N_r}{P_T}\mathbf{I}_{N_c}\right)^{-1} \mathbf{Q}_k(d)^H \tilde{\mathbf{H}}_k^H \tilde{\mathbf{H}}_k \mathbf{Q}_k(d) \tag{19}$$

To this end, applying the SVD decouples the effective channel into parallel subchannels for carrying parallel data streams as $\tilde{\mathbf{H}}_k \mathbf{Q}_k(d) \mathbf{P}_k^{\text{AMB}}(d) = \tilde{\mathbf{U}}_k(d) \tilde{\mathbf{\Lambda}}_k(d) \tilde{\mathbf{V}}_k(d)^H$. Ultimately, the precoder and receive filter of the proposed AMB technique are expressed as

$$\mathbf{F}_k^{\text{AMB}} = \mathbf{Q}_k(d)\mathbf{P}_k^{\text{AMB}}(d)\tilde{\mathbf{V}}_k(d) \text{ and } \mathbf{W}_k^{\text{AMB}} = \tilde{\mathbf{U}}_k(d)^H \mathbf{\Gamma}_k \tag{20}$$

IV. Performance Analysis

In this section, we carry out an analysis of the performance of the proposed adaptive precoding approaches. We consider an analysis in terms of the sum-rate and computational complexity.

A. Achievable Sum-Rate Analysis

Note that the AMB precoding regularizes the inversion by a regularization factor, which is inversely proportional to the SNR operating point of the downlink. Therefore, in the high SNR regime the regularization factor approaches zero, and consequently the MMSE based precoding method converges to the ZF based precoding and therefore exhibits a similar sum-rate [5], [7]. However, in the low SNR regime, we expect an achievable sum-rate in favor of AMB precoding outperforming the AZB solution due to the fact that the regularization factor mitigates the degradation caused by the noise term. Unlike fixed precoding based conventional methods, i.e. CR-ZF-BD and CR-MMSE-BD, the proposed adaptive MMSE based precoding scheme calculates a set of precoding matrices (precoding diversity) for the kth CU and therefore exhibits a MUI diversity at each CU. Intuitively speaking, precoding diversity likely improves the SNR and the spectral efficiency gains of the CUs. Furthermore, we emphasize that those gains grow proportionally in terms of the number of antennas per CU because of the antenna coordination at the receiver. To calculate the maximum SNR and achievable sum-rate of the proposed schemes, we count on the output signal of the kth CU receive filter considering the optimal pair $\mathbf{F}_k(\hat{d}), \mathbf{W}_k(\hat{d})$ obtained from (13) as

$$\tilde{\mathbf{y}}_k = \mathbf{W}_k(\hat{d})\mathbf{H}_k\mathbf{F}_k(\hat{d})\mathbf{s}_k + \mathbf{W}_k(d)\mathbf{H}_k \sum_{j=1, j\neq k}^{K} \mathbf{F}_j(\hat{d})\mathbf{s}_j + \mathbf{W}_k(\hat{d})\mathbf{n}_k \tag{21}$$

By substituting (7), (11), and (20) in (21), we get

$$\tilde{\mathbf{y}}_k^{\text{AZB}} = \hat{\mathbf{\Lambda}}_k(\hat{d})(\mathbf{P}_k^{\text{ZB}})^{\frac{1}{2}}\mathbf{s}_k + \hat{\mathbf{U}}_k(\hat{d})^H \tilde{\mathbf{H}}_k \sum_{j=1, j\neq k}^{K} \mathbf{F}_j(\hat{d})\mathbf{s}_j + \hat{\mathbf{U}}_k(\hat{d})^H \tilde{\mathbf{n}}_k \tag{22}$$

$$\tilde{\mathbf{y}}_k^{\text{AMB}} = \tilde{\mathbf{\Lambda}}_k(\hat{d})\mathbf{s}_k + \tilde{\mathbf{U}}_k(\hat{d})^H \tilde{\mathbf{H}}_k \sum_{j=1, j\neq k}^{K} \mathbf{F}_j(\hat{d})\mathbf{s}_j + \tilde{\mathbf{U}}_k(\hat{d})^H \tilde{\mathbf{n}}_k \tag{23}$$

