

# Scheduling for Massive MIMO with Few Excess Antennas

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**Abstract**—Scheduling all user equipments and using a linear transmission scheme is optimal for massive MIMO with sufficiently many base station antennas. For a smaller excess of base station antennas scheduling provides gains. Using as less base station antennas helps saving cost and array space. We determine the regime where scheduling provides gains for massive MIMO scenarios with linear precoding methods. We apply semi-orthogonal user selection to massive MIMO and propose two scheduling algorithms with improved performance.

## I. INTRODUCTION

The capacity bound of multiple-input multiple-output (MIMO) communication systems is achieved by dirty paper coding (DPC) [1]. However in practical systems linear precoding methods are preferred. The performance of DPC is approached by linear precoding methods with the help of scheduling [2].

We refer to conventional MIMO as communication systems where the number of base station (BS) antennas is less or equal to the number of served user equipments (UEs). A conventional MIMO base station serves at one time a subset of all UEs. The scheduling algorithm selects the subset according to an objective (e.g. maximize sum rate or fairness). Many scheduling algorithms have been proposed [3]. A widely considered scheduling approach is semi-orthogonal user selection (SUS) [2]. It was originally developed for conventional MIMO.

Massive MIMO is a key idea to increase the spectral efficiency in new mobile communication standards (e.g. 5G). It refers to systems where the number of BS antennas exceeds the number of served UEs. For massive MIMO with sufficient randomness in the channel and sufficiently many BS antennas the channels hardens [4]. This means that scheduling all UEs and using a linear transmission scheme like zero forcing beamforming (ZFBF) is optimal. In this regime scheduling does not provide gains. However scheduling increases performance when the channels to the UEs are correlated [2] or when the excess of BS antennas is small [5].

We analyze the gain of scheduling with increasing numbers of BS antennas using the suboptimal transmission scheme ZFBF. We obtain results for i.i.d. Rayleigh-fading channel coefficients and plan to extend the analysis to channels with correlation. As expected ZFBF and all UEs being scheduled

all the time approaches the capacity achieved by DPC with increasing excess of BS antennas. In the regime of less than twice as many BS antennas as served UEs the gap to capacity is large. However this regime is favorable for practical implementations as more antennas mean higher cost and higher space consumption. Scheduling helps to bridge the gap to capacity.

We apply SUS to massive MIMO and achieve a performance close to capacity. To further reduce the gap we propose two scheduling algorithm. The simulation results show that the gap to optimal scheduling is reduced by 50%. We plan to analyze the complexity of the proposed scheduling algorithms and expect it to be less than the complexity of the original SUS algorithm.

In [6] scheduling serves a different purpose. The idea, which is called Joint Spatial Division and Multiplexing (JSDM), is to partition UEs into groups based on their channel's covariance matrices. This allows to divide precoding into two stages. In the pre-beamforming stage the groups are separated using the dominant eigenvectors of each group's channel covariance matrix. The precoding of the second stage combats the inter-group interference based on the effective channels after the first stage, which have a reduced dimensionality. Hence the required channel state information (CSI) is reduced as the first stage requires longterm knowledge only.

The works [7] and [8] on scheduling for massive MIMO communication systems are based on JSDM. There the focus is somewhat different to our work as the number of UEs is larger than the number of BS antennas. The task of the scheduler is to select UEs which form well separated groups for JSDM. In [8] it is also shown that random beamforming [9] performs poorly for a finite number of UEs.

In [10] a fixed number of heterogeneous UEs are scheduled based on the norm of their instantaneous CSI. The work targets, in contrast to our work, scenarios where the excess of BS antennas to served UEs is large. The channel is then asymptotically orthogonal and transmitting to the UEs with largest channel norm is optimal.

## II. SCHEDULING

We describe briefly the original SUS algorithm and our two proposed algorithms. A detailed description will be included in the full paper.

### A. Original Semiorthogonal User Selection

The SUS algorithm [2] was originally designed for conventional MIMO where the number of BS antennas is less or equal to the number of served UEs. It finds a suboptimal user group with the objective of maximizing the sum rate. It achieves the same asymptotic performance as DPC [2].

The idea when selecting the user group goes as follows: First the UE with the largest channel norm is scheduled. In each following iteration the SUS algorithm schedules the UE with the largest orthogonal component to the subspace spanned by the already scheduled UEs. The key novelty of the SUS algorithm is that after each iteration the semi-orthogonality of each unscheduled UE to the current scheduled UE is determined. When the degree of semi-orthogonality is too small the UE is removed from the set of unscheduled UEs.

A variable  $\alpha$  characterizes the degree of required semi-orthogonality between two channel vectors. For smaller  $\alpha$  more UEs are removed. The optimal  $\alpha$  is determined with numerical simulations. An analysis of the optimal  $\alpha$  for massive MIMO scenarios will be included in the full paper.

The result of the algorithm is a set of scheduled UEs. The channel vectors of the UEs are as orthogonal as possible to each other and their norms are as large as possible.

### B. Massive MIMO Pair-wise SUS Algorithm

The SUS algorithm presented in the previous section starts with an empty set of scheduled UEs and adds a user in every step. A natural question to ask is whether it is possible to improve the performance and/or the complexity by exploiting the fact that the set of scheduled UEs can be initialized as all UEs for a massive MIMO scenario.

