A Study of Pilot-Aided Channel Estimation in MIMO-GFDM Systems

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***Abstract*—Generalized frequency division multiplexing (GFDM) is a promising candidate waveform for next generation wireless communications systems. Unlike conventional orthogonal frequency division multiplexing (OFDM) based systems, it is a non-orthogonal waveform subject to inter-carrier and inter-symbol interference. In multiple-input multiple-output (MIMO) systems, the additional inter-antenna interference also takes place. The presence of such three-dimensional interference challenges the receiver design. This paper addresses the MIMO-GFDM channel estimation problem with the aid of structurally inverted pilot symbols on the transmitter side. Specifically, the received signal is expressed as the joint effect of the pilot part, unknown data part and noise part. On top of this formulation, least squares (LS) and linear minimum mean square error (LMMSE) estimators are presented, while their performance is evaluated for various pilot arrangements.**

I. Introduction

Ultra low latency, very high reliability and robustness, low out-of-band (OOB) emission and very high data capacity are among the challenges for the 5th generation (5G) of wireless systems, e.g., [1], [2], [3]. The well-known orthogonal frequency division multiplexing (OFDM) has reached to its boundaries in addressing the above various requirements. Hence, several non-orthogonal waveform candidates have been proposed and rediscovered for the new air interface of 5G, e.g. filter bank multi-carrier (FBMC) [4], universal filtered multi-carrier (UFMC) [5], filtered-OFDM [6] as well as generalized frequency division multiplexing (GFDM) [7].

This paper considers GFDM, since it is equipped with necessary flexibility to address a wide range of requirements envisioned for 5G, e.g., latency, data rates, reliability and OOB emission. Relying on it, a unified air interface can be provided for various service types. The combination of GFDM with multiple antennas, i.e., multiple-input multiple-output (MIMO) GFDM, can further enhance the system performance e.g. [8], [9], [10]. For the MIMO-GFDM receiver design, channel estimation is a critical functional unit. The prior work [11] relied on preamble which is spectrally efficient for continuous transmission over slow fading channels. This paper aims to deliver accurate estimates of channel state information (CSI) for coherent detection by scattered pilot symbols. This type of data-aided channel estimation is more suitable for time and frequency dispersive channels.

In pilot-aided channel estimation, pilot symbols and information-bearing data symbols are multiplexed and transmitted within the same time-frequency resource block, e.g., Fig. 1. At the receiver side, the task of channel estimation is to estimate CSI based on the knowledge of pilot symbols. To this end, different channel estimation techniques have been developed for conventional OFDM systems e.g. [12], [13], [14], [15] and reference therein. The extension of OFDM-based channel estimation methods for GFDM are not straightforward, because the orthogonality of OFDM ensures clean pilot observations without interference from unknown data symbols. This property is not valid for GFDM which is a non-orthogonal waveform in general. Moreover, in OFDM many narrow-band subcarriers allow one-tap equalization while on the contrary, in GFDM depending on the transmit signal configuration (e.g. low latency requirement) the subcarriers might have broader bandwidth and consequently, they become frequency selective.

Given the knowledge of data symbols at the transmitter side, it is possible to design pilots such that the interference from data symbols can be properly pre-cancelled. This idea has been applied for channel estimation in a single carrier transmission system over a frequency selective fading channel [16] as well as a GFDM-based system [17]. However, the approach proposed in [17] was developed under the assumption of a nearly flat and slow fading channel, which is unrealistic with respect to broadband communication.

This paper tackles the MIMO-GFDM channel estimation problem for rich multipath fading channels. Two well known estimation techniques, namely least squares (LS) and linear minimum mean square error (LMMSE), are respectively tailored for pilot-aided MIMO-GFDM channel estimation. We evaluate and analyze their performance in accordance with pilot arrangement and correspondingly, we examine their complexity. The LS approach is an unbiased estimator which does not require any probabilistic assumption and therefore, it is being widely used due to its ease of implementation [18]. Nevertheless, the performance loss in LS estimation needs significant attention. On the other hand, LMMSE estimation is a Bayesian approach which exploits the a-priori knowledge of channel statistics in order to improve the estimation quality at the cost of further implementation complexity.