It is worth noting that the statistical characteristics of $\tilde{\mathbf{n}}_k$ do not change when multiplied by a unitary matrix $\mathbf{U}_k^H$. Therefore, the maximum achievable sum-rate of the proposed AZB and AMB precoding schemes can be expressed, respectively, as follows:

$$R_{\text{AZB}} = \sum_{k=1}^{K} \sum_{i=1}^{N_c} \log_2 \left(1 + \frac{\hat{\lambda}_{k,i}(\hat{d})^2 p_{k,i}^{\text{ZB}}}{1 + \sum_{j=1, j\neq k}^{K} \left\| \mathbf{w}_{k,i}(\hat{d}) \mathbf{H}_k \mathbf{F}_j(\hat{d}) \right\|^2}\right) \tag{24}$$

$$R_{\text{AMB}} = \sum_{k=1}^{K} \sum_{i=1}^{N_c} \log_2 \left(1 + \frac{\tilde{\lambda}_{k,i}^2(\hat{d})}{1 + \sum_{j=1, j\neq k}^{K} \left\| \mathbf{w}_{k,i}(\hat{d}) \mathbf{H}_k \mathbf{F}_j(\hat{d}) \right\|^2}\right) \tag{25}$$

where $\hat{\lambda}_{k,i}(\hat{d})$, $\tilde{\lambda}_{k,i}(\hat{d})$, and $p_{k,i}^{\text{ZB}}$ refer to the ith diagonal element of $\hat{\mathbf{\Lambda}}_k(\hat{d})$, $\tilde{\mathbf{\Lambda}}_k(\hat{d})$, and $\mathbf{P}_k^{\text{ZB}}$, respectively. $\mathbf{w}_{k,i}(\hat{d})$ is the ith row vector of $\mathbf{W}_k(\hat{d})$.

B. Power Allocation of the AZB Precoder

In this section, we illustrate the derivation of the waterfilling power $\mathbf{P}_k^{\text{ZB}}, \forall k$ employed in the proposed AZB precoder. Note that the objective function to be optimized is the sum-rate subjected to a single constraint: The power budget. First, the Lagrangian can be written as

$$\mathcal{L}_{\text{ZB}}(\{\mathbf{P}_k\}, \mu) = \sum_{k=1}^{K} \log_2 \left| \mathbf{I}_{N_c} + \hat{\mathbf{\Lambda}}_k^2 \mathbf{P}_k \right| - \mu \left(\sum_{k=1}^{K} \text{Tr}\left(\mathbf{P}_k\right) - P_T \right) \tag{26}$$

where μ is a Lagrange multiplier associated with the power constraint and both $\hat{\mathbf{\Lambda}}_k$ as well as $\mathbf{P}_k$ are diagonal matrices. Note that the MUI vanishes in the ZF-BD technique. Then we obtain the optimal power by applying the Karush-Kuhn-Tucker (KKT) conditions [10]. The gradient $\frac{\partial \mathcal{L}_{\text{ZB}}}{\partial \mathbf{P}_k} = 0$ can be easily derived from which the optimal power is written as

$$\mathbf{P}_k^{\text{ZB}} = \left[\left\{\ln 2\left(\mu \mathbf{I}_{N_c}\right)\right\}^{-1} - \hat{\mathbf{\Lambda}}_k^{-2}\right]^+, \forall k \tag{27}$$

The power is obtained after n iterations for the dual variable μ using a numerical method like the bisection method [10].

