Pilot Contamination Precoding Assisted Sum Rate Maximization for Multi-cell Massive MIMO Systems

(Extended Abstract)

Jianhua Liu, †Haixia Zhang, Shuaishuai Guo and Dongfeng Yuan
Shandong provincial key laboratory of wireless communication technologies, Shandong University, China.

Summary: *This paper focuses on the pilot contamination precoding (PCP) assisted sum rate maximization for multi-cell massive MIMO system with finite number of antennas. By jointly considering the noise, the channel estimation errors, the channel uncertainty caused by the usage of statistical channel state information (CSI), the channel non-orthogonality of users due to the finite number of the base station antennas as well as the pilot contamination among cells, we formulate a PCP optimization problem to maximize the sum rate of all users. Solving the problem, the expression of the PCP matrix maximizing sum rate (MSR-PCP) is derived. Based on the expression, an iterative algorithm is proposed to get the suboptimal solution. Simulations are done to verify its superiority and results show that the proposed MSR-PCP outperforms the existing zero forcing PCP (ZF-PCP) considerably, especially for the case that users are located at the cell edges and suffer from strong interference.*

A. Related Work and Our Contributions

Pilot contamination precoding (PCP) is a newly proposed technique combating the effect of the pilot contamination in multi-cell massive MIMO systems [1-4]. By exploiting the information shared at all the base stations (BSs), the sum rate can be greatly improved through PCP. The authors in [1] propose a ZF-PCP scheme based on the estimated channel state information (CSI), and it is shown that when the number of antennas at BS is infinite, ZF-PCP can eliminate the effect of pilot contamination. As precoding based on more accurate CSI remits better performance, [2] considers the channel training into account and proposes a beamforming training (BT)-PCP scheme to improve the system sum rate. Both the PCP schemes in [1] and [2] are relied on the ideal assumption that the number of the base station antennas is infinite or goes to infinity. Considering that the number of base station antennas is alway finite, [5] proposed a PCP scheme to maximize the minimal transmission rate of an individual user of all the cells, thus offers better fairness among users. In this work, we try to maximize the sum rate of the whole system through PCP. Analysis has shown that the sum rate is jointly determined by the noise, the channel estimation errors, the channel uncertainty caused by the usage of statistical channel state information (CSI), the channel non-orthogonality of users due to the finite number of the base station antennas as well as the pilot contamination among cells. By jointly considering all these factors, an optimization problem is formulated and the expression of the optimal precoding matrix is derived. Based on the expression, an iterative algorithm is proposed to get the suboptimal solutions for the formulated problem. The imposed computational complexity is analyzed and the convergence of the algorithm is investigated through simulations. The performance of the system in terms of the sum rate is also investigated for different system layouts. Results show that the proposed scheme can greatly improve the system sum rate.

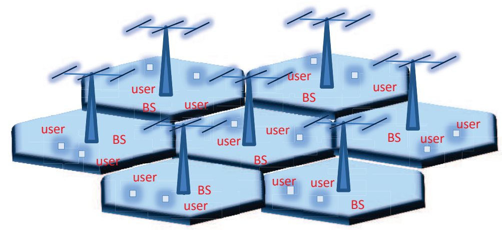

Fig. 1. System Model

B. System model

We consider the downlink of a Massive MIMO system where there are L cells and each serves K single antenna users as illustrated in Fig. 1. The BSs are all equipped with M antennas. Let $\mathbf{g}_j^{[kl]} = \sqrt{\beta_j^{[kl]}}\mathbf{h}_j^{[kl]}$ denote the downlink channel vector from BS j to user k in the cell l, in which $\beta_j^{[kl]}$ represents the large scale fading coefficient and $\mathbf{h}_j^{[kl]} = [h_{j1}^{[kl]}, h_{j2}^{[kl]}, \cdots, h_{jM}^{[kl]}]$ represents the small scale fading vector. Each entry of the channel vector is complex Gaussian variable with zero mean and unit variance, i.e., $\mathcal{CN}\ (0,1)$. Assume that the maximum ration transmission (MRT) beamforming is employed and let $\alpha_j^{[ni]}q^{[ni]}$ be the pilot contamination precoded transmitted signal for user n in the cell i, after transmission, the received signal of user k in cell l writes