The rest of this extended abstract is organized as follows: Section II describes the GFDM modulation, pilots insertion and also the assumptions taken into account for the MIMO channel. Section III applies the LS channel estimation method and calculates the closed form expression of the mean squared error (MSE). Based on the computations provided in Sec. III, the LMMSE estimator is then obtained in Section IV. A short summary of this abstract and further plans for the full paper are provided in Sec. V.

A. Notations

Column-vectors are denoted by vector sign $\vec{X}$ and matrices by boldface $\mathbf{X}$. Time and frequency domain representations are separated by lowercase and uppercase letters respectively. $\mathbb{E}[\cdot]$ is the expectation operator. The trace of a square matrix $\boldsymbol{X}$ is $\mathrm{Tr}(\boldsymbol{X})$. The transpose and Hermitian conjugate of $\boldsymbol{X}$ are $\boldsymbol{X}^T$ and $\boldsymbol{X}^H$ respectively. The Frobenius norm of a matrix $\boldsymbol{X}$ is $\|\boldsymbol{X}\|$ and its square can be written as $\|\boldsymbol{X}\|^2 = \mathrm{Tr}(\boldsymbol{X}\boldsymbol{X}^H)$. The vectorization of a matrix $\boldsymbol{X}$ (i.e. stacking its columns on top of one another from left to right) is denoted by $\mathrm{vec}(\boldsymbol{X})$. The Kronecker and Hadamard products [19] of matrices $\boldsymbol{X}$ and $\boldsymbol{Y}$ are denoted as $\boldsymbol{X}\otimes\boldsymbol{Y}$ and $\boldsymbol{X}\circ\boldsymbol{Y}$ respectively. $\mathrm{diag}(\vec{X})$ is a diagonal matrix whose diagonal entries are the entries of the column vector $\vec{X}$. Furthermore, $\mathrm{diag}(\boldsymbol{X},\cdots,\boldsymbol{Y})$ is a block diagonal matrix according to its matrix entries with $\boldsymbol{X}$ being the top-left and $\boldsymbol{Y}$ being the bottom-right blocks. The matrix $\boldsymbol{I}_n$ is the identity matrix of size n. $\vec{0}_n$ is a column vector of size n with all zero entries. $\sqrt{\mathbf{X}}$ is the element-wise square root of matrix $\mathbf{X}$.

II. System Model

A. GFDM Modulation

We assume a GFDM block of length $N = MK$ samples where M complex valued subsymbols are being transmitted on K subcarriers. In GFDM, the entries of vector $\vec{d} \in \mathbb{C}^{N\times 1}$ are filtered through circularly time and frequency shifted versions of a prototype filter $g[n]$. Hence, we define

$$g_{k,m}[n] \triangleq g\left[(n - mK) \bmod N\right] \exp\left[j2\pi\frac{k}{K}n\right], \quad (1)$$

where the circular time shift is acquired via the modulo operation and frequency shift is obtained through the complex exponential term corresponding to subsymbol index m and subcarrier index k respectively.

The superposition of pulse shaped data symbols will then provide the GFDM transmit sample:

$$x[n] = \sum_{k=0}^{K-1}\sum_{m=0}^{M-1} g_{k,m}[n]d_{k,m},\ n = 0,\cdots,N-1 \quad (2)$$

where $d_{k,m}$ is the symbol transmitted on subcarrier k and subsymbol m.

