Achievable Rate Performance of TDD Multi-cell Massive MIMO with Non-Orthogonal Pilots

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***Abstract*—This paper investigates the downlink throughput performance of the Grassmannian line packing (GLP)-based pilot sequence design for time-duplex-division (TDD) massive multiple-input-multiple-output (MIMO) systems. When the length of the uplink pilot sequence is less than the number of single-antenna user equipments (UEs), the GLP-based scheme may perform better than the pilot reuse scheme. With given number of UEs and pilot sequence length, the pilot design problem can be considered as the line packing problem in Grassmannian manifold. We propose a closed-form achievable rate expression for the multi-cell scenario and evaluate the throughput improvement of GLP, which is compared with the conventional pilot reuse scheme. Numerical results demonstrate the GLP-based pilot design can provide significant improvements in the system throughput over the pilot reuse scheme.**

I. Introduction

Massive multiple-input-multiple-output (MIMO) system deploys a large-scale array of antennas at the base station (BS). It is designed to support increased numbers of user equipments (UEs) and considered as a promising technology which attracted great interests recently. It is shown that both spectrum and energy efficiency can be greatly improved by benefiting the increasing number of multiplexing gain and the pairwise orthogonal channel property [1]. However, the limited length of the coherence interval may cause *pilot contamination*, which seriously affects estimation of the channel state information (CSI) and consequently might degrade the system performance. The pilot design schemes such as reuse are proposed to combat contamination [2]. In [3], the Grassmannian line packing (GLP)-based pilot sequence design is proposed to reduce contamination other than the reuse scheme. However, the discussion focuses on the estimation error performance and does not provide the theoretical analysis of the pilot length on the throughput. The authors investigate the throughput performance of different pilot schemes [4], but the study is only in a single-cell system.

Contribution: In this study, the focus is on the downlink achievable rate performance of a TDD multi-cell massive MIMO system with GLP-based pilot sequences. The closed-form achievable sum rate expression for the multi-cell system is proposed and comparisons are made with the conventional pilot reuse scheme. The numerical results show that the GLP-based pilot design significantly outperforms the pilot reuse scheme and it brings large gains to the system throughput.

II. System model

Consider a multi-cell massive MIMO cellular network with L cells. Each cell includes a central BS equipped with M antennas serving K single antenna UEs where $M \gg K$. The channel vector from the UE k in the i-th cell to the BS in the j-th cell is defined as

$$\mathbf{g}_{k,i,j} = \sqrt{\beta_{k,i,j}}\mathbf{h}_{k,i,j}, \tag{1}$$

where $\beta_{k,i,j}$ denotes the large-scale fading coefficient that models the effect of path-loss and shadowing, and $\mathbf{h}_{k,i,j}$ represents the small-scale fading and the vector contains independent and identically distributed (i.i.d) complex Gaussian random variables with zero-mean and unit variance.

We assume the block-fading channel model and the spectrum sharing among all the UEs. In TDD cellular communication environment, the coherence interval can be mainly divided into three phases: uplink training, uplink data transmission and downlink data transmission. In uplink phase, each UE simultaneously transmit its own pilot sequence to its BS for channel training. Let the vector $\mathbf{s}_{i,j} \in \mathbb{C}^{\tau\times 1}$ denote the uplink pilot sequence from UE i in the j-th cell with a pilot signal length of τ and $\mathbb{E}[|\mathbf{s}_{i,j}|^2] = 1$. The correlation coefficient of the pilot sequences between UE i and UE k in the jth-cell is defined as $\rho^2_{i,k,j} \triangleq |\mathbf{s}^{\mathrm{H}}_{i,j}\mathbf{s}_{k,j}|^2$, where $\rho^2_{i,k,j} \in [0, 1]$. For the BS in cell j, it receives the uplink training sequences from the UEs in all the cells. Herein, the least squares method is utilized for channel estimation.

Thanks to the channel reciprocity in the TDD operation, the estimated uplink CSI can be utilized to express the downlink CSI. The maximum ratio transmission (MRT) precoding is employed in the downlink transmission. The received noisy downlink signal at the UE i in the j-th cell can be expressed as

$$\mathbf{r}_{i,j} = \sum_{l=1}^{L}\sqrt{p_{i,j,l}\beta_{i,j,l}}\mathbf{h}_{i,j,l}\sum_{k=1}^{K}\mathbf{q}_{i,k,l}t_{i,k,l} + z_{i,j}, \tag{2}$$

where $\mathbf{r}_{i,j}$ contains the signals from both BS in cell j and the others, and $p_{i,j,l}$ denotes the downlink transmission power, and $\mathbf{q}_{i,k,l}$ denotes the MRT precoding vector and $t_{i,k,l}$ denotes the data symbols transmitted by the BS in the l-th cell, and $z_{i,j}$ is the additive white Gaussian noise with $z_{i,j} \in \mathcal{CN}(0, \sigma^2_i)$.

$$SINR_{i,j}^{G} = \frac{\beta_{i,j}p_{i,j}}{\beta_{i,j}(\rho_G^2 \sum_{k\neq i}^{K} p_{k,j} + \frac{K\rho_G^2+\sigma_w^2}{M}\sum_{m=1}^{K} p_{m,j}) + \sum_{l\neq j}^{L}\sum_{\overline{k}=1}^{K}\beta_{i,l}p_{\overline{k},l}\frac{(M+K)\rho_G^2+\sigma_w^2}{M} + \frac{K\rho_G^2+\sigma_w^2}{M}\sigma_i^2} \quad (5)$$