$$x^{[kl]} = \sqrt{\frac{\rho_d}{\gamma}}\sum_{j=1}^{L}\sum_{n=1}^{K}\sum_{i=1}^{L}\mathbf{g}_j^{[kl]}\mathbf{w}^{[nj]}\alpha_j^{[ni]}q^{[ni]} + n^{[kl]} \tag{1}$$

where $\mathbf{w}^{[nj]}$ is the MRT beamforming vector, ρ_d is the downlink transmission power, $n^{[kl]}$ denotes the complex Gaussian noise obeying $\mathcal{CN}\ (0,1)$ and γ is the scaling factor. Considering the power constraint for each BS, the scale factor is set as [1]

$$\gamma = \frac{M}{L}\sum_{k=1}^{K}\sum_{j=1}^{L}\sum_{l=1}^{L}(1+\rho_u\tau\sum_{s=1}^{L}\beta_j^{[ks]})|\alpha_j^{[kl]}|^2 \tag{2}$$

C. Problem Formulation and MSR-PCP Design

Both the MRT beamforming vectors and the PCP are designed based on imperfect CSI. Specifically, denoting $\hat{\mathbf{g}}_j^{[kl]}$ as the estimated CSI, MRT precoder is expressed as $\mathbf{w}^{[kl]} = (\hat{\mathbf{g}}_l^{[kl]})^*$. The PCP is designed based on the statistical CSI. In TDD mode, the downlink channel vector can be got by uplink training due to the channel reciprocity. Assume that the pilot sequences of users in a cell are orthogonal and reused among cells, and employ minimum mean square error (MMSE) channel estimation [5], the estimated channel vector can be expressed as

$$\hat{\mathbf{g}}_j^{[kl]} = \frac{\sqrt{\rho_u \tau}\beta_j^{[kl]}}{1+\rho_u\tau\sum_{s=1}^{L}\beta_j^{[ks]}}(\sqrt{\rho_u\tau}\sum_{i=1}^{L}\sqrt{\beta_j^{[ki]}}\mathbf{h}_j^{[ki]} + \mathbf{n}_j^{[k]}) \quad (3)$$

where ρ_u is the average uplink training power of every pilot symbol, τ denotes the length of pilot sequence and $\mathbf{n}_j$ denotes the Gaussian noise with identical independent distributed entries. Let $\tilde{\mathbf{g}}_j^{[kl]}$ denote the channel estimation errors, we have $\mathbf{g}_j^{[kl]} = \hat{\mathbf{g}}_j^{[kl]} + \tilde{\mathbf{g}}_j^{[kl]}$. Therefore, the received signal in (1) rewrites

$$x^{[kl]} = \bar{g}^{[kl]}q^{[kl]} + n_1^{[kl]} + n_2^{[kl]} + n_3^{[kl]} + n_4^{[kl]} + n^{[kl]} \quad (4)$$