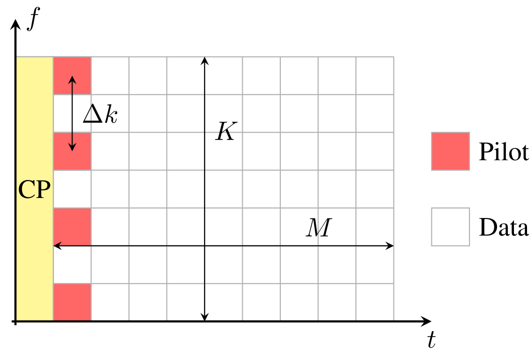

Fig. 1: Pilot positions in time-frequency grid

In terms of matrix and vector notations, the above expression (2) can be rewritten as

$$\vec{x} = \mathbf{A}\vec{d}, \quad (3)$$

where $\vec{d} = (\vec{d}_k[m])^T_{m=0:M-1}$, $\vec{d}_k[m] = (d_{k,m})^T_{k=0:K-1}$ and $\vec{x} = (x[n])^T_{n=0:N-1}$. The GFDM transmit matrix $\mathbf{A}$ follows:

$$\mathbf{A} = \left(\vec{g}_{0,0},\cdots,\vec{g}_{K-1,0},\vec{g}_{0,1},\vec{g}_{1,1},\cdots,\vec{g}_{K-1,M-1}\right), \quad (4)$$

with column vector $\vec{g}_{k,m} = \left(g_{k,m}[n]\right)^T_{n=0,1,\cdots,N-1}$.

Furthermore, the vector $\vec{d} = \vec{d}_p + \vec{d}_d$ is generated from the summation of pilots sequence $\vec{d}_p \in \mathbb{C}^{N\times 1}$ and data vector $\vec{d}_d \in \mathbb{C}^{N\times 1}$. The pilots sequence $\vec{d}_p$ contains one pilot subsymbol every Δk subcarrier and the rest of subsymbols which are the position of data samples from $\vec{d}_d$ are kept zero. Note that each time-frequency resource element is associated to either pilots or data leading $\vec{d}_p \circ \vec{d}_d = \vec{0}_N$. Fig. 1 shows an example of pilot positions in the time-frequency grid.

B. MIMO Wireless Channel

Consider a multi-path MIMO block fading channel with $\mathsf{n_T}$ transmit and $\mathsf{n_R}$ receive antennas. Due to the cyclic prefix (CP), the receive signal $\vec{y}_{i_\mathsf{R}}$ (at Rx antenna i_R) in time is the circular convolution of transmit signal $\vec{x}_{i_\mathsf{T}}$ (from Tx antenna i_T) and the channel impulse response $\vec{h}_{i_\mathsf{T},i_\mathsf{R}}$ (between the antennas i_T and i_R) plus the AWGN process $\vec{w}_{i_\mathsf{R}}$

$$\vec{y}_{i_\mathsf{R}} = \sum_{i_\mathsf{T}=1}^{\mathsf{n_T}} \vec{x}_{i_\mathsf{T}} \circledast \vec{h}_{i_\mathsf{T},i_\mathsf{R}} + \vec{w}_{i_\mathsf{R}}. \quad (5)$$

In the above expression, it is assumed that all the channels have shorter lengths L compared to the CP length. Moreover, the channel impulse response between the antennas i_T and i_R is defined as

$$\vec{h}_{i_\mathsf{T}i_\mathsf{R}} \triangleq \sqrt{\mathrm{diag}(\vec{P}_{i_\mathsf{T}i_\mathsf{R}})}\vec{\mathfrak{g}}_{i_\mathsf{T}i_\mathsf{R}}, \quad (6)$$

where $\vec{P}_{i_\mathsf{T}i_\mathsf{R}} \in \mathbb{R}^{L\times 1}$ is the power delay profile between the Tx antenna i_T and Rx antenna i_R; and $\vec{\mathfrak{g}}_{i_\mathsf{T}i_\mathsf{R}} \in \mathbb{C}^{L\times 1}$ is a vector of zero mean complex Gaussian random variables with unit variance, representing independent Rayleigh fading for different Tx-Rx antenna pairs.

