Distributed Assignment and Resource Allocation for Energy Efficiency in MIMO Wireless Networks

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***Abstract*—This paper deals with the problem of distributed resource allocation in multi-carrier MIMO networks, for energy efficiency maximization. The user-subcarrier assignment is jointly allocated together with the users' transmit powers. To this end, a novel approach is proposed which merges the popular Dinkelbach's algorithm with the framework of distributed auction theory. The resulting algorithm can be implemented in a distributed fashion, with very limited feedback overhead, and is guaranteed to converge to the global optimum of the system energy efficiency, within a predefined threshold which can be chosen arbitrarily small. Numerical results compare the proposed distributed algorithm to the optimal, centralized allocation, showing its merits both in terms of performance and computational complexity.**

I. Introduction

Energy efficiency is considered one key requirement of future 5G cellular networks in order to keep the energy consumption at today's levels. While the energy efficiency of a communication network can be optimized in a centralized manner, it requires the presence of a central controller with global channel state information (CSI) knowledge. This leads to large overheads, especially in large networks with many devices, which is anticipated to be the typical scenario for 5G networks. In order to minimize the overhead involved in energy efficiency optimization it is desirable to preform most of the computation locally at the mobile units. In this case distributed protocols are needed to maximize the energy efficiency of the network.

Many approaches have been recently suggested for distributed allocation and subcarrier selection to optimize the total rate of orthogonal multi-carrier systems. A non-exhaustive, short list of the works most closely related to the approach we propose in this paper is reviewed here. The study of decentralized techniques for spectrum allocation was tackled in [1]–[3] by means of non-cooperative game theory, whereas references [4], [5] combine the tools of stable matching with an opportunistic version of carrier sensing [6] to propose distributed algorithms for sum-rate maximization. Instead, [7] employs bargaining theory, a branch of cooperative game theory, in multi-channel ALOHA systems. Finally, [8] employs the theory of distributed auction, always in conjunction with the opportunistic carrier sensing approach from [6], to obtain distributed, near-optimal sum-rate maximization algorithms.

However, all these previous works, and references therein, do not consider the issue of energy efficiency optimization. Motivated by this background, in this work we consider a single-cell, multi-carrier system, and propose a fully distributed algorithm for energy efficiency maximization. Specifically, the following main contributions are made:

- A distributed algorithm for energy efficiency maximization is developed, by combining fractional programming theory with the tool of distributed auction. The base station and the mobile units work in a master slave mode, where the base station collects minimal amount of information from the mobile units and sends in response a parameter. The mobile units use this parameter to solve a local problem using a fully distributed protocol based on carrier sensing. This approach fits very well to multi-frequency access points in Wi-Fi as well as uplink cellular networks.
- The performance of the proposed distributed method is compared with the global optimum solution, both in terms of achieved energy efficiency and of overhead requirements. It is shown that the proposed algorithm is guaranteed to be globally optimal within a pre-defined tolerance which can be made arbitrarily small, while at the same time requiring much less feedback overhead.

II. System model and problem statement

Consider a single-cell multi-carrier wireless network with K users and N available resource blocks. Each user is equipped with N_T antennas, whereas N_R antennas are deployed at the base station. Each user is allowed to transmit over one resource block, and each resource block can be assigned to only one user. Denote by $\boldsymbol{Q}_{k,n}$ and $\boldsymbol{H}_{k,n}$ the $N_T \times N_T$ transmit covariance matrix and the $N_R \times N_T$ propagation channel of user k over resource block n, respectively. Finally, define $\alpha_{k,n}$ as the binary variable which equals 1 if user k transmits on resource block n, and 0 otherwise. Given this notation, the k-th user's achievable rate on resource block n is expressed as:

$$R_{k,n} = \alpha_{k,n} B \log_2 \left| \boldsymbol{I}_{N_R} + \rho \boldsymbol{H}_{k,n} \boldsymbol{Q}_{k,n} \boldsymbol{H}_{k,n}^H \right| \,, \qquad (1)$$

wherein B is the subcarrier bandwidth and $\rho = 1/\sigma^2$, with σ^2 the noise power at the receiver. In order to guarantee the achievable rate in (1), user k needs to consume the following power on resource block n:

$$P_{k,n} = \alpha_{k,n} (\mu_{k,n} \mathrm{tr}\left(\boldsymbol{Q}_{k,n}\right) + \theta_{k,n}) \,, \qquad (2)$$

wherein $\mu_{k,n}$ is the inverse of the amplifier efficiency on resource block n, while $\theta_{k,n}$ is the static power dissipated in all other hardware blocks associated to the n-th transmit-receive chain.