We propose an approach along the idea which we call massive MIMO pair-wise semi-orthogonal user selection (pair-wise SUS). For massive MIMO the BS can transmit data to all UEs. At each iteration the scheduling algorithm removes one of the UEs. To determine this UE the pair-wise SUS algorithm finds the UE pair with the smallest degree of orthogonality. From this pair the UE with the smaller channel vector norm is removed. This continues until a stopping condition is met.

Note that removing a UE based on the orthogonality to individual other UEs is usually suboptimal when maximizing the sum rate.

### C. Massive MIMO Subspace SUS Algorithm

The idea of the massive MIMO subspace semi-orthogonal user selection (subspace SUS) algorithm is similar to the approach of the pair-wise SUS algorithm. The main difference is the removal criterion. In each iteration the algorithm calculates for all UEs the component of their channel vector orthogonal to the subspace spanned by the other scheduled UEs. Then the UE with the smallest orthogonal component is removed from the set of scheduled UEs.

### D. Optimal Scheduling

The optimal schedule is found by exhaustive search. We calculate the achieved sum rates of all combinations of UEs and find the combination with the maximal sum rate. This approach is limited to few UEs as the number of combinations increases exponentially with the number of UEs.

## III. FIRST SIMULATION RESULTS

Consider a communication system with 10 UEs and one BS. The number of BS antennas is varied between 10 and 100. The channel coefficients are i.i.d. zero-mean circularly symmetric complex Gaussian random variables. The SNR is 10 dB. We average over 500 channel realizations. The capacity is obtained as in [1]. Figure 1 shows the sum-rate versus the number of BS antennas.

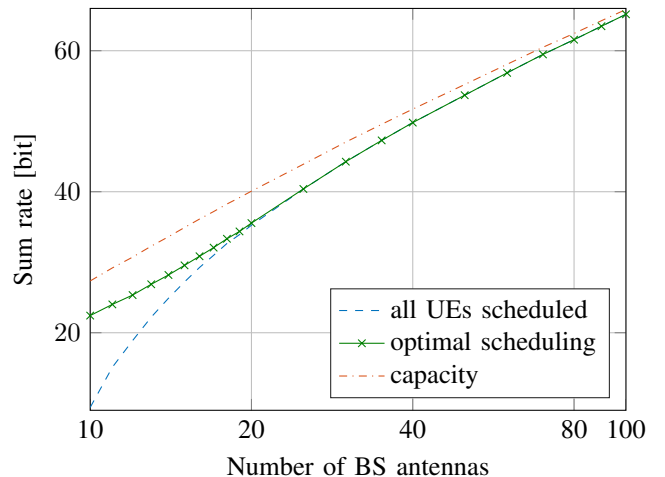


Fig. 1. Capacity compared to suboptimal linear precoding for 10 UEs and an SNR of 10 dB. Note that the x-axis is logarithmic.

The gap between optimal scheduling and capacity for lower number of BS antennas is due to suboptimal ZFBF precoding. For higher number of BS antennas the gap vanishes. For more than twice as many BS antennas as served UEs all scheduling algorithms perform the same. Here scheduling all UEs is optimal. Hence scheduling can help saving costs by operating efficiently in the regime of less than twice as many BS antennas as served UEs.

Figure 2 shows the sum rate results for the different scheduling algorithms for the regime between 10 and 20 BS antennas. All presented scheduling algorithms achieve a performance close to optimal scheduling. A sum rate of 25 bit is achieved by optimal scheduling with 12 BS antennas, by pair-wise SUS or by subspace SUS with 13 BS antennas, while original SUS requires 14 BS antennas. Here the gap to optimal scheduling is reduced by 50% compared to SUS by the proposed algorithms.

## IV. CONCLUSIONS

We compared the performance of the SUS algorithm, the two proposed algorithms, optimal scheduling, and capacity in a massive MIMO scenario. We show that for a smaller excess of BS antennas to served UEs scheduling provides gains over

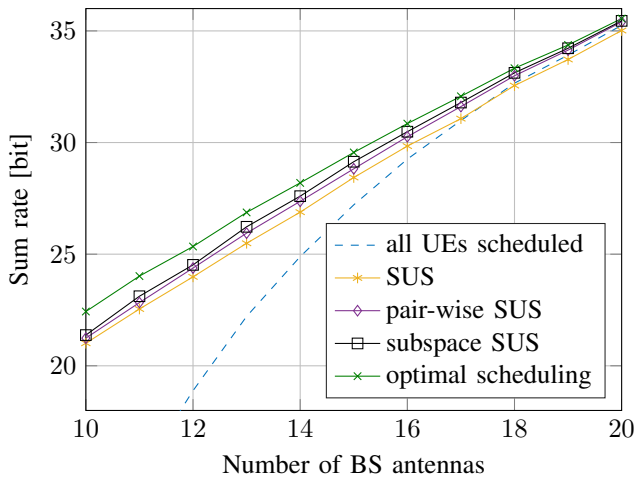


Fig. 2. Comparison of the scheduling algorithms for 10 UEs and an SNR of 10 dB. Note that here the x-axis is linear.

scheduling all UEs. The two proposed scheduling algorithms outperform SUS.

Further extensions for the full paper include a proof of the asymptotic optimality of pair-wise SUS and subspace SUS and an analysis of the complexity of the proposed algorithms.

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