Due to the circular convolution in (5), the individual channels are diagonal in frequency domain and therefore, the observed signal on Rx antenna i_{R} is characterized by the following linear equation:

$$\vec{Y}'_{i_{\mathsf{R}}} = \sum_{i_{\mathsf{T}}=1}^{\mathsf{n_T}} (\boldsymbol{X}'_{p,i_{\mathsf{T}}} + \mathbf{X}'_{d,i_{\mathsf{T}}})\vec{H}'_{i_{\mathsf{T}},i_{\mathsf{R}}} + \vec{W}'_{i_{\mathsf{R}}}, \tag{7}$$

with $\vec{H}'_{i_{\mathsf{T}},i_{\mathsf{R}}} = \mathbf{F}'_L \vec{h}_{i_{\mathsf{T}},i_{\mathsf{R}}}$. Furthermore, $\boldsymbol{X}'_{s,i_{\mathsf{T}}} = \mathrm{diag}(\vec{X}'_{s,i_{\mathsf{T}}})$ is a diagonal matrix associated either to pilots p or data sequences d (i.e. $s \in \{p, d\}$). $\vec{X}'_{s,i_{\mathsf{T}}}$ is being transmitted on Tx antenna i_{T} and it is defined as $\vec{X}'_{s,i_{\mathsf{T}}} \triangleq (\mathbf{F}'_t \mathbf{A} \vec{d}_s)_{i_{\mathsf{T}}}$. $\mathbf{F}'_t \in \mathbb{C}^{N\times N}$ is the DFT matrix and $\mathbf{F}'_L \in \mathbb{C}^{N\times L}$ contains only the first L columns of $\mathbf{F}'_t$ where L is the channel length. $\vec{W}'_{i_{\mathsf{R}}}$ is the frequency domain counterpart of AWGN process on receive antenna i_{R}.

If the number of pilot subcarriers is smaller than the number of data subcarriers, i.e., the subcarrier spacing $\Delta k > 1$, only a subset of observations in frequency domain that contain the information of pilots will be used for pilot-aided channel estimation. In equations, the received signal at pilot-bearing subcarriers follows:

$$\vec{Y}_{i_{\mathsf{R}}} = \sum_{i_{\mathsf{T}}=1}^{\mathsf{n_T}} (\boldsymbol{X}_{p,i_{\mathsf{T}}} + \mathbf{X}_{d,i_{\mathsf{T}}})\vec{H}_{i_{\mathsf{T}},i_{\mathsf{R}}} + \vec{W}_{i_{\mathsf{R}}}, \tag{8}$$

where $\vec{H}_{i_{\mathsf{T}},i_{\mathsf{R}}} = \mathbf{F}_L \vec{h}_{i_{\mathsf{T}},i_{\mathsf{R}}}$, $\vec{W}_{i_{\mathsf{R}}} = \mathbf{F}_t \vec{w}_{i_{\mathsf{R}}}$, $\boldsymbol{X}_{s,i_{\mathsf{T}}} = \mathrm{diag}(\vec{X}_{s,i_{\mathsf{T}}})$ and $\vec{X}_{s,i_{\mathsf{T}}} = (\mathbf{F}_t \mathbf{A} \vec{d}_s)_{i_{\mathsf{T}}}$. Here, $\mathbf{F}_t \subseteq \mathbf{F}'_t$ and $\mathbf{F}_L \subseteq \mathbf{F}'_L$ are $\frac{N}{\Delta k} \times N$ and $\frac{N}{\Delta k} \times L$ matrices that take the DFT at pilot subcarriers respectively i.e. every $m + kM$ row of $\mathbf{F}_t, \mathbf{F}_L$ corresponds to $m + kM\Delta k$ row of $\mathbf{F}'_t, \mathbf{F}'_L$ respectively. Note that an estimation of the channel exists if and only if the number of pilot-bearing subcarriers is larger than the channel length i.e. $\frac{K}{\Delta k} > L$.