The system global energy efficiency (GEE) is defined as the ratio between the system achievable sum-rate and total power consumption [9]. Based on (1) and (2), this leads to:

$$\text{GEE} = \frac{B\sum_{k=1}^{K}\sum_{n=1}^{N}\alpha_{k,n}\log_2\left|\boldsymbol{I}_{N_R}+\rho\boldsymbol{H}_{k,n}\boldsymbol{Q}_{k,n}\boldsymbol{H}_{k,n}^H\right|}{\sum_{k=1}^{K}\sum_{n=1}^{N}\alpha_{k,n}(\mu_{k,n}\text{tr}\left(\boldsymbol{Q}_{k,n}\right)+\theta_{k,n})} \, . \tag{3}$$

It should be remarked that the GEE in (3) is measured in bit/Joule, and represents the system benefit-cost ratio in terms of amount of bits reliably transmitted, and corresponding total consumed power.

In this context, the GEE maximization problem can be cast as the problem of finding the optimal assignment $\{\alpha_{k,n}\}_{k,n}$ of the K users to the N available resource blocks, as well as the optimal users' transmit covariance matrices $\{\boldsymbol{Q}_{k,n}\}_{k,n}$ in order to maximize (3). Mathematically, the problem is formulated as the mixed-integer optimization program:

$$\max_{\boldsymbol{\alpha},\boldsymbol{Q}} \; \frac{B\sum_{k=1}^{K}\sum_{n=1}^{N}\alpha_{k,n}\log_2\left|\boldsymbol{I}_{N_R}+\rho\boldsymbol{H}_{k,n}\boldsymbol{Q}_{k,n}\boldsymbol{H}_{k,n}^H\right|}{\sum_{k=1}^{K}\sum_{n=1}^{N}\alpha_{k,n}(\mu_{k,n}\text{tr}\left(\boldsymbol{Q}_{k,n}\right)+\theta_{k,n})} \tag{4a}$$

$$\text{s.t.} \; \sum_{k=1}^{K}\alpha_{n,k}=1 \;, \forall\, n=1,\ldots,N \tag{4b}$$

$$\sum_{n=1}^{N}\alpha_{n,k}=1 \;, \forall\, k=1,\ldots,K \tag{4c}$$

$$\alpha_{k,n}\in\{0,1\} \;\forall\, k=1,\ldots,K \;,\; n=1,\ldots,N \tag{4d}$$

$$\text{tr}\left(\boldsymbol{Q}_{k,n}\right)\leq P_{max_k} \;, \forall\, k=1,\ldots,K \;,\; n=1,\ldots,N \;, \tag{4e}$$

wherein $\boldsymbol{\alpha}=\{\alpha_{k,n}\}_{k,n}$ and $\boldsymbol{Q}=\{\boldsymbol{Q}_{k,n}\}_{k,n}$, with $k=1,\ldots,K$ and $n=1,\ldots,N$. In (4), Constraints (4b) and (4c) ensure that each resource block is assigned to only one user and that each user transmit on only one resource block, Constraint (4d) accounts for the binary nature of the assignment variables, while Constraint (4e) represents a per-user maximum power constraint.

The main goal of this paper is to provide a fully distributed algorithm to solve Problem (4) in a near-optimal way. This will be accomplished in the coming Section. The global solution of (4) is also derived by a centralized approach, which will be used for benchmarking purposes.

III. Distributed GEE maximization

Problem (4) is an instance of a single-ratio fractional problem, and therefore it can be tackled by means of fractional programming tools [9]. One challenge in deriving a distributed solution of (4) is related to the particular structure of the objective which is the ratio of two sums, thereby making it difficult to separate the terms associated to different users or resource blocks. However, this difficulty can be overcome by exploiting one fractional programming method, namely Dinkelbach's algorithm.

The theoretical foundation of Dinkelbach's algorithm is the following result from [10].