III. Grassmannian line packing based pilot design

The GLP is a well known applied mathematics problem which aims to find the optimal packing of N-dimensional vectors in a Grassmannian manifold by maximising the minimum pairwise distance of the vectors. For massive MIMO system with the case where $\tau < K$, it is not possible to assign each UE with an orthogonal pilot for the uplink training. The *pilot reuse scheme* assigns part of the UEs with orthogonal pilots and the rest reuse these training sequences. As it is desired to assign low cross-correlation pilots to UEs, inspired by the GLP, it can also be utilized in pilot sequence design. Different from pilot reuse scheme, the GLP can be applied to get the non-orthogonal pilots by minimizing the cross-correlation. To be more specific, the pilot sequences can be treated as packing K one-dimensional vectors in a $\mathcal{G}(\tau, 1)$ Grassmann manifold. The purpose is to maximize the pairwise distance between pilots which reduces pilot contamination. As a result, these GLP-based pilot sequences might achieve better performance with limited τ. The correlation between pilot sequence $\mathbf{s}_1$ and $\mathbf{s}_2$ can be interpreted as

$$\rho_{\mathbf{s}_1,\mathbf{s}_2}^2 = |\mathbf{s}_1^{\mathrm{H}}\mathbf{s}_2|^2 = 1 - d_c^2(\mathbf{s}_1, \mathbf{s}_2), \quad (3)$$

where $d_c(\mathbf{s}_1, \mathbf{s}_2)$ denotes the chordal distance with a definition of $d_c(\mathbf{s}_1, \mathbf{s}_2) = \sqrt{1-|\mathbf{s}_1^{\mathrm{H}}\mathbf{s}_2|^2}$. It builds the relation between the chordal distance and the correlation. Therefore, based on the GLP-based pilot sequence design, the downlink achievable sum rate can be obtained as follows.

Proposition 1. *For a L cells TDD massive MIMO system serving K UEs with the length of training sequence of τ, the downlink achievable sum rate of the system with the GLP pilot sequence design can be expressed as*

$$R_G = \sum_{l=1}^{L}\sum_{i=1}^{K}(1 - \frac{\tau}{T})\log_2(1 + SINR_{i,l}^{G}) \quad (4)$$

where T is the length of coherence interval; $SINR_{k,l}^{G}$ is given by equation (5) at the top of the page and ρ_G^2 is represented as

$$\rho_G^2 = \begin{cases} 1 - (\frac{\tau-1}{\tau}\cdot\frac{K}{K-1}), & \text{if } K \leq \tau(\tau+1)/2 \\ 1 - (\frac{\tau-1}{\tau}), & \text{if } K > \tau(\tau+1)/2, \end{cases}$$

Proof. Proof will be given in the full manuscript. □

IV. Numerical results

Consider a TDD massive MIMO system with seven equal size hexagon shaped cells with 1 centre cell and 6 neighbour cells. The UEs are randomly distributed in each cell with minimum 35 meters distance from BS. The centre cell is set as a hotspot region which contains more UEs than the neighbour cells. The simulation is to investigate the achievable sum rates of the GLP-based pilot sequence design and compare with the pilot reuse scheme with $\tau < K$. The pilot reuse scheme provides τ UEs with orthogonal pilots and the remaining $(K - \tau)$ UEs reuse these pilot waveforms.

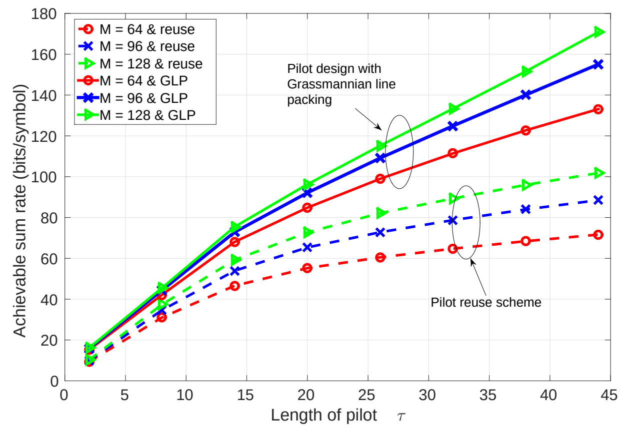

Fig. 1. Comparison of GLP based pilot design and pilot reuse scheme with $M = 64, 96, 128$

Fig.1 shows the achievable sum rate versus the length of the pilot τ with different numbers of antennas at BS where $M = 64, 96$ or 128. The centre cell contains 15 UEs and the other 6 cells contain 5 UEs. With $\tau < K$, the longer pilot sequence indicates that more UEs can be assigned with orthogonal pilots and it causes less pilot contamination. In the figure it can be observed that the GLP-based pilot design has much higher achievable rates compared to the pilot reuse scheme as both M and τ increase.

V. Conclusion

This extended abstract has investigated the performance of pilot design based on Grassmannian line packing in multi-cell TDD massive MIMO system. A closed-form achievable sum rate is proposed and a numerical comparison is made with the conventional pilot reuse scheme. The numerical result clearly show the Grassmannian line packing based pilot design provides significant gains in system throughput. The full version of this paper will present the proof of Proposition 1 and more numerical results.

References

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