On Physical Limits of Massive MISO Systems

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***Abstract*—We analyze the performance of a fixed-size uniform circular array which is used to transmit information to a single antenna receiver as a function of the number of transmit side antennas. We find that the minimum necessary energy to transfer one information bit stops decreasing once the number of antennas grows over a certain bound. We relate this bound to the diameter of the array and show that there is an optimum and finite number of antennas for every such array. For large array diameters this optimum number grows linearly with the diameter, the factor of proportionality primarily depending on whether the receiver is inside or outside of the transmit uniform circular array.**

I. Introduction

Massive MIMO systems are currently considered a possible key technology for the next generation of wireless communication systems [1]. The idea is that the number of antennas at the base station is much larger than the total number of antennas of the served user terminals. This may require hundreds or even thousand of antennas at the base station.

There are a number of possible advantages of such an approach. For example, Russek et al. point out in [2] that massive MIMO systems 1) allow linear signal processing techniques to reach near optimum performance, 2) provide a natural stage for improved analysis based on random matrix theory [3], and 3) allow that thermal noise can be averaged out since coherent averaging offered by a receive antenna array would eliminate quantities that are uncorrelated between the antenna elements, and especially thermal noise. In [1] Larsson et al. additionally point out that 4) if an antenna array were serving a single terminal, then it could be shown that the total necessary transmit power could be made inversely proportional to the number of antennas at the transmitter.

While assertions 1) and 2) above might ring true, the assertions 3) and 4) look problematic. Since increasing the number of antennas in a fixed space requires the average antenna separation to decrease, the inevitable electromagnetic interaction of the antennas leads to correlated instead of uncorrelated thermal noise, which violates the basic assumption of assertion 3). Similarly, electromagnetic interaction leads to the effect that the power which is radiated by an antenna array is not proportional to the sum of squares of the antenna's excitation [4]. A consequence of this is that, when the number of antennas grows beyond a certain bound, the radiated power to ensure a preset signal quality at the receiver does not drop any more by adding still more antennas. This is the subject of this paper.

II. System Model

Figure 1 schematically shows the system under investigation. It consists of a uniform circular array (UCA) of N quarter wavelength monopoles used for transmission, and one single such monopole, located in the center of the circle, used for reception. The center monopole is loaded with a resistance of $R = 35\,\Omega$, while the N UCA monopoles are fed by linear generators with the same output impedance of R. All monopoles reside over an infinite groundplane in an otherwise empty half-space, while the half-space below the groundplane contains the generators and the termination resistance. Denoting with r the radius of the UCA, the distance between neighboring monopoles equals $\Delta l = 2r \sin \pi/N$. Keeping r constant, Δl must decrease with increasing N towards zero. This ever increasing proximity creates strong mutual electromagnetic interaction between all antennas and has to be modeled carefully. To this end, the multiport model shown in Figure 1 is used. The coupling between the N+1 antennas is described by the impedance matrix $\boldsymbol{Z}$, which is partitioned into four blocks according to:

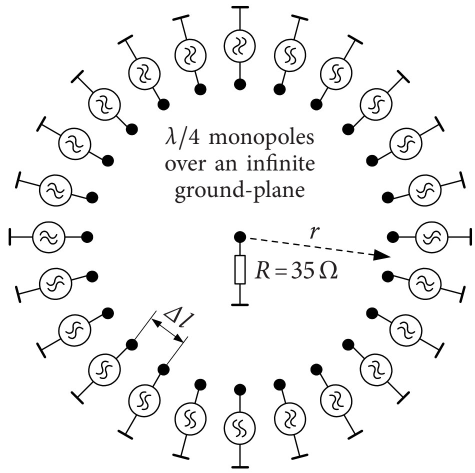

Figure 1: Uniform circular array transmits to a single antenna receiver located in the formers center.

$$\begin{bmatrix} \boldsymbol{u}_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} \boldsymbol{Z}_\mathrm{T} & \boldsymbol{z} \\ \boldsymbol{z}^\mathrm{T} & Z_\mathrm{R} \end{bmatrix} \begin{bmatrix} \boldsymbol{i}_1 \\ i_2 \end{bmatrix} + \begin{bmatrix} \boldsymbol{u}_{\mathrm{N},1} \\ u_{\mathrm{N},2} \end{bmatrix},$$

where $\boldsymbol{u}_1$, $\boldsymbol{i}_1$ and $\boldsymbol{u}_{\mathrm{N},1}$ are N-dimensional vectors of the complex envelopes of the port voltages, currents and open-circuit noise voltages of the UCA antenna ports, while u_2, i_2 and $u_{\mathrm{N},2}$ are the respective quantities for the receiver's single antenna port. The N complex envelopes of the open-circuit generator voltages are collected into the vector $\boldsymbol{u}_\mathrm{G}$, while the vector $\boldsymbol{i}_{\mathrm{N},R}$ contains the N complex envelopes of the noise currents which model the thermal agitation of the electrons in the generators' internal resistances R. At the receiver side, u_N and i_N are the complex envelopes of the equivalent input noise voltage and current that model the noise of the receiver's low-noise amplifier and following stages. The input resistance of the receiver is assumed to be equal to R. Finally, we denote by u the complex

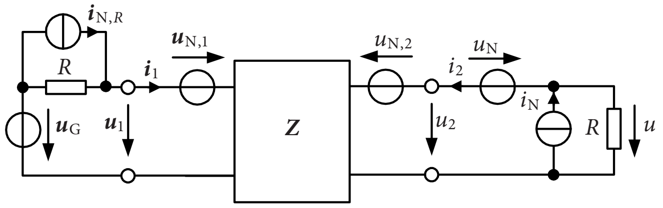

Figure 2: Multiport model of the system from Figure 1.