Proposition 1. *Consider Problem* (4) *and denote by $\mathcal{F}$ its feasible set. Define also the auxiliary function $F:\lambda\in\mathbb{R}\to F(\lambda)$ as*

$$F(\lambda)=\max_{(\boldsymbol{\alpha},\boldsymbol{Q})\in\mathcal{F}}\sum_{k=1}^{K}\sum_{n=1}^{N}R_{k,n}(\boldsymbol{Q}_{k,n},\alpha_{k,n})-\lambda P_{k,n}(\boldsymbol{Q}_{k,n},\alpha_{k,n}). \tag{5}$$

Then, a pair $(\boldsymbol{\alpha}^*,\boldsymbol{Q}^*)$ is a global solution of (4) *if and only if $F(\lambda^*)=0$, with λ^* being the maximum value of the objective of* (4)*, i.e. $\lambda^*=\text{GEE}(\boldsymbol{\alpha}^*,\boldsymbol{Q}^*)$.*

In words, this result establishes that solving a fractional problem is equivalent to finding the zero of the auxiliary function $F(\lambda)$. One remark is in order.

Remark 1. *The original result from [10] assumed that the numerator and denominator of the fractional function to maximize be continuous, and the constraint set compact. These assumption are clearly not fulfilled for Problem* (4)*, since the assignment variables are discrete. However, in our case the result is still valid. Indeed, the continuity and compactness assumption in the original results from [10] were required to make sure that both the original fractional problem and the auxiliary function F are well-defined. For the case at hand, this is still true, because both* (4) *and* (5) *admit a maximizer. Indeed, since the objectives are continuous in $\boldsymbol{Q}$ and the set of the feasible $\boldsymbol{Q}$ is compact, it holds that for each fixed assignment $\bar{\boldsymbol{\alpha}}$, an optimal $\bar{\boldsymbol{Q}}$ exists. In turn, this implies the existence of a solution for both* (4) *and* (5)*, because the number of possible subcarrier assignments is finite.*

Dinkelbach's algorithm is an iterative algorithm able to find the zero of the auxiliary function $F(\lambda)$, by solving a sequence of Problems of the form in (5), updating the parameter λ after each iteration. The formal pseudo-code is reported here.

Algorithm 1 Dinkelbach's algorithm

Set $\varepsilon>0$; $j=0$; $\lambda_j=0$; $F(\lambda_j)=c>\varepsilon$;

while $F(\lambda_j)\geq\varepsilon$ **do**

$(\boldsymbol{\alpha}^*,\boldsymbol{Q}^*) = \arg\max_{\{\boldsymbol{\alpha},\boldsymbol{Q}\}\in\mathcal{F}} \sum_{k=1}^{K}\sum_{n=1}^{N}\Big\{R_{k,n}(\boldsymbol{Q}_{k,n},\alpha_{k,n}) - \lambda P_{k,n}(\boldsymbol{Q}_{k,n},\alpha_{k,n})\Big\}$;

$F(\lambda_j)=\sum_{k=1}^{K}\sum_{n=1}^{N}\Big\{R_{k,n}(\boldsymbol{Q}_{k,n}^*,\alpha_{k,n}^*)-\lambda P_{k,n}(\boldsymbol{Q}_{k,n}^*,\alpha_{k,n}^*)\Big\}$;

$\lambda_{j+1}=\frac{\sum_{k=1}^{K}\sum_{n=1}^{N}R_{k,n}(\boldsymbol{Q}_{k,n}^*,\alpha_{k,n}^*)}{\sum_{k=1}^{K}\sum_{n=1}^{N}P_{k,n}(\boldsymbol{Q}_{k,n}^*,\alpha_{k,n}^*)}$;

$j=j+1$;

end while

At a first sight, Proposition 1 does not seem to make (4) easier, since it requires to solve a sequence of mixed-integer problems of the form of (5). However, unlike the fractional form in (4), the sum-based objective in (5) can be maximized in a decentralized fashion. The first step towards this goal is to observe that thanks to its sum-based form, the objective in (5) can be decoupled with respect to the users' covariance matrices. Indeed, for any given $\boldsymbol{\alpha}$, each user k will transmit over the assigned subcarrier n with the covariance matrix which maximizes Summand (k,n) in (5). Otherwise stated, for any (k,n), the optimal covariance matrix $\boldsymbol{Q}_{k,n}$ is the solution of the convex problem:

$$\max_{\boldsymbol{Q}_{k,n}} B\log_2\left|\boldsymbol{I}_{N_R}+\rho\boldsymbol{H}_{k,n}\boldsymbol{Q}_{k,n}\boldsymbol{H}_{k,n}^H\right|-\lambda\mu_{k,n}\mathrm{tr}\left(\boldsymbol{Q}_{k,n}\right) \tag{6a}$$

$$\text{s.t. } \mathrm{tr}\left(\boldsymbol{Q}_{k,n}\right)\leq P_{max}\ ,\ \boldsymbol{Q}_{k,n}\succeq\boldsymbol{0}\ . \tag{6b}$$

