# Factor Graph based Equalizer for Two Way Relaying Channels with General Waveforms

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***Abstract*—Multi-carrier schemes with general waveforms are flexible bandwidth efficient transmission schemes offering a robust design regarding practical impacts like carrier frequency offsets and timing offsets. Especially for two way relay channels (TWRCs) where two users simultaneously transmit data on the same resources, a robust design in presence of practical constraints is important. Equalization within general multi-carrier schemes is a required tool to reduce the intrinsic interference introduced by non-orthogonal waveforms. Therefore, we fully describe an TWRC transmission as factor graphs for general multi-carrier transmissions, which gives us a framework on iterative detection of the relay message based on the sum-product algorithm. The focus of this work is the equalizer, which is mainly influenced by the chosen waveforms and the channels. The edges and therefore the complexity within a factor graph can be configured by choosing waveforms. It turns out that well localized waveforms achieve the best BER performance-complexity trade-off under practical constraints.**

## I. Introduction

We consider a two-phase physical-layer network coding (PLNC) scheme, where two users A and B exchange data via a relay node R [1]. All nodes are equipped with one antenna and are restricted to the half duplex constraint. In the multiple access (MA) phase both users transmit their data to the assisting relay. Based on the superposition of both user signals, the relay estimates a network coded message and transmits it to the users in the broadcast (BC) phase. The users are able to estimate the message of the other user bb the XOR operation of the relay message with their own message.

Practical transmission schemes like WLAN, Long Term Evolution (LTE) or Digital Video Broadcast (DVB)-Terristic (DVB-T) implement cyclic prefix OFDM (CP-OFDM) as flexible system offering time-frequency resource mapping, simple synchronization and low complex one-tap equalization. Thus, CP-OFDM has been applied to two way relay channels (TWRCs) and analyzed, due to the flexible resource mapping. However, it suffers from high out of band (OoB) radiation and high peak to average power ratio (PAPR). Additionally, the performance of CP-OFDM degrades significantly, if time spreads exceed the cyclic prefix (CP) or carrier frequency offsets (CFOs) occur within the transmission. Within a single user scenario, these problems can be mitigated by synchronization and offset compensation. In a TWRC, the influences of timing offset (TO) and CFO cannot be resolved individually leading to severe performance degradations [2], [3]. However, other transmission scheme than CP-OFDM are more robust against impairments like CFO, TO and channel influences [4], especially, if well localized waveforms are used. The additional interference introduced by QAM/filter bank multi-carrier (FBMC) (QAM/FBMC) [5] applying non-orthogonal waveforms requires equalizer with higher complexity compared to the simple one-tap equalizer in CP-OFDM.

In this extended abstract, we illustrate the overall TWRC transmission as a factor graphs [7], [8], offering a framework on iterative detection based on the sum-product algorithm to estimate and generate the broadcast message at the relay. The iterative sum product algorithm (SPA) is widely implemented for doubly dispersive channels [9] as well as for MIMO-OFDM [10]. The ideas are here extended to general waveforms in QAM/FBMC especially in TWRC which offers control on the complexity of the SPA within the factor graph in the equalization step at the relay. Our main contributions are the general description of multi-carrier schemes within TWRCs, the flexible structure of the equalizer and furthermore trade-off between performance, complexity, channel impact and further practical impairments like TOs and CFOs, controllable with proper chosen waveforms. The QAM/FBMC transmission scheme applying a Gaussian waveform is compared with OFDM (without CP), where both apply the factor graph equalizer (FGE). First results show that QAM/FBMC outperforms OFDM with comparable complexity.

## II. System model

Fig. 1 illustrates of the TWRC transmission including encoding, modulation and channel influences to the relay as factor graph. Each user $i$ with $i \in \{\mathrm{A}, \mathrm{B}\}$ encodes an information word $\mathbf{b}_i$ to a code word $\mathbf{c}_i$ with a linear code (e.g., LDPC). The elements of the code words are modulated to symbols $d_i \in \mathcal{A}$ given a linear modulation scheme with alphabet $\mathcal{A}$. Each symbol is mapped to physical resources with index $(k, \ell)$ of a general waveform by shifting the symbol $d_i^{(k,\ell)}$ to the corresponding sub-carrier $kF$ and time instance $\ell T$ [5], where $F$ is the sub-carrier spacing and $T$ is the symbol timing. Both user A and B transmit their symbols simultaneously on the same resources to an assisting relay R in the MA phase. Each transmit signal is affected by individual time variant channels and the received signal $\mathbf{y}_\mathrm{R}$ at the relay after matched filtering and sampling is given by

$$\mathbf{y}_\mathrm{R} = \mathbf{V}_\mathrm{A} \cdot \mathbf{d}_\mathrm{A} + \mathbf{V}_\mathrm{B} \cdot \mathbf{d}_\mathrm{B} + \mathbf{n}_\mathrm{R}. \quad (1)$$