We rearrange the expression (8) into matrix form as

$$\mathbf{Y} = (\boldsymbol{X}_p + \boldsymbol{X}_d)\mathbf{F}\mathbf{h} + \mathbf{W}, \ \text{with} \begin{cases} \mathbf{Y}, \mathbf{W} \in \mathbb{C}^{N\times \mathsf{n_R}} \\ \boldsymbol{X}_p, \boldsymbol{X}_d \in \mathbb{C}^{N \times N\mathsf{n_T}} \\ \mathbf{F} \in \mathbb{C}^{N\mathsf{n_T} \times L\mathsf{n_T}} \\ \mathbf{h} \in \mathbb{C}^{L\mathsf{n_T}\times \mathsf{n_R}} \end{cases} \tag{9}$$

herein, each of the above parameters are defined as

$$\mathbf{Y} \triangleq (\vec{Y}_1, \ \cdots, \vec{Y}_{i_{\mathsf{R}}}, \ \cdots, \vec{Y}_{\mathsf{n_R}}), \tag{9a}$$

$$\boldsymbol{X}_s \triangleq (\boldsymbol{X}_{s,1}, \ \cdots, \boldsymbol{X}_{s,i_{\mathsf{T}}}, \ \cdots, \boldsymbol{X}_{s,\mathsf{n_T}}), \tag{9b}$$

$$\mathbf{F} \triangleq \mathbf{I}_{\mathsf{n_T}} \otimes \mathbf{F}_L, \tag{9c}$$

$$\mathbf{h} \triangleq \begin{pmatrix} \vec{h}_{11} & \cdots & \vec{h}_{1\mathsf{n_R}} \\ \vdots & \ddots & \vdots \\ \vec{h}_{\mathsf{n_T}1} & \cdots & \vec{h}_{\mathsf{n_T}\mathsf{n_R}} \end{pmatrix}, \tag{9d}$$

$$\mathbf{W} \triangleq (\vec{W}_1, \ \cdots, \vec{W}_{i_{\mathsf{R}}}, \ \cdots, \vec{W}_{\mathsf{n_R}}). \tag{9e}$$

Eq. (9) depicts that the observed matrix $\mathbf{Y}$ contains a deterministic term $\boldsymbol{X}_p\mathbf{F}\mathbf{h}$, an interference term due to useful information $\boldsymbol{X}_d\mathbf{F}\mathbf{h}$ and the WGN $\mathbf{W}$. Moreover, Fig. 2 shows an example of matrix structures for $\mathsf{n_T} = 2$ by $\mathsf{n_R} = 2$ antennas. In Fig. 2 it is illustrated that $\boldsymbol{X}_s$ is a wide matrix composed of individual diagonal matrices of transmit signals

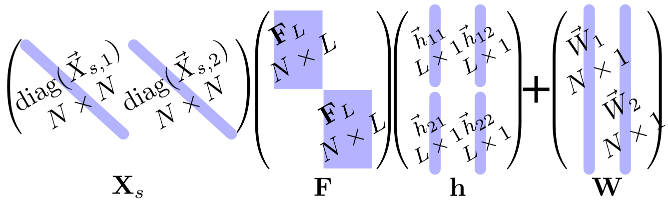

Fig. 2: Overview of the matrix sturctures for a 2×2 MIMO channel

associated to different Tx antennas. Furthermore, the matrix of channel impulse responses $\mathbf{h}$ is structured as $\mathsf{n_T} \times \mathsf{n_R}$ column vectors. Such matrix structure brings an advantage for mathematical analysis when vectorizing the channel matrix. It is plane from Fig. 2 that $\vec{h} = \mathrm{vec}(\mathbf{h})$ will consist of $\mathsf{n_T}\mathsf{n_R} = 4$ independent column vectors of channel impulse responses, and thus, considering Rayleigh fading channels with no spatial correlation the covariance matrix of all channel impulse responses $\mathbb{E}\left[\vec{h}\vec{h}^H\right]$ becomes block diagonal.