envelope of the voltage at the receiver's output scaled down by its voltage amplification factor. For such a scaling does not change the signal to noise ratio, no harm is done to take u itself as the output quantity. The components $\boldsymbol{Z}_\mathrm{T}$, $\boldsymbol{z}$ and Z_R, of the antenna system's impedance matrix, can be determined for the antenna configuration of Figure 1, by following classical antenna theory [5]. The transmit power P_T, is defined as the sum of active powers which flow into the N ports of the UCA, not counting the contribution of noise:

$$P_\mathrm{T} = \mathrm{E}\left[\mathrm{Re}\left\{\boldsymbol{u}_1^\mathrm{H}\boldsymbol{i}_1\right\} \mid \text{no noise}\right],$$

where $\mathrm{E}[\cdot]$ and $(\cdot)^\mathrm{H}$ are the expectation and the complex conjugate transpose operations, respectively, while $\mathrm{Re}\{\cdot\}$ returns the real-part of its argument. To fix ideas, we assume the noise properties of the receiver to be given as:

$$\mathrm{E}\left[|u_\mathrm{N}|^2\right] = 2\mathrm{k_B}TWR,\ \mathrm{E}\left[|i_\mathrm{N}|^2\right] = 2\mathrm{k_B}TW/R,\ \mathrm{E}\left[u_\mathrm{N} i_\mathrm{N}^*\right] = 0,$$

where $\mathrm{k_B}$ is Boltzmann's constant, W is the (small) noise bandwidth, and T the antenna noise temperature, while * and $|\cdot|$ are the complex conjugation and magnitude operations, respectively. We note in passing that the minimum noise figure of this receiver is equal to 3 dB and is achieved when it is connected to a source of impedance R [6]. Regarding the remaining noise sources in Figure 2, we assume that they generate thermal equilibrium noise [7]. Moreover, we assume that the receiver's noise is uncorrelated with the other noise sources and that all noise is Gaußian distributed.

III. Shannon Limit

Let C be the channel capacity of the communication system from Figure 2. Then

$$E_\mathrm{b} = \min_C \; P_\mathrm{T}/C$$

is the minimum required energy per information bit, the so-called Shannon-limit [8]. In the full paper, we will derive that E_b for the described system is given by:

$$\frac{E_\mathrm{b}}{\mathrm{k_B}T} = \frac{|R+Z_\mathrm{R}|^2}{\boldsymbol{z}^\mathrm{H}\left(\mathrm{Re}\left\{\boldsymbol{Z}_\mathrm{T} - \dfrac{\boldsymbol{z}\boldsymbol{z}^\mathrm{T}}{R+Z_\mathrm{R}}\right\}/R\right)^{-1}\boldsymbol{z}} \cdot \log_\mathrm{e} 4, \qquad (1)$$

where $\log_\mathrm{e}(\cdot)$ refers to the natural logarithm function. Figure 3 shows $E_\mathrm{b}/(\mathrm{k_B}T)$ as a function of the number N of UCA antennas for a number of different fixed radii. Starting from a single antenna at the transmitter, we see that E_b first drops with increasing N, approximately reducing to half its value when N is increased twofold. However, when N climbs over a certain (radius dependent) number (e.g. about 5 for $r = 10\lambda$)

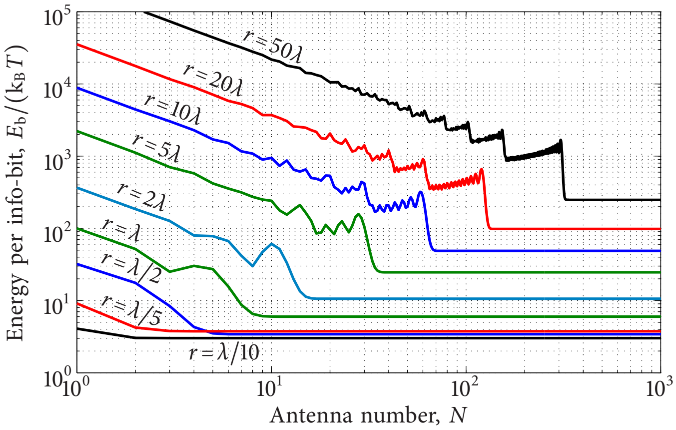

Figure 3: Energy per information bit as function of the number of transmit side antennas. Exact results up to round-off errors.

we observe a more and more irregular and non-monotonic behavior of E_b with respect to N. When another critical number of antennas is reached (e.g., about 60 for $r = 10\lambda$), E_b *sharply* decreases (e.g., by more than a factor of 5 for a 12% increase of the antenna number when $r = 10\lambda$). After this steep descent, E_b levels off almost immediately and remains at the same value (e.g., $48.74\,\mathrm{k_B}T$ for $r = 10\lambda$), no matter how the antenna number is increased further. Calling this critical antenna number N_sat where E_b saturates, it turns out that

$$\text{for } r \gg \lambda, \quad N_\mathrm{sat} \approx \left\lfloor 2\pi\frac{r}{\lambda}\right\rfloor, \qquad (2)$$

where λ denotes the wavelength. We note in passing that, for $r = \lambda/2$, the antenna spacing corresponding to N_sat is also $\Delta l = \lambda/2$, which makes the UCA a hexagonal array which achieves an array gain of 9 from 6 antennas and an E_b which is 1.6 dB larger than the absolute minimum.

IV. Outlook

In the full paper, we will derive (1) and apply it also to different positions of the receiver, both inside and outside of the circle of the transmitter array. It will turn out that similar relationships as (2) can be found depending on whether the receiver is located inside or outside of the transmitter's UCA.

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