The received signal vector $\mathbf{y}_\mathrm{R}$ and each user data $\mathbf{d}_i$ is represented by stacked vectors containing all symbols of one frame

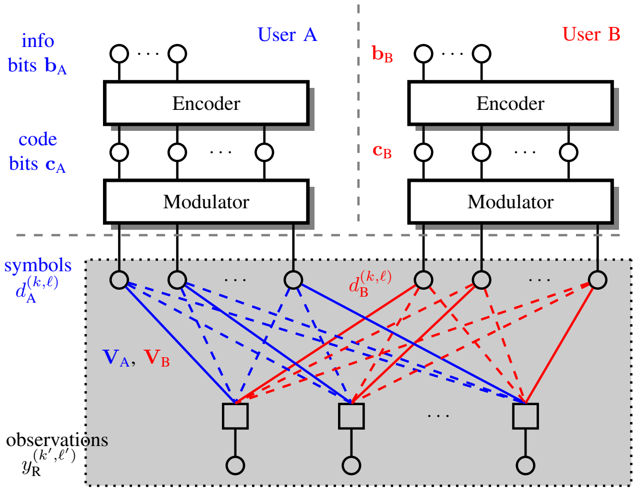


Fig. 1. Factor graph of the MA phase. Edges between the symbol variable of user A and channel observations in red, for user B in blue.

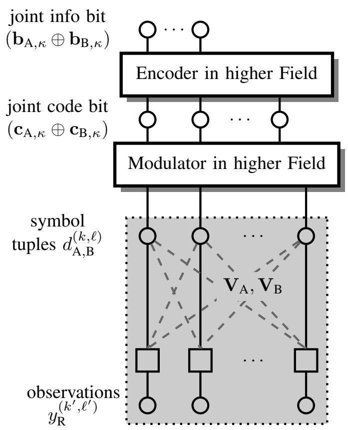


Fig. 2. Factor graph used at the relay. The goal at the relay is the estimation of the joint code bits $\left(\mathbf{c}_{\mathrm{A},\kappa} \oplus \mathbf{c}_{\mathrm{B},\kappa}\right)$ to generate the joint message for the BC phase

(i.e., time and frequency components). Each element of the received signal vector $\mathbf{y}_\mathrm{R}$ contains channel observations $y_\mathrm{R}^{(k',\ell')}$ sampled at sub-carrier $k'$ and time instant $\ell'$. The effective channel matrix $\mathbf{V}_i$ includes the influence of transmit/receive filters and of the channel of user $i$ to the relay [5], [11]. Furthermore, edges between the variable nodes $d_i^{(k,\ell)}$ and the channel observations $y_\mathrm{R}^{(k',\ell')}$ are given by the channel matrix $\mathbf{V}_\mathrm{A}$ and $\mathbf{V}_\mathrm{B}$, i.e., each non-zero element in $\mathbf{V}_i$ gives an edge in the factor graph in Fig. 1. Thus, by applying the SPA, the number of edges represents the number of messages exchanged within the factor graph and therefore is an indicator for computational complexity.

*CP-OFDM:* In case of CP-OFDM without CFO and time invariant delay spread within the CP, each matrix $\mathbf{V}_i$ is diagonal. For that, in Fig. 1 only the straight solid edges between symbols and channel function nodes $(k = k', \ell = \ell')$ exist. If time variant channels or additional CFOs are present a dense connection between all nodes [9] will occur, i.e., also the dashed edges get present within the factor graph. Hence, the complexity will severely change depending on the channel realization.