Resorting to the matrix identity $\mathrm{vec}(\mathsf{ABC}) = (\mathsf{C}^T \otimes \mathsf{A})\mathrm{vec}(\mathsf{B})$ [19], the corresponding vectorization of the observed matrix $\mathbf{Y}$ yields the following equation:

$$\vec{Y} = \mathrm{vec}(\mathbf{Y}) = \widetilde{\mathbf{x}}\vec{h} + \vec{W}, \tag{10}$$

where $\vec{h} = \mathrm{vec}(\mathbf{h})$, $\vec{W} = \mathrm{vec}(\mathbf{W})$ and $\widetilde{\mathbf{x}} = (\mathbf{I}_{\mathsf{n_R}} \otimes \mathbf{X}\mathbf{F})$.

III. Least Squares Estimation

The structure of the transmit signal matrix $\mathbf{X}_s$ does not allow to provide a least squares estimate of the channel in frequency domain. As mentioned in Sec. II-B, $\mathbf{X}_s$ is a wide matrix of diagonal matrices and therefore, the product of $\mathbf{X}_s^H\mathbf{X}_s$ or specifically $\mathbf{X}_p^H\mathbf{X}_p$ is always singular. Although, one can obtain the LS estimate of the channel impulse response by minimizing $\|\mathbf{Y} - \boldsymbol{X}_p\mathbf{F}\mathbf{h}\|^2$ with respect to $\mathbf{h}$. This yields

$$\hat{\mathbf{h}}_{\mathrm{LS}} = \boldsymbol{Q}_{\mathrm{LS}}\mathbf{Y} = \mathbf{h} + \mathbf{E}, \tag{11}$$

where $\boldsymbol{Q}_{\mathrm{LS}} = \left((\mathbf{X_p}\mathbf{F})^H(\mathbf{X_p}\mathbf{F})\right)^{-1}(\mathbf{X_p}\mathbf{F})^H$. The above estimation yields the following interference and noise terms:

$$\mathbf{E} = \boldsymbol{Q}_{\mathrm{LS}}\boldsymbol{\Psi} + \boldsymbol{Q}_{\mathrm{LS}}\mathbf{W}. \tag{12}$$

Here, $\boldsymbol{\Psi} = \boldsymbol{X}_d\mathbf{F}\mathbf{h}$ leads to an error floor due to the confront of the pilots and useful information.

Accordingly, the result of the MSE calculation follows:

$$\begin{aligned} \mathrm{MSE} &= \mathbb{E}\left[\|\hat{\mathbf{h}}_{\mathrm{LS}} - \mathbf{h}\|^2\right] \\ &= \mathrm{Tr}\left(\left(\mathbf{I}_{\mathsf{n_R}} \otimes (\boldsymbol{Q}_{\mathrm{LS}}^H\boldsymbol{Q}_{\mathrm{LS}})\right)\boldsymbol{\Sigma}_{\Psi\Psi}\right) \\ &\quad + \sigma_w^2 \,\mathrm{Tr}\left(\mathbf{I}_{\mathsf{n_R}} \otimes (\boldsymbol{Q}_{\mathrm{LS}}^H\boldsymbol{Q}_{\mathrm{LS}})\right), \end{aligned} \tag{13}$$

where σ_w^2 is the noise variance. Then, we compute the covariance matrix of the interference term as