C. Computational Complexity Analysis

In this section, we present an analysis of the computational complexity of the proposed method. Relying on the floating point operations (flops) stated in [11], the flops of the required matrix operations are described as follows:

- Multiplication of $m \times n$ by $n \times p$ complex matrices: $8mnp - 2mp$
- QR decomposition of an $m \times n (m \leq n)$ complex matrix: $16(n^2 m - nm^2 + \frac{1}{3}m^3)$
- SVD of an $m \times n (m \leq n)$ complex matrix where only $\mathbf{\Lambda}$ and $\mathbf{V}$ are obtained: $32(nm^2 + 2m^3)$
- SVD of an $m \times n (m \leq n)$ complex matrix where $\mathbf{U}, \mathbf{\Lambda}$, and $\mathbf{V}$ are obtained: $8(4n^2 m + 8nm^2 + 9m^3)$
- Inversion of an $m \times m$ matrix using Gauss-Jordan elimination: $\frac{4}{3}m^3$
- Hadamard product of $m \times n$ real and $m \times n$ complex matrices: mn

We illustrate the required flops for the proposed adaptive linear preprocessing techniques AZB-SC, AZB-CPC, and AMB in TABLE III, TABLE IV, and TABLE V, respectively.

TABLE III
COMPUTATIONAL COMPLEXITY OF AZB-SC

Steps	Operations	Flops $\times(\lvert\mathcal{D}\rvert = \mathcal{R}_k + 1)$
1	SVD of $\bar{\mathbf{H}}_k$	$32K(N_{CB}\mathcal{R}_k^2 + 2\mathcal{R}_k^3)/\lvert\mathcal{D}\rvert$
2	$\tilde{\mathbf{H}}_k \bar{\mathbf{H}}_k^{(d)}$	$K(8N_c^2 N_{CB} + 8N_c N_{CB} d - 2N_c^2 - 2N_c d)$
3	SVD of $\tilde{\mathbf{H}}_k \bar{\mathbf{H}}_k^{(d)}$	$8K(18N_c^3 + 10N_c^2 d + N_c d^2)$
4	Calculation of $\mathbf{P}_k^{\text{ZB}}$	$6KnN_c/\lvert\mathcal{D}\rvert$

TABLE IV
COMPUTATIONAL COMPLEXITY OF AZB-CPC

Steps	Operations	Flops $\times(\lvert\mathcal{D}\rvert = 2^{\mathcal{R}_k} - 1)$
1	$\mathbf{M}_k^{(d)} \circ \bar{\mathbf{H}}_k$	$K\mathcal{R}_k N_{CB}$
2	SVD of $\tilde{\mathbf{H}}_k(d)$ in (9)	$32K(N_{CB}\mathcal{R}_k^2 + 2\mathcal{R}_k^3)$
3	$\tilde{\mathbf{H}}_k \tilde{\mathbf{H}}_k^{(0)}(d)$	$K(8N_c^2 N_{CB} - 2N_c^2)$
4	SVD of $\tilde{\mathbf{H}}_k \tilde{\mathbf{H}}_k^{(0)}(d)$	$8K(21N_c^3)$
5	Calculation of $\mathbf{P}_k^{\text{ZB}}$	$6KnN_c/\lvert\mathcal{D}\rvert$

TABLE V
COMPUTATIONAL COMPLEXITY OF AMB

Operations	Flops $\times(\lvert\mathcal{D}\rvert = 2^{N_r} - 1)$
$\mathbf{M}^{(d)} \circ \underline{\tilde{\mathbf{H}}}$	$N_r N_{CB}$
$\underline{\hat{\mathbf{F}}}(d)$ in (15)	$\frac{4}{3}N_{CB}^3 + N_{CB}(2 - 2N_r) + N_{CB}^2(24N_r + 8N_p - 6)$
QR of $\hat{\mathbf{F}}_k(d)$	$16K(N_{CB}^2 N_c - N_c^2 N_{CB} + \frac{1}{3}N_c^3)$
Calculation of $\mathbf{P}_k^{\text{AMB}}(d)$	$K\{N_c^3(\frac{28}{3} + 8) + N_c^2(16N_{CB} - 7) + N_c(16N_{CB}^2 - 4N_{CB} + 1 + 8N_{CB}^2/\lvert\mathcal{D}\rvert) - 2N_{CB}^2/\lvert\mathcal{D}\rvert\}$
$\tilde{\mathbf{H}}_k \mathbf{Q}_k(d) \mathbf{P}_k^{\text{AMB}}(d)$	$8KN_c^3 + KN_c^2(8N_{CB} - 4)$
SVD of $\tilde{\mathbf{H}}_k \mathbf{Q}_k(d) \mathbf{P}_k^{\text{AMB}}(d)$	$8K(21N_c^3)$