Since the linear term in (6b) does not depend on the eigenvectors of $\boldsymbol{Q}_{k,n}$, it follows that the optimal eigenvectors of $\boldsymbol{Q}_{k,n}$ should diagonalize $\boldsymbol{H}_{k,n}$. Moreover, the optimal eigenvalues of $\boldsymbol{Q}_{k,n}$ can be determined by solving the resulting water-filling problem [11]. After solving (6) for each k and n, each user k is left with a set of N optimal covariance matrices $\{\bar{\boldsymbol{Q}}_{k,n}\}_{n=1}^N$, wherein $\bar{\boldsymbol{Q}}_{k,n}$ represents the optimal covariance matrix should user k transmit over subcarrier n. As a consequence, the joint problem of allocating both the assignment vector $\boldsymbol{\alpha}$ and the covariance matrices $\boldsymbol{Q}$, can be recast as a maximization with respect to $\boldsymbol{\alpha}$ only:

$$\max_{\boldsymbol{\alpha}}\sum_{k=1}^K\sum_{n=1}^N\alpha_{k,n}\Big[B\log_2\left|\boldsymbol{I}_{N_R}+\rho\boldsymbol{H}_{k,n}\bar{\boldsymbol{Q}}_{k,n}\boldsymbol{H}_{k,n}^H\right| -\lambda(\mu_{k,n}\mathrm{tr}\left(\bar{\boldsymbol{Q}}_{k,n}\right)+\theta_{k,n})\Big]\ . \tag{7}$$

As a result, the joint maximization with respect to $(\boldsymbol{\alpha},\boldsymbol{Q})$ in (5), has been reduced to a pure assignment problem in which the utility of each user k over resource block n is given by

$$u_{k,n}=B\log_2\left|\boldsymbol{I}_{N_R}+\rho\boldsymbol{H}_{k,n}\bar{\boldsymbol{Q}}_{k,n}\boldsymbol{H}_{k,n}^H\right| -\lambda(\mu_{k,n}\mathrm{tr}\left(\bar{\boldsymbol{Q}}_{k,n}\right)+\theta_{k,n})\ . \tag{8}$$

The advantage of this reformulation is that assignment problems of the form of (7) can be globally solved in a fully decentralized and parallel fashion, by means of the Distributed Auction Algorithm [8].

A. Distributed auction algorithm

In this section we describe the main idea behind the distributed auction algorithm. For the full details, we refer to [8]. The distributed auction is part of the more general auction algorithm framework, which provides a method for user-subcarrier association inspired to auction dynamics. Like in a real auction, the algorithm is composed of two stages which are iteratively repeated, the *bidding stage* and the *assignment stage*. In the bidding stage, each user who is not yet assigned to a subcarrier raises the price of the subcarrier he wishes to acquire. In the assignment stage, every subcarrier is assigned to the highest bidder. To elaborate further, let us define the $K\times N$ matrix $\boldsymbol{B}$, which collects the bids of all users for all subcarriers at a given round of the auction. Then, the price ρ_n of subcarrier n is expressed as

$$\rho_n=\max_k\boldsymbol{B}_{n,k}\ , \tag{9}$$

i.e. the highest bid among the users. User k is happy with subcarrier n_k when

$$u_{k,n_k}-\rho_{n_k}\geq\max_n\{u_{k,n}-\rho_n\}-\delta\ , \tag{10}$$

with $u_{k,n}$ given by (8). Thus, user k is satisfied with subcarrier n_k, when the profit he makes by choosing subcarrier n_k is higher than the profit he would obtain with any other subcarrier, up to some threshold δ. The auction terminates when, after an assignment stage, all users are happy with their subcarriers. In [12], this approach was proved to converge in a finite number of steps[1], and to achieve the optimal assignment within the threshold δ. However, such an approach is centralized, in the sense that it requires full CSI to be implemented, because each user needs to know the price of each subcarrier. Instead, in [8] a fully distributed implementation of the auction algorithm was proposed, in which each user makes bids based only on local prices. This distributed implementation of the auction algorithm is called distributed auction, and it retains the pleasant property of converging to the optimal assignment up to a threshold δ, in a finite number of steps.