$$\begin{aligned}\mathbf{\Sigma}_{\Psi\Psi} &= \mathbb{E}\left[\text{vec}(\boldsymbol{X}_d\mathbf{F}\mathbf{h})\text{vec}(\boldsymbol{X}_d\mathbf{F}\mathbf{h})^H\right] \\ &= \mathbb{E}_{\boldsymbol{X}_d}\left[(\mathbf{I}_{\mathsf{n_R}}\otimes\boldsymbol{X}_d\mathbf{F})\,\mathbb{E}_{\mathbf{h}}\left[\vec{h}\vec{h}^H|\boldsymbol{X}_d\right](\mathbf{I}_{\mathsf{n_R}}\otimes\boldsymbol{X}_d\mathbf{F})^H\right] \\ &= \mathbb{E}_{\boldsymbol{X}_d}\left[(\mathbf{I}_{\mathsf{n_R}}\otimes\boldsymbol{X}_d\mathbf{F})\,\mathbf{\Sigma}_{\mathbf{hh}}\,(\mathbf{I}_{\mathsf{n_R}}\otimes\boldsymbol{X}_d\mathbf{F})^H\right]. \quad (14)\end{aligned}$$

Here, an important fact arises that both of the above matrices $(\mathbf{I}_{\mathsf{n_R}}\otimes\boldsymbol{X}_d\mathbf{F})$ and $\mathbf{\Sigma}_{\mathbf{hh}}$ have block diagonal structures as

$$\begin{aligned}\mathbf{I}_{\mathsf{n_R}}\otimes\boldsymbol{X}_d\mathbf{F} = \text{diag}([&\boldsymbol{X}_{d,1}\mathbf{F}_L,\cdots,\boldsymbol{X}_{d,\mathsf{n_T}}\mathbf{F}_L],\cdots, \\ &[\boldsymbol{X}_{d,1}\mathbf{F}_L,\cdots,\boldsymbol{X}_{d,\mathsf{n_T}}\mathbf{F}_L]), \quad (15)\end{aligned}$$

$$\begin{aligned}\mathbf{\Sigma}_{\mathbf{hh}} = \text{diag}(&\mathbf{\Sigma}_{h_{11}},\cdots,\mathbf{\Sigma}_{h_{\mathsf{n_T}1}}, \\ &\cdots,\mathbf{\Sigma}_{h_{(\mathsf{n_T}-1)\mathsf{n_R}}},\mathbf{\Sigma}_{h_{\mathsf{n_T}\mathsf{n_R}}}), \quad (16)\end{aligned}$$

where $\mathbf{\Sigma}_{h_{i_\mathsf{T} i_\mathsf{R}}} \in \mathbb{R}^{L\times L}$ is the diagonal covariance matrix of channel impulse response, computed as

$$\begin{aligned}\mathbf{\Sigma}_{h_{i_\mathsf{T} i_\mathsf{R}}} &= \mathbb{E}\left[\vec{h}_{i_\mathsf{T} i_\mathsf{R}}\vec{h}^H_{i_\mathsf{T} i_\mathsf{R}}\right] \\ &= \text{diag}(\vec{P}_{i_\mathsf{T} i_\mathsf{R}}). \quad (17)\end{aligned}$$

The product of (15), (16) and the hermitian conjugate of (15) will then provide a block diagonal structure for the interference covariance matrix $\mathbf{\Sigma}_{\Psi\Psi}$ as expressed in (14). This is due to the fact that independent Rayleigh fading has been considered for the individual channels (see Sec. II-B). As a result, it is possible to perform the computations separately for the individual blocks. Hence, for the Tx antenna i_T and Rx antenna i_R we have [19]:

$$\begin{aligned}\mathbf{\Sigma}_{\Psi\Psi_{i_\mathsf{T} i_\mathsf{R}}} &= \mathbb{E}_{\boldsymbol{X}_{d,i_\mathsf{T}}}\left[\boldsymbol{X}_{d,i_\mathsf{T}}\mathbf{F}_L\mathbb{E}_h\left[\vec{h}_{i_\mathsf{T} i_\mathsf{R}}\vec{h}^H_{i_\mathsf{T} i_\mathsf{R}}|\boldsymbol{X}_{d,i_\mathsf{T}}\right]\mathbf{F}_L^H\boldsymbol{X}^H_{d,i_\mathsf{T}}\right] \\ &= \mathbf{\Upsilon}_{i_\mathsf{T} i_\mathsf{R}}\circ\mathbf{\Sigma}_{X_dX_d,i_\mathsf{T}}, \quad (18)\end{aligned}$$