Note that the computational complexity primarily counts on the system antenna configuration. For instance the CR configurations 8×(3,3) and 8×(2,2,2) for a given PR configuration 2×2, although they have a similar N_r, present different complexity and performance due to the contrast in K and $\mathcal{R}_k$. Specifically, the configuration considering large N_c and little K is preferred over the opposite one since it provides less complexity as well as better sum-rate per CU due to the antenna coordination at the receiver.

V. Simulation Results

A. Channel Model and System Configuration

In the simulations, we conduct experiments for a multi-user MIMO CR based broadcast channel model suffering from frequency-selective fading. The path loss, shadowing, and Rayleigh-distributed multipath fading are combined in the following model [12]:

$$\frac{\mathrm{P}_r}{\mathrm{P}_t} = \mathrm{Z}_{PL}\ \mathrm{Z}_S\ \mathrm{Z}_M = \mathrm{Z}_0 \left(\frac{d_0}{d}\right)^{\gamma}\ \mathrm{Z}_S\ \mathrm{Z}_m^2 = \lvert\text{gain}\rvert^2$$

where $\mathrm{Z}_0 = \left(\frac{\lambda}{4\pi d_0}\right)^2$, γ is the path loss (PL) exponent, $\frac{d_0}{d}$ is the reference-to-destination distance ratio. The shadowing component is modeled as $\mathrm{Z}_S = 10^{\mathrm{Z}_{S(\text{dB})}/10}$, where $\mathrm{Z}_{S(\text{dB})} \sim \mathcal{N}(0, \sigma_{S(\text{dB})}^2)$. The Rayleigh multipath component is modeled as $\mathrm{Z}_m = \left|\mathrm{X}_I + \mathrm{j}\ \mathrm{X}_Q\right|$, where X_I and $\mathrm{X}_Q \sim \mathcal{N}(0, \sigma_m^2)$. The elements of the channels of the cognitive links $\mathbf{H}_k, \forall k$ and the interference link $\mathbf{G}$ are generated as i.i.d random variables as described in the above channel model. The channel parameters are configured as follows. We assume a normalized free space loss, i.e. $\mathrm{Z}_0 = 1$, $\gamma = 2$, shadowing variance $\sigma_{S(dB)}^2 = 0$, and Rayleigh multipath fading variance $\sigma_m^2 = 1$.

The SNR of the PU link is defined as $\text{SNR}_{PR} = P_{PB}/\sigma_n^2$, and the CUs link as $\text{SNR}_{CR} = P_T/\sigma_n^2$ with noise variance $\sigma_n^2 = 1$.

The CR link is configured as $14 \times (4{,}4{,}4)$, i.e. three CUs with $N_c = 4$, and the PR link as 2×2 unless otherwise stated.

B. Example 1: Comparison of the Achievable Sum-rates

In this section, simulations are conducted for six different precoders including the proposed techniques as illustrated in Fig. 2. Generally speaking, the proposed adaptive precoders outperform the conventional precoders in the low and moderate SNR regions which are of practical importance. Also, it exhibits the superiority of the AMB precoder with achievable spectral efficiency gain of 5 bps/Hz at $\text{SNR}_{CR} = 5$ dB and a SNR gain about 3 dB at a spectral efficiency of 10 bps/Hz. Furthermore, it shows that employing a larger number of antenna per CU improves the sum-rate as a consequence for the antenna coordination at the receiver. From another perspective, setting more antennas per CU causes a bigger capacity gap between the proposed AMB precoder and the conventional schemes and, however, smaller gap towards the AZB precoder curves.