B. Distributed implementation

By embedding the distributed auction into Dinkelbach's algorithm, it is possible to implement Algorithm 1 in a decentralized fashion, with very limited feedback requirements. The distributed implementation is based on four main steps:

- Each user computes the optimal covariance matrices $\{\bar{\boldsymbol{Q}}_{k,n}\}_{n=1}^N$ over the N possible subcarrier choices. This step can be performed locally and in parallel by the different users, since it only requires each user k to know his own channels $\{\boldsymbol{H}_{k,n}\}_{n=1}^N$, which are locally available.
- The distributed auction algorithm is used to compute the optimal assignment $\boldsymbol{\alpha}^*$ for Problem (7). Let n_k^* be the optimal subcarrier choice by user k, then the k-th user's optimal covariance matrix is given by $\boldsymbol{Q}_k^*=\bar{\boldsymbol{Q}}_{k,n_k^*}$.
- Each user k computes and feeds back $R_{k,n_k^*}=\log_2\left|\boldsymbol{I}_{N_R}+\rho\boldsymbol{H}_{k,n_k^*}\bar{\boldsymbol{Q}}_{k,n_k^*}\boldsymbol{H}_{k,n_k^*}\right|$ and $P_{k,n_k^*}=\mu_{k,n_k^*}\mathrm{tr}(\bar{\boldsymbol{Q}}_{k,n_k^*})+\theta_{k,n_k^*}$.
- The base station updates and broadcasts λ. The process loops until the base station does not broadcast λ any more because convergence has been reached.

C. Global optimum of the GEE

After describing how Algorithm 1 can be implemented in a distributed fashion, let us briefly describes how to globally solve the GEE maximization problem by a centralized implementation of Algorithm 1. To this end, let us consider Problem (7) to be solved in the generic iteration of Dinkelbach's algorithm and recall that the feasible set $\mathcal{F}$ is composed of Constraints (4b), (4c), and (4d). Now, the difficulty in solving (7) directly is due to the integer constraint (4d). However, it can be observed that the constraint matrix of (4b), (4c), and (4d) is totally unimodular, thus implying that no loss

[1]The number of requires step can be upper-bounded by a quantity inversely proportional to δ.

of optimality is incurred by relaxing (4d) to the continuous constraint $\alpha_{k,n} \geq 0$, for all $k = 1, \ldots, K$ and $n = 1, \ldots, N$. Upon doing this, Problem (7) reduces to a simple linear problem in $\boldsymbol{\alpha}$, which can be solved by standard methods. However, it should be stressed that in order to implement Algorithm 1 in this fashion, a centralized controller with global CSI is required.

D. Overhead and performance comparison

The distributed implementation of Algorithm 1 as described in Section III-B requires to feedback $2K + 1$ real numbers for each iteration. So, denoting by I the total number of iterations of Algorithm 1, the total amount of data to feedback is $I(2K + 1)$ real numbers. As for the value of I, although general formulas are not available, it should be stressed that Dinkelbach's algorithm is guaranteed to have a super-linear convergence rate, which typically results in convergence in a handful of iterations. The numerical results to be provided in Section IV corroborate this point.

On the other hand, a centralized implementation would require the knowledge of all channels $\{\boldsymbol{H}_{k,n}\}_{k,n}$, for a total of $2N_R N_T K N$ real numbers to feedback[2]. So, for typical values of N and K, the feedback required for a centralized implementation of Algorithm 1 is much higher than for the proposed distributed implementation.

Finally, as for the performance of Algorithm 1, the following result holds.

Proposition 2. *Algorithm 1 converges to the optimal solution of* (4)*, up to the tolerance δ with which the distributed auction algorithm converges to the optimal subcarrier assignment of* (7).

As a consequence of this result, we have that the distributed implementation of Dinkelbach's algorithm converges to the global maximum of the system GEE, up to a pre-defined tolerance which can be made small at will.