where $\mathbf{\Upsilon}_{i_\mathsf{T} i_\mathsf{R}} = \mathbf{F}_L\text{diag}(\vec{P}_{i_\mathsf{T} i_\mathsf{R}})\mathbf{F}_L^H$. Furthermore, the covariance matrix of data is being calculated as

$$\begin{aligned}\mathbf{\Sigma}_{X_dX_d,i_\mathsf{T}} &= \mathbb{E}[(\mathbf{F}\mathbf{A}\vec{d}_d)_{i_\mathsf{T}}(\mathbf{F}\mathbf{A}\vec{d}_d)^H_{i_\mathsf{T}}] \\ &= (\mathbf{F}\mathbf{A}\text{diag}(\vec{\sigma}_d^2)\mathbf{A}^H\mathbf{F}^H)_{i_\mathsf{T}}, \quad (19)\end{aligned}$$

where, $\vec{\sigma}_d^2$ is the vector of data variances with zero entries at pilot positions.

Consequently, for each Rx antenna i_R we calculate the individual diagonal blocks of $\mathbf{\Sigma}_{\Psi\Psi}$ as

$$\mathbf{\Sigma}_{\Psi\Psi(i_\mathsf{R})} = \sum_{i_\mathsf{T}=1}^{\mathsf{n_T}}\mathbf{\Upsilon}_{i_\mathsf{T} i_\mathsf{R}}\circ\mathbf{\Sigma}_{X_dX_d,i_\mathsf{T}}. \quad (20)$$

Hence, the full interference covariance matrix follows:

$$\mathbf{\Sigma}_{\Psi\Psi} = \text{diag}(\mathbf{\Sigma}_{\Psi\Psi(i_\mathsf{R}=1)},\mathbf{\Sigma}_{\Psi\Psi(i_\mathsf{R}=2)},\cdots,\mathbf{\Sigma}_{\Psi\Psi(i_\mathsf{R}=\mathsf{n_R})}). \quad (21)$$

IV. LMMSE

The LMMSE estimation calculates the coefficients of a linear filter aiming at minimum mean squared error. In accordance with (9) and the corresponding vectorization in (10), we formally have:

$$\hat{\vec{h}}_{\text{LMMSE}} = \mathbf{\Sigma}_{hY}\mathbf{\Sigma}^{-1}_{YY}\vec{Y}, \quad (22)$$

with the matrices defined as

$$\begin{aligned}\mathbf{\Sigma}_{YY} &= \widetilde{\mathbf{x}}_p\mathbf{\Sigma}_{hh}\widetilde{\mathbf{x}}_p^H + \mathbf{\Sigma}_{\Psi\Psi} + \sigma_w^2\mathbf{I}_{N\mathsf{n_R}}, \quad (23) \\ \mathbf{\Sigma}_{hY} &= \mathbf{\Sigma}_{hh}\widetilde{\mathbf{x}}_p^H, \quad (24)\end{aligned}$$

where $\widetilde{\mathbf{x}}_p = (\mathbf{I}_{\mathsf{n_R}}\otimes\boldsymbol{X}_p\mathbf{F})$. Note that $\hat{\vec{h}}_{\text{LMMSE}}$ is a column vector containing $\mathsf{n_T n_R}$ individual column vectors of size L, associated to the LMMSE estimates of the individual channel impulse responses.

V. Short Summary

This paper presents a system model (9), serving as a general framework for deriving various pilot-aided channel estimators for MIMO-GFDM systems. Both LS and LMMSE criterion-based estimators are derived and their resulting MSE performance is analyzed. The symbol error rate (SER) performance of the MIMO-GFDM channel estimation will be simulated and compared to OFDM in the full paper. Additionally, simulation results for different pilot arrangements as well as numerical analysis of the analytical computations vs simulation will be provided accordingly.

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