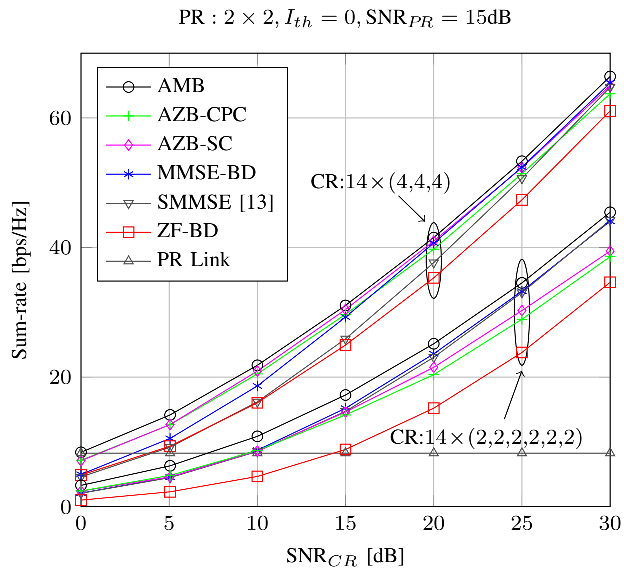

Fig. 2. Sum-rate versus SNR$_{CR}$ for two antenna configurations.

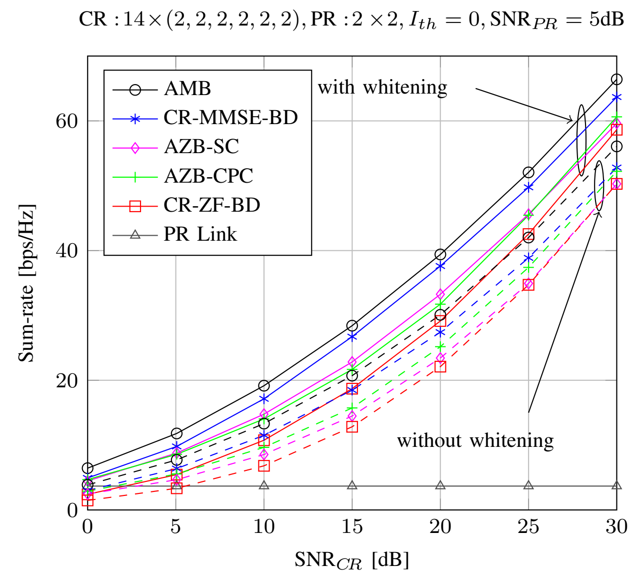

Fig. 3. Effect of the whitening process on sum-rate

C. Example 2: Effect of the Whitening Process and PR SNR on the CR Sum-rate

In this example, an illustration is presented about the effect of the whitening process on the sum-rate of a CR link configured as 14×(2,2,2,2,2,2) at the operating point SNR$_{PR}$ = 5 dB. As mentioned in the system model, the whitening process reduces the effect of the interference produced by the PBS and received by the CUs enhancing the CR sum-rate. Fig. 3 verifies this fact for which the whitening process takes the advantage of the PBS interference to improve the SNR. Note that a small SNR$_{PR}$ produces small PR sum-rate and little PBS interference causing little improvement for the whitened CR sum-rate over the non-whitened. Non-whitening can be implemented by setting $\mathbf{\Gamma}_k, \forall k$ as identity matrix. Moreover, it is worth to note that the proposed AMB precoder can achieve a spectral efficiency gain up to 3 bps/Hz in the high SNR region when the number of the antennas per CU decreases whether with or without whitening process.

D. DoF and Complexity Analysis

In this example, we present an illustration on the precoding DoF diversity and its influence on the capacity and complexity of the proposed AMB precoder. Concerning the capacity aspect, Fig. 4 exhibits an instantaneous CR sum-rate conducted at SNR$_{CR}$ = 25 dB in terms for the dth index of DoF, $\forall d \in \mathcal{D}$. It illustrates that the DoF diversity causes MUI and PUI diversities. Furthermore, it demonstrates that the optimal DoF selected for the AMB precoder meets the PUI constraint while achieving a spectral efficiency gain.