*General multi-carrier:* In contrast to CP-OFDM general multi-carrier schemes offer the possible configuration of general transmit/receive filters, which can be either orthogonal or non-orthogonal. Thus the representation of the factor graph in general multi-carrier schemes is directly controlled by the choice of waveforms. Well-localized waveform, will only affecting direct neighbors within the time-frequency grid $(k, \ell)$. Hence, only edges between variable nodes and function nodes in the direct neighborhood are present. Even under constraints like CFOs or TOs, there is practically no change of affected neighbors within the time-frequency grid. Thus, the number of edges within the factor graph and the complexity remains similar.

In the full paper, a complexity analysis will be given for different channel realization as well as for different transmit and receive filters.

*Relay processing:* The goal in TWRCs is the estimation of the joint code word with the bitwise operation $c_\mathrm{R} = c_\mathrm{A} \oplus c_\mathrm{B}$ from the corresponding observation from $\mathbf{y}_\mathrm{R}$.

The factor graph in Fig. 1 can be restructured as in Fig. 2, if the overall transmission is reinterpreted by symbol pairs $d_{\mathrm{A,B}}^{(k,\ell)} = \left(d_\mathrm{A}^{(k,\ell)}, d_\mathrm{B}^{(k,\ell)}\right)$ of user A and B. Here, the detection of the relay code word $\mathbf{c}_\mathrm{R}$ is done jointly. In this work, we focus on the generalized joint channel decoding and physical-layer network coding (G-JCNC) detection scheme [12] and we will give further analyses within the full paper.

*Complexity/Performance trade-off:* The complexity of the equalizer is dominated by the number of exchanged messages within the factor graph given by the edges in $\mathbf{V}_i$ unequal to zero.

An approximated matrix can be defined by

$$\tilde{v}_i(\kappa,\lambda) = \begin{cases} v_i(\kappa,\lambda) & , |v_i(\kappa,\lambda)| > t_\mathrm{threshold} \\ 0 & , \text{else} \end{cases}, \tag{2}$$

containing only elements of the matrix having a larger amplitude than a given threshold $t_\mathrm{threshold}$.

Thus, if the edges are generated by $\tilde{\mathbf{V}}_i$, the complexity can be reduced at the expense of performance. Therefore, the complexity can be configured by $t_\mathrm{threshold}$. In the full paper, we give a deep numerically performance analysis on different threshold values and waveforms w.r.t. bit error rate (BER) and complexity.

## III. Performance Evaluation

For performance evaluation, a multi-carrier system with BPSK modulation is considered, where Rayleigh fading channels with $N_\mathrm{h}$ complex exponentially decaying channel coefficient $h_\iota$ with $\iota = 0, \ldots, N_\mathrm{h}-1$ are assumed. The time delay $\tau_\iota$ and the Doppler shift of channel tap $h_\iota$ are equally distributed within $[0\tau_\mathrm{max}]$ and $[-\nu_\mathrm{max}\nu_\mathrm{max}]$, respectively. For simulations, the maximum time delay and maximum Doppler shift are restricted to $\tau_\mathrm{max} = 0.2T$ and $\nu_\mathrm{max} = 0.2F$, respectively. In

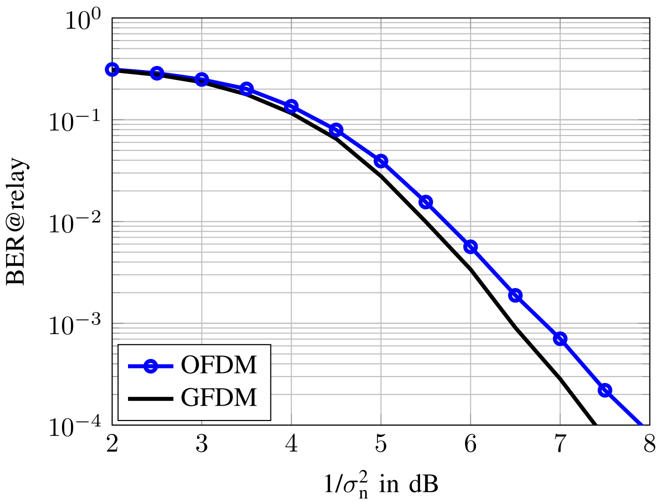


Fig. 3. BER performance at the relay of OFDM and QAM/FBMC in a TWRC applying the SPA based approach.

total, $N_k = 32$ sub-carriers and $N_\ell = 10$ time symbols are used to generate a frame containing 320 data symbols, thus, the matrix $\mathbf{V}_i \in \mathcal{C}^{N_k N_\ell \times N_k N_\ell}$ contains 102400 elements. Fig. 3 shows the BER performance of OFDM and QAM/FBMC with the Gaussian waveform applying the proposed equalizer structure and an LDPC code with a rate of 0.3. To keep the complexity constant, the threshold $t_{\text{threshold}}$ is chosen such that only the 3200 largest values in $|\mathbf{V}_i|$ are considered to generate the corresponding factor graph. It can be observed that QAM/FBMC outperforms OFDM by around 0.4dB with the same amount of complexity.

