Abstract:
Channel Parameter Estimation for LOS MIMO Systems

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I. Introduction

Channel estimation and synchronization are fundamental tasks that need to be solved for every communications system. For MIMO systems this can become very complex due to the possibly high number of parameters that need to be estimated depending on the system setup. For fading channels there have been several investigations over years determining fundamental limits and viable training schemes that allow the estimation of the channel, as well as common/individual frequency offsets [1], [2], [3]. It was shown that performances close (or asymptotically equal) to the Cramér-Rao Bound (CRB) can be achieved, depending on the complexity of the algorithm involved and the length of the training sequence. In principal, the same techniques used in the literature can also be applied to line-of-sight (LOS) MIMO systems for parameter estimation. However, the inherent structure of the channel can be exploited in order to reduce estimation complexity and length of training sequences.

In this paper we will show some results on how to estimate channel coefficients and frequency offsets specifically for LOS channel MIMO systems that can use spatial multiplexing [4], which has rarely been considered in the literature. We will discuss suitable pilot sequences, estimation schemes and compare them to the fundamental limits.

II. System Model

Consider the narrowband received signal of a MIMO system in baseband to be defined by

$$y_m(t) = \sum_{n=1}^{N} h_{mn} \cdot x_n(t) \cdot e^{j2\pi \Delta f_{mn} t} e^{j\Delta\phi_{mn}} + n_m(t) \quad (1)$$

where $n = 1 \dots N$ and $m = 1 \dots M$ describes the index and number of transmit and receive antennas. Furthermore, h_{mn} is the channel coefficient between the corresponding antennas and $x_n(t)$ is the continuous information carrying waveform transmitted from the nth antenna. Additionally, Δf_{mn} and $\Delta\phi_{mn}$ denote phase and frequency differences between the different transmit and receiving antennas, given that each of them has a dedicated oscillator. The term $n_m(t)$ is additive noise with complex Gaussian distribution at the mth antenna. Note that the frequency offsets correspond to the normalized angular value, i.e., $\Delta f_{mn} = \frac{f_n - f_m}{f_s}$ where f_s is the symbol rate and f_n, f_m are the frequencies of the corresponding oscillators.

For the case of a common oscillator at transmitter and receiver this reduces to

$$y_m(t) = e^{j2\pi\Delta f t} e^{j\Delta\phi} \cdot \sum_{n=1}^{N} h_{mn} \cdot x_n(t) + n_m(t) \quad (2)$$

which should generally perform better as less parameters have to be estimated and compensated, but might not always be realizable in practice, e.g., due to a large number of antennas.

For a pure LOS channel the coefficients are determined through

$$\begin{aligned} h_{mn} &= a_{mn} \cdot \exp\left(-j2\pi f_n \cdot \tau_{mn}\right) && (3) \\ &= a_{mn} \cdot \exp\left(-j2\pi \frac{r_{mn}}{\lambda_n}\right) && (4) \end{aligned}$$

where a_{mn} is the corresponding attenuation coefficient and τ_{mn} is the propagation delay between antenna n and antenna m, which is given by the distance between the antennas r_{mn} and the wavelength of the nth transmit oscillator $\lambda_n = c/f_n$ where c is the speed of light. The value of a_{mn} should in a LOS scenario be approximately equal across the different paths and can thus be neglected for the further analysis.

III. Cramér-Rao Bound

The CRB offers the fundamental limit that an estimator can possibly achieve. To derive it first assume that $x_n(t)$ is now a training signal that is going to be used to estimate the unknown parameters of the channel. Using P discrete samples, we can write the mth received signal as a vector with

$$\mathbf{y}_m = \underbrace{(\mathbf{\Omega}_m \odot \mathbf{X})}_{\mathbf{X}_{m,\omega}} \underbrace{\mathbf{\Phi}_m \mathbf{h}_m}_{\mathbf{h}_{m,\phi}} + \mathbf{n}_m \quad (5)$$

where $\mathbf{y}_m = [y_m(1), \dots, y_m(P)]^T$, $\mathbf{n}_m = [n_m(1), \dots, n_m(P)]^T$, $\mathbf{h}_m = [h_{m1}, \dots, h_{mN}]^T$,

$\mathbf{\Phi}_m = \operatorname{diag}\left(e^{j\Delta\phi_{m1}}, \ldots, e^{j\Delta\phi_{mN}}\right)$, $\odot$ is the Hadamard product, and

$$\mathbf{X} = \begin{bmatrix} x_1(1) & \cdots & x_N(1) \\ \vdots & \ddots & \vdots \\ x_1(P) & \cdots & x_N(P) \end{bmatrix},$$

$$\mathbf{\Omega}_m = \begin{bmatrix} e^{j\Delta\omega_{m1}} & \ldots & e^{j\Delta\omega_{mN}} \\ e^{j2\Delta\omega_{m1}} & \ldots & e^{j2\Delta\omega_{mN}} \\ \vdots & \ddots & \vdots \\ e^{jP\Delta\omega_{m1}} & \ldots & e^{jP\Delta\omega_{mN}} \end{bmatrix},$$

with $\Delta\omega_{mn} = 2\pi\Delta f_{mn}$. Note that for the common oscillator setup, there is no dependence on m and n in the matrices $\mathbf{\Phi}_m$ and $\mathbf{\Omega}_m$.

We can finally also write the complete received vector as

$$\mathbf{y} = \mathbf{X}_\omega \mathbf{h}_\phi + \mathbf{n} \tag{6}$$

where $\mathbf{y} = \left[\mathbf{y}_1^T, \ldots, \mathbf{y}_M^T\right]^T$, $\mathbf{n} = \left[\mathbf{n}_1^T, \ldots, \mathbf{n}_M^T\right]^T$, $\mathbf{h}_\phi = \left[\mathbf{h}_1^T\mathbf{\Phi}_1, \ldots, \mathbf{h}_M^T\mathbf{\Phi}_M\right]^T$, $\mathbf{X}_\omega = \operatorname{blkdiag}\left(\mathbf{\Omega}_1 \odot \mathbf{X}, \ldots, \mathbf{\Omega}_M \odot \mathbf{X}\right)$. As noted in other works [1], [2], the estimation of the parameters for each of the receiving antennas is decoupled (FIM is block diagonal, CRB is block diagonal) and can be carried out independently and hence we will in the following focus on (5) rather than (6).

A. No Frequency Offset

Let us first investigate the case when $\Delta\omega_{mn} = 0$. Then, the model reduces to

$$\mathbf{y}_m = \mathbf{X}\mathbf{h}_{m,\phi} + \mathbf{n}_m \tag{7}$$

where the parameter vector to be estimated is $\boldsymbol{\theta}_m = \left[\operatorname{Re}\{\mathbf{h}_{m,\phi}^T\} \quad \operatorname{Im}\{\mathbf{h}_{m,\phi}^T\}\right]^T$, and since $\mathbf{n}_m$ is a white Gaussian noise vector, the CRB is readily