Regarding to the complexity aspect, the antenna configuration and number of CUs K to be served both affect the number of the required flops. Fig. 5 and Fig. 6 exhibits the number of flops as a function of K and N_c, respectively. It shows that the proposed AZB-SC scheme is the least

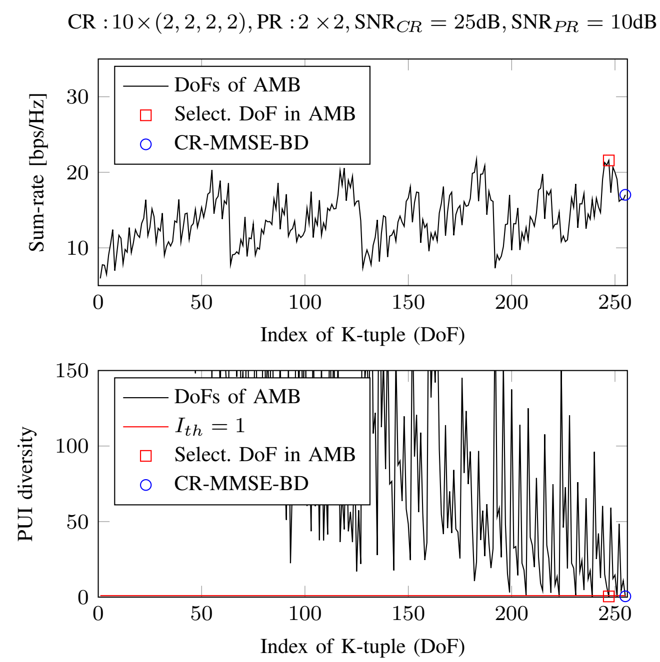

Fig. 4. Effect of the DoF diversity on the sum-rate

complex as it has linear complexity, but both AMB and AZB-CPC schemes are expensive computationally as they have exponential complexities. It is also obvious that the growth of N_c increases the complexities more than the growth of K.

VI. Conclusion

In this paper, we have developed a non-iterative linear adaptive MMSE based precoder dubbed “AMB” suitable for multiuser MIMO CR based broadcasting. The proposed AMB precoder employs a new concept called precoding diversity