## IV. Summary And Outlook

In this extended abstract we applied an equalizer based on factor graphs for general multi-carrier systems in TWRC to reduce the impact of time and frequency impairments. The complexity in QAM/FBMC is limited due to the good localization of Gaussian waveforms, whereas OFDM suffers from complexity limitations due to the dense factor graph. The performance results indicate, that the QAM/FBMC system with Gaussian waveforms achieves similar results as OFDM with rectangular waveforms after only two iterations. In the final paper, we will provide a detailed derivation on the factor graph based system model in connection to PLNC detection methods and provide further performance investigations. Furthermore, an extended bibliography will be provided.

## References


[1] S. Zhang, S. Liew, and P. Lam, "*Physical Layer Network Coding,*" in Proc. International Conference on Mobile Computing and Networking (MobiCom), Los Angeles, USA, 2006, pp. 358-365.

[2] I.W.-H. Ho, S.C. Liew, and L. Lu, "*Feasibility Study of Physical-Layer Network Coding in 802.11p VANETs,*" ISIT'14, Honolulu, HI, USA, June 2014

[3] M. Wu, F. Ludwig, M.Woltering, D. Wübben, A. Dekorsy, and S. Paul, "*Analysis and Implementation for Physical-Layer Network Coding with Carrier Frequency Offset,*" WSA'2014, Erlangen, Germany, Mar. 2014

[4] G. Wunder, P. Jung, M. Kasparick, T. Wild F. Schaich, et. al. "*5GNOW: Non-Orthogonal, Asynchronous Waveforms for Future Mobile Applications,*" IEEE Communications Magazine, vol. 52, issue 2, pp.97-105, Feb. 2014

[5] S. Schedler, M. Woltering, D. Wübben, V. Kühn, and A. Dekorsy "*Investigation on Gaussian Waveforms to Improve Robustness in Physical Layer Network Coding,*" InOWo'14, Essen, Germany, Aug. 2014

[6] M. Woltering, D. Wübben, A. Dekorsy, "*Physical Layer Network Coding with Gaussian Waveforms using Soft Interference Cancellation,*" IEEE 81st VTC Spring 2015, Glasgow, Scotland, UK, May 2015

[7] F.R. Kschischang, B.J. Frey, and H.-A. Loeliger, "*Factor Graphs and the Sum-Product,*" Algorithm. IEEE Transactions on Information Theory, 47(2):498-519, Feb 2001

[8] H. Wymeersch "*Iterative receiver design,*" Cambridge University Press 2007

[9] P. Ochandiano, H. Wymeersch, M. Mendicute, L. Martínez I. Sobrón "*Factor Graph based Detection Approach for high-mobility OFDM Systems with large FFT Modes,*" EURASIP Journal on Wireless Communications and Networking, Vol. 2012

[10] X. Xu and R. Mathar, "*Factor Graph based Detection and Channel Estimation for MIMO-OFDM Systems in doubly selective Channel,*" ISWCS 2010, York, UK, Sept 2010

[11] M. Woltering, D. Wübben, A. Dekorsy, S. Schedler, and V. Kühn, "*Physical Layer network Coding using Gaussian Waveforms: A Link Level performance analysis,*" SCC'2015, Hamburg, Germany Feb. 2015

[12] D. Wübben and Y. Lang, "*Generalized Sum-Product Algorithm for Joint Channel Decoding and Physical-Layer Network Coding in Two-Way Relay Systems,*" GLOBECOM 2010, Miami, FL, USA, Dec 2010

[13] D. Wübben, "*Joint Channel Decoding and Physical-Layer Network Coding in Two-Way QPSK Relay Systems by a Generalized Sum-Product Algorithm,*" ISWCS 2010, York, UK, Sept. 2010