IV. Numerical Results

In our numerical simulations we have considered the uplink of a cellular system in which $K = 10$ users are randomly placed in a circular area of radius $R = 500\,\text{m}$. The service base station is placed at the center of the area to cover, and the number of available resource blocks is $N = 10$. Each user is equipped with $N_T = 3$ antennas, and $N_R = 3$ antennas are deployed at the base station. The channel from the generic user k to the base station over sub-carrier n has been generated as

$$\boldsymbol{H}_{k,n} = f(d_k, \eta)\boldsymbol{\Sigma}_{k,n} \;, \tag{11}$$

wherein $\boldsymbol{\Sigma}_{k,n}$ is a realization of an $N_R \times N_T$ random matrix with zero-mean and unit-variance complex Gaussian entries, which accounts for the fast fading between user k and the base station over sub-carrier n, whereas $f(d_k, \eta)$ is a scalar coefficient modeling the path-loss as a function of the distance d_k between user k and the base station, and of the power decay factor η. In particular, the path-loss model in [13] has been used, with $\eta = 3.5$. The remaining system parameters have been set as in typical LTE systems [14]. Specifically, the receive noise power has been generated as $\sigma^2 = \mathcal{N}_0 BF$, wherein $\mathcal{N}_0 = -174\,\text{dBm/Hz}$ is the noise power spectral density, $B = 180$ kHz is the communication bandwidth, and $F = 3\,\text{dB}$ is the receiver noise figure. All power amplifier efficiency factors and static circuit power consumption terms have been assumed equal across users and sub-carriers, namely $\mu_{k,n} = 3.8$, $\theta_{k,n} = -20$ dBW, for all k, n. All numerical illustrations have been obtained by averaging over 10^3 independent system scenarios.

In Fig. 1, the maximum feasible transmit power has been assumed equal for all users, i.e. $P_{max,k} = P_{max}$, and the achieved GEE versus P_{max} is illustrated for the following schemes:

(a) Joint assignment and covariance matrix optimization by the proposed distributed implementation of Algorithm 1 described in Section III-B
(b) Joint assignment and covariance matrix optimization by the centralized, optimal implementation of Algorithm 1 described in Section III-C
(c) GEE maximization by covariance matrix optimization for a fixed assignment. In this scenario, the user-resources assignment is randomly selected, and based on this assignment, optimal covariance matrix allocation is performed.

The results indicate that schemes (a) and (b) significantly outperform scheme (c), thereby showing that assignment optimization can bring a relevant performance improvement. Moreover, schemes (a) and (b) perform virtually the same, thus implying that the proposed distributed allocation algorithm is able to achieve the same performance as the optimal, centralized scheme, while at the same time requiring a much lower feedback overhead.

Next, Table I considers the complexity of the proposed distributed GEE maximization algorithm, in comparison with its centralized counterpart. Specifically, Table I reports the number of outer iterations required for the two algorithms to reach convergence, for different values of P_{max}. For both algorithms, the tolerance on the auxiliary parameter λ is set to $\varepsilon = 10^{-3}$. Similarly, the threshold value for the distributed auction has been set to $\delta = 10^{-3}$. It is seen that both algorithms converge in a comparable and limited number of iterations. Thus, the proposed distributed implementation of Algorithm 1 easily lends itself to being implemented in practical networks.

TABLE I
$N_T = N_R = 3$, $N = K = 10$; Average number of iterations needed for the distributed and centralized implementation of Algorithm 1 to converge versus P_{max}. Convergence is declared when $\varepsilon \leq 10^{-3}$.

Algorithm 1	Distributed	Centralized
$P_{max} = -40\,\text{dBW}$	2	2
$P_{max} = -35\,\text{dBW}$	2	2
$P_{max} = -30\,\text{dBW}$	2.01	2.01
$P_{max} = -25\,\text{dBW}$	2.49	2.52
$P_{max} = -20\,\text{dBW}$	4.01	4.04
$P_{max} = -15\,\text{dBW}$	4.98	4.99
$P_{max} = -10\,\text{dBW}$	6.03	6.05

[2]Recall that the entries of the channel matrices are complex numbers.