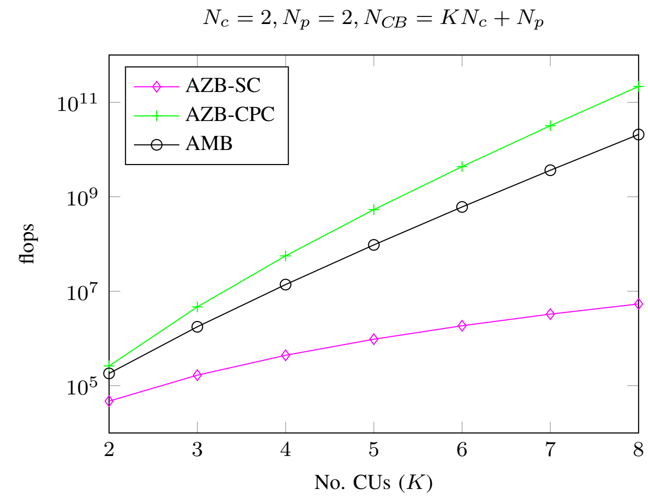

Fig. 5. Effect of the DoF diversity on the complexity for fixed N_c

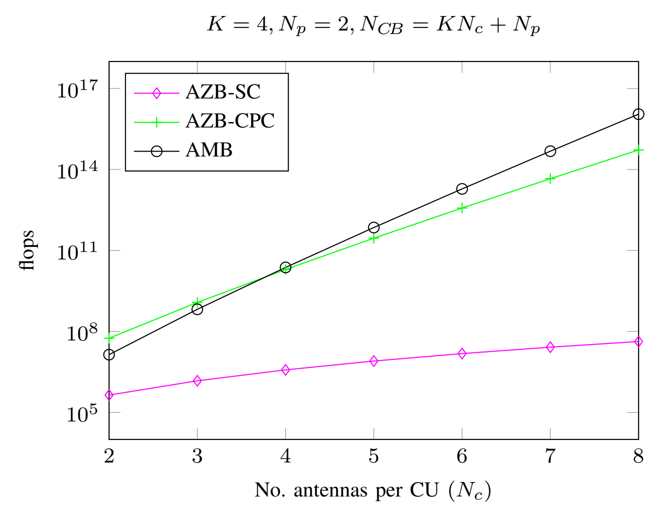

Fig. 6. Effect of the DoF diversity on the complexity for fixed K

in which the multiple antenna structure is utilized more efficient compared to conventional precoders. We have also extended the conventional ZF based precoding approach into an adaptive precoding dubbed "AZB" exploiting precoding diversity. Roughly speaking, the proposed AMB precoder resolves notable spectral and SNR gains over the state-of-the-art and the AZB precoder in the SNR region of interest: The low SNR region. In the high SNR region, the AMB precoder is characterized by an increment of spectral efficiency gain when the number of antennas per CU decreases. Unlike non-linear iterative precoders, the proposed precoders are linear non-iterative and therefore provide less complexity along with a spectral efficiency achievement. The degrees of freedom of the proposed AMB precoder can be handled independently by parallel computing. The characteristics of our proposed precoder make it a candidate for future mobile CR networks.

References

[1] J. Mitola, "Cognitive radio: An integrated agent architecture for software defined radio," *PhD dissertation*, KTH, Stockholm, Sweden, Dec. 2000.

[2] S. Haykin, "Cognitive radio: Brain-empowered wireless communications," *IEEE J. Sel. Areas Commun.*, vol. 23, no. 2, pp. 201-220, Feb. 2005.

[3] L. Zhang, Y. Xin, and Y.-C. Liang, "Weighted sum rate optimization for cognitive radio MIMO broadcast channels," *IEEE Trans. Wireless Commun.*, vol. 8, no. 6, pp. 2950-2959, June 2009.

[4] A. Yaqot and P. A. Hoeher, "Efficient resource allocation for MIMO-OFDM cognitive networks with adaptive precoding," in *Proc. 18th Int. OFDM Workshop*, Aug. 2014, pp. 85-91.

[5] K.-J. Lee and I. Lee, "MMSE based block diagonalization for cognitive radio MIMO broadcast channels," *IEEE Trans. Wireless Commun.*, vol. 10, no. 10, pp. 3139-3144, Oct. 2011.

[6] Q. H. Spencer, A. L. Swindelhurst and M. Haardt, "Zero-forcing methods for downlink spatial multiplexing in multiuser MIMO channels," *IEEE Trans. Signal Process.*, vol. 52, no. 2, pp. 461-471, Feb. 2004.

[7] H. Sung, S. Lee, and I. Lee, "Generalized channel inversion methods for multiuser MIMO systems," *IEEE Trans. Commun.*, vol. 57, no. 11, pp. 3489-3499, Nov. 2009.

[8] M. Joham, W. Utschick and J. A. Nossek, "Linear transmit processing in MIMO communications systems," *IEEE Trans. Signal Process.*, vol. 53, no. 8, pp. 2700-2712, Aug. 2005.

[9] R. Zhang, Y.-C. Liang, and S. Cui, "Dynamic resource allocation in cognitive radio networks," *IEEE Signal Process. Magazine*, vol. 27, no. 3, pp. 102-114, May 2010.

[10] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge, UK: Cambridge Univ. Press, 2004.

[11] G. Golub and C.V. Loan, *Matrix Computations*. The Johns Hopkins Univ. Press, 1996.

[12] A. Goldsmith, *Wireless Communication*. Cambridge, UK: Cambridge Univ. Press, 2005.

[13] H. Lee, K. Lee, B.M. Hochwald, and I. Lee, "Regularized channel inversion for multiple-antenna users in multiuser MIMO downlink," in Proc. *IEEE ICC*, 2008, pp. 351-355