where $\bar{g}^{[kl]} = \sqrt{\frac{\rho_d}{\gamma}}\sum_{j=1}^{L}E\{\hat{\mathbf{g}}_j^{[kl]}\mathbf{w}^{[kj]}\}\alpha_j^{[kl]}$ is the statistical effective channel estimated by user k in cell l, $n_1^{[kl]} = \sqrt{\frac{\rho_d}{\gamma}}\sum_{j=1}^{L}\sum_{i\neq l}^{L}E\{\hat{\mathbf{g}}_j^{[kl]}\mathbf{w}^{[kj]}\}\alpha_j^{[ki]}q^{[ki]}$ represents the interference caused by pilot contamination caused by pilot reuse, $n_2^{[kl]} = \sqrt{\frac{\rho_d}{\gamma}}\sum_{j=1}^{L}\sum_{i=1}^{L}\sum_{n=1}^{K}\tilde{\mathbf{g}}_j^{[kl]}\mathbf{w}^{[nj]}\alpha_j^{[ni]}q^{[ni]}$ is caused by channel estimation error, $n_3^{[kl]} = \sqrt{\frac{\rho_d}{\gamma}}\sum_{j=1}^{L}\sum_{i=1}^{L}\sum_{n\neq k}^{K}\hat{\mathbf{g}}_j^{[kl]}\mathbf{w}^{[nj]}\alpha_j^{[ni]}q^{[ni]}$ is caused by channel non-orthogonality of users when the number of antennas at the BS is not large enough, $n_4^{[kl]} = \sqrt{\frac{\rho_d}{\gamma}}\sum_{j=1}^{L}\sum_{i=1}^{L}(\hat{\mathbf{g}}_j^{[kl]}\mathbf{w}^{[kj]} - E\{\hat{\mathbf{g}}_j^{[kl]}\mathbf{w}^{[kj]}\})\alpha_j^{[ki]}q^{[ki]}$ is from the channel uncertainty brought by the statistical average of CSI. The signal-to-noise-ratio (SINR) with jointly considering the above interferences and noise can be expressed as

$$SINR^{[kl]} = \frac{\left|\sum_{j=1}^{L}\beta_j^{[kl]}\alpha_j^{[kl]}\right|^2}{D_1+\frac{1}{M}D_2} \triangleq \frac{\eta^{[kl]}}{\sigma^{[kl]}} \quad (5)$$

where $D_1 = \sum_{i=1,i\neq l}^{L}\left|\sum_{j=1}^{L}\beta_j^{[kl]}\alpha_j^{[ki]}\right|^2$, $D_2 = \sum_{j=1}^{L}\sum_{n=1}^{K}\left(\frac{1}{L\rho_d}+\beta_j^{[kl]}\right)\left(\frac{1}{\rho_u\tau}+\sum_{s=1}^{L}\beta_j^{[ns]}\right)\sum_{i=1}^{L}|\alpha_j^{[ni]}|^2$.

The lower bound on achievable sum rate of the system can be expressed as

$$R = \sum_{l=1}^{L}\sum_{k=1}^{K}\log\left(1+\frac{\eta^{[kl]}}{\sigma^{[kl]}}\right) \quad (6)$$

Define $\mathbf{A}$ as the PCP matrix which is a block diagonal matrix with $\mathbf{A}_{(k-1)L+j,(k-1)L+l} = \alpha_j^{[kl]}$, the PCP matrix $\mathbf{A}$ maximizing the sum rate can be designed as

$$\mathbf{A}_{\text{MSR}} = \arg\max_{\mathbf{A}} R \quad (7)$$

Theorem 1: The optimal $\mathbf{A}_{\text{MSR}}$ is of the expression $\mathbf{A}_{\text{MSR}} = (\mathbf{B}^*\mathbf{DB}+\mathbf{CN})^{-1}\mathbf{E}$, where the block diagonal matrices $\mathbf{B}$, $\mathbf{E}$ and the diagonal matrices $\mathbf{C}$, $\mathbf{D}$, $\mathbf{N}$ are defined as

$$\begin{aligned}
&\mathbf{B}_{(k-1)L+l,(k-1)L+j} = \beta_j^{[kl]},\\
&\mathbf{B} \triangleq \text{diag}\left[\mathbf{B}^{[1]},\mathbf{B}^{[2]},\cdots,\mathbf{B}^{[K]}\right],\\
&[\mathbf{P}_j]_{((k-1)L+l,(k-1)L+l)} = \tfrac{1}{L\rho_d}+\beta_j^{[kl]},\\
&\delta^{[kl]} = \tfrac{\sum_{j=1}^{L}\beta_j^{[kl]}\alpha_j^{[kl]}}{den^{[kl]}},\\
&\Delta^{[k]} = \text{diag}\left[\delta^{[k1]},\delta^{[k2]},\cdots,\delta^{[kL]}\right],\\
&\mathbf{E}=\left[\mathbf{B}^{[1]}\Delta^{[1]},\mathbf{B}^{[2]}\Delta^{[2]},\cdots,\mathbf{B}^{[K]}\Delta^{[K]}\right]^T,\\
&d^{[kl]} = \tfrac{\eta^{[kl]}}{\sigma^{[kl]}(\eta^{[kl]}+\sigma^{[kl]})},\ \mathbf{D}_{(k-1)L+l,(k-1)L+l} = d^{[kl]},\\
&n^{[kl]} = \tfrac{1}{\rho_u\tau}+\sum_{s=1}^{L}\beta_j^{[ks]},\ \mathbf{N}_{(k-1)L+l,(k-1)L+l} = n^{[kl]},\\
&\mathbf{O}_{j,j} = \text{Tr}(\mathbf{P}_j\mathbf{D}),\ \mathbf{C} = \text{diag}\left[\mathbf{O},\mathbf{O},\cdots,\mathbf{O}\right].
\end{aligned}$$

Proof: If $\mathbf{A}_{\text{MSR}}$ is the optimal solution of (7), it must satisfies

$$\begin{aligned}
\frac{\partial R}{\partial\alpha_{[a]}^{[bc]}} =& \sum_{l=1}^{L}\sum_{k=1}^{K}\frac{(\eta^{[kl]})'\sigma^{[kl]} - \eta^{kl}(\sigma^{[kl]})'}{\left(\eta^{[kl]}+\sigma^{[kl]}\right)\sigma^{[kl]}}\\
=&\frac{\beta_a^{[bc]}\left|\sum_{j=1}^{L}\beta_j^{[bc]}\alpha_j^{[bc]}\right|}{\sigma^{[bc]}}\\
&-\sum_{l=1}^{L}\frac{\eta^{[bl]}\beta_a^{[bl]}\left|\sum_{j=1}^{L}\beta_j^{[bc]}\alpha_j^{[bc]}\right|}{\sigma^{[bl]}(\sigma^{[bl]}+\eta^{[bl]})}\\
&-\sum_{k=1}^{K}\sum_{l=1}^{L}\frac{1}{M}d^{[kl]}n^{ba}|\alpha_a^{[bc]}|\left(\frac{1}{L\rho_f}+\beta_a^{[kl]}\right)\\
=&0
\end{aligned}$$

Rewrite the above equation in matrix form as $\mathbf{E}-\mathbf{B}^*\mathbf{DBA}-\mathbf{CNA}=\mathbf{0}$ and we can get $\mathbf{A}=(\mathbf{B}^*\mathbf{DB}+\mathbf{CN})^{-1}\mathbf{E}$. ■

Since the PCP matrix $\mathbf{A}$ is embedded in $\mathbf{C}$ $\mathbf{D}$ and $\mathbf{E}$, it is very difficult to solve $\mathbf{A}$ out. Inspired by the methodology in [6], we develop an iterative algorithm to search for a suboptimal solution, which is summarized in Algorithm 1. The complexity mainly depends on the iterative calculations of $KL \times KL$ matrix inverse, which need around $\mathcal{O}(N_{iter}K^3L^3)$ operations with N_{iter} denoting the number of iterations.

Algorithm 1 Iterative MSR-PCP design algorithm

Given $\mathbf{B}$, initiate $\mathbf{A}_0 = \mathbf{B}^{-1}, i=1, R_{-2}=10, R_{-1}=100$,
Repeat while $|R_{i-1}-R_{i-2}| \geq 10^{-3}$.
1) $\mathbf{A}_i = (\mathbf{B}^*\mathbf{D}_i\mathbf{B}+\mathbf{C}_i\mathbf{N})^{-1}\mathbf{E}_i$.
2) $\eta^{[kl]} = \left|[\mathbf{BA}_i]_{(k-1)L+l,(k-1)L+l}\right|^2$,
$\mathbf{Z}_{j,j}^{[kl]} = \frac{1}{\rho_d L}+\beta_j^{[kl]}$, $\mathbf{Q}^{[kl]} = \text{diag}\left[\mathbf{Z}^{[kl]},\mathbf{Z}^{[kl]},\cdots,\mathbf{Z}^{[kl]}\right]$.
$\sigma^{[kl]} = \sum_{v\neq l}\left|(\mathbf{BA}_i)_{(k-1)L+l,(k-1)L+v}\right|^2 + \frac{1}{M}\text{Tr}((\mathbf{A}_i)^*\mathbf{Q}^{[kl]}\mathbf{NA}_i)$.
3) $R_i = \sum_{l=1}^{L}\sum_{k=1}^{K}\log\left(1+\frac{\eta^{[kl]}}{\sigma^{[kl]}}\right)$,
4) $d_{i+1}^{[kl]} = \frac{\eta^{[kl]}}{\sigma^{[kl]}(\eta^{[kl]}+\sigma^{[kl]})}$, $\delta_{i+1}^{[kl]} = \frac{\sum_{j=1}^{L}\beta_j^{[kl]}\alpha_j^{[kl]}}{\sigma^{[kl]}}$.
5) Calculate $\mathbf{D}_{i+1}$,$\mathbf{C}_{i+1}$,$\mathbf{E}_{i+1}$.
6) $i=i+1$
end

TABLE I. SIMULATION SETUPS

Number of cells	$L = 3$
Users per cell	$K = 5$
Radius of cell	1 km
ρ_d	10 W
ρ_u	1 W
Large scale fading factor $\beta_j^{[kl]}$	$\left(d_j^{[kl]}\right)^{-3.5}$

$d_j^{[kl]}$ is the distance between user k in cell l and BS j

D. Simulation results and analysis

The performance of MSR-PCP of a multi-cell massive MIMO system whose setups are listed in Table 1 is investigated via Monte carlo simulations. in addition, we assume that the length of the pilot sequences is equal to the number of users in one cell and the same pilot sequences are reused among all the cells. The sum rate performance is included in Fig. 2. For comparison purpose, performance of ZF-PCP is also included as a baseline. All the sum rates are obtained under the assumption that all the users are uniformly distributed inside the cells. It is observed that MSR-PCP brings considerable improvement on the same rate for all the depicted antenna number regime compared with ZF-PCP and no PCP schemes. This is because the proposed MSR-PCP considers not only the interference but also the noise, while the ZF-PCP aims only to cancel the interference. Considering that if the pilot sequences are reused among cells, the users allocated in the cell edge area suffer most, we also simulated the sum rate performance when users are located at the cell edges, results are included in Fig. 3. Similar conclusion can be made and much more improvement on sum rate can be achieved by the proposed MSR-PCP. The superiority of the proposed scheme is most obvious when there is small number BS antennas compared with ZF-PCP, especially when users suffer from strong interference i.e., all the users are located in the cell edge area. This is because ZF-PCP may enhance the interferences, but the MSR-PCP will not. This is attribute to the fact that MSR-PCP design maximizes the sum rate considering all the factors that limits the transmission rate of users.

The coverage property of Algorithm 1 is also investigated. Since we cannot theoretically prove that the algorithm is convergent. We show the convergence through five different channel realizations, results are in Fig. 4. It shows that Algorithm converges after 30 to 80 iterations. Therefore, compared with ZF-PCP whose complexity is $\mathcal{O}(KL^3)$, the increased complexity of MSR-PCP is still acceptable.

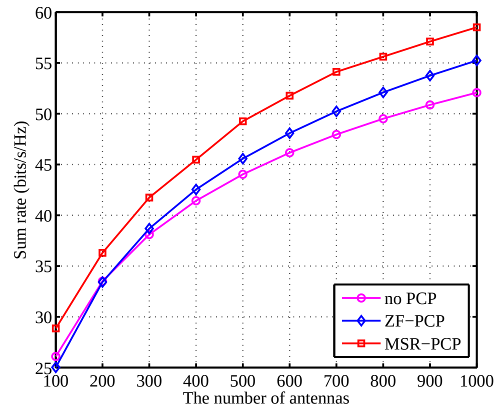

Fig. 2. Sum rate comparison of systems with various PCP techniques when users are uniformly distributed inside the cells.

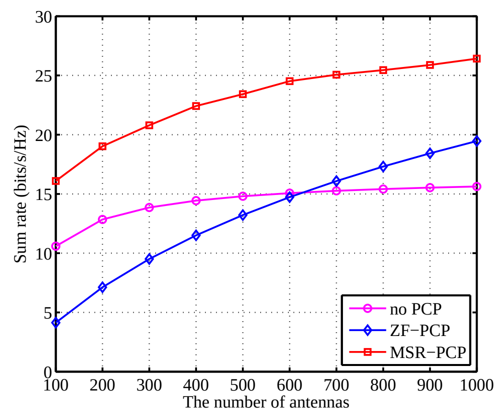

Fig. 3. Sum rate comparison of systems with various PCP techniques when users are located at the cell edges.

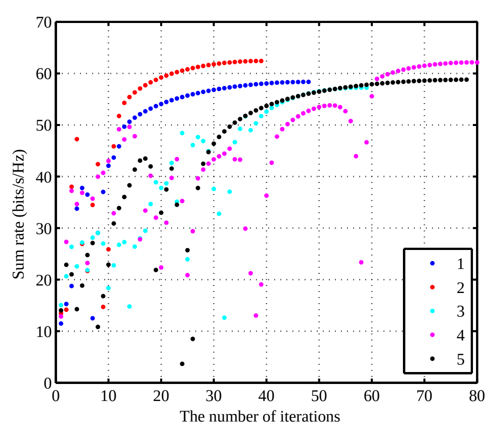

Fig. 4. Sum rate values during the procedure of Algorithm 1

REFERENCES

[1] A. Ashikhmin and T. L. Marzetta,"Pilot contamination precoding in multi-cell large scale antenna systems," in *Proc. IEEE Int. Symp. on Inf. Theory (ISIT12)*, Cambridge, MA,, pp. 1137-1141, July 2012.

[2] J. Zuo, J. Zhang, C. Yuen, W. Jiang, and W. Luo, "Multi-cell multi-user massive MIMO transmission with downlink training and pilot contamination precoding," *IEEE Trans. Vec. Technol.* , 2015. [online].Available: http://ieeexplore.ieee.org/stamp/stamp.jsp?tp=&arnumber=7234921.

[3] J.Choi and J.Ha,"On the Estimation of Slow-Fading Coefficients for Pilot Contamination Precoding," in *Proc. IEEE VTC (Spring)*, pp. 1 - 5, May 2014.

[4] M.Mazrouei-Sebdani and W.A.Krzymien,"Massive MIMO with clustered pilot contamination precoding," in *Proc. IEEE ACSSC*, pp. 1218 - 1222, Nov. 2013.

[5] L. Li, A. Ashikhmin and T. L. Marzetta. "Pilot contamination precoding for interference reduction in large scale antenna systems." *Communication, Control, and Computing (Allerton)*, pp. 226- 232, Oct. 2013.

[6] M. Stojnic, H. Vikalo, and B. Hassibi, "Rate maximization in multi-antenna broadcast channels with linear preprocessing," *IEEE Trans. on wireless commun.*, vol. 5, no. 9, pp. 2338-2342, Sep. 2006.