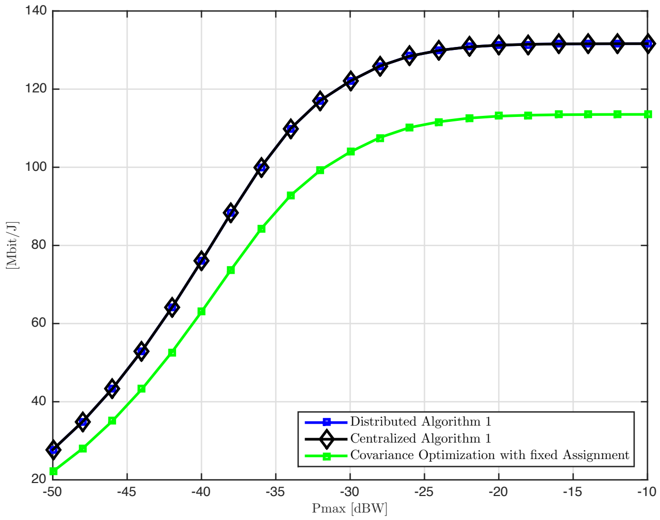

Fig. 1. $N_T = N_R = 3$, $N = K = 10$; Achieved GEE versus P_{max} for: (a) Distributed GEE maximization; (b) Centralized GEE maximization; (c) Covariance optimization for fixed assignment.

References

[1] Z. Han, Z. Ji, and K. Liu, “Fair multiuser channel allocation for ofdma networks using nash bargaining solutions and coalitions,” *Communications, IEEE Transactions on*, vol. 53, no. 8, pp. 1366–1376, 2005.

[2] A. Leshem and E. Zehavi, “Game theory and the frequency selective interference channel,” *Signal Processing Magazine, IEEE*, vol. 26, no. 5, pp. 28 –40, september 2009.

[3] E. Larsson, E. Jorswieck, J. Lindblom, and R. Mochaourab, “Game theory and the flat-fading gaussian interference channel,” *Signal Processing Magazine, IEEE*, vol. 26, no. 5, pp. 18 –27, september 2009.

[4] Y. Yaffe, A. Leshem, and E. Zehavi, “Stable matching for channel access control in cognitive radio systems,” in *Cognitive Information Processing (CIP), 2010 2nd International Workshop on*, june 2010, pp. 470 –475.

[5] A. Leshem, E. Zehavi, and Y. Yaffe, “Multichannel opportunistic carrier sensing for stable channel access control in cognitive radio systems,” *Selected Areas in Communications, IEEE Journal on*, vol. 30, no. 1, pp. 82–95, 2012.

[6] Q. Zhao and L. Tong, “Opportunistic carrier sensing for energy-efficient information retrieval in sensor networks,” *EURASIP Journal on Wireless Communications and Networking*, vol. 2005, no. 2, pp. 231–241, 2005.

[7] K. Cohen, A. Leshem, and E. Zehavi, “Game theoretic aspects of the multi-channel aloha protocol in cognitive radio networks,” *Selected Areas in Communications, IEEE Journal on*, vol. 31, no. 11, pp. 2276–2288, 2013.

[8] O. Naparstek and A. Leshem, “Fully distributed optimal channel assignment for open spectrum access,” *IEEE Transactions on Signal Processing*, vol. 62, no. 2, pp. 283–294, January 2014.

[9] A. Zappone and E. Jorswieck, “Energy efficiency in wireless networks via fractional programming theory,” *Foundations and Trends® in Communications and Information Theory*, vol. 11, no. 3-4, pp. 185–396, 2015.

[10] W. Dinkelbach, “On nonlinear fractional programming,” *Management Science*, vol. 13, no. 7, pp. 492–498, Mar. 1967.

[11] E. Jorswieck, H. Boche, and S. Naik, “Energy-aware utility regions: Multiple-access pareto boundary,” *IEEE Transactions on Wireless Communications*, vol. 9, no. 7, pp. 2216–2226, July 2010.

[12] D. Bertsekas, “A distributed algorithm for the assignment problem,” *Lab. for Information and Decision Systems Working Paper, MIT*, 1979.

[13] G. Calcev, D. Chizhik, B. Goransson, S. Howard, H. Huanga, A. Kogiantis, A. Molisch, A. Moustakas, D. Reed, and H. Xu, “A wideband spatial channel model for system-wide simulations,” *IEEE Trans. Veh. Technol.*, vol. 56, no. 2, March 2007.

[14] G. Auer, V. Giannini, C. Desset, I. Godor, P. Skillermark, M. Olsson, M. Imran, D. Sabella, M. Gonzalez, O. Blume *et al.*, “How much energy is needed to run a wireless network?” *IEEE Wireless Communications*, vol. 18, no. 5, pp. 40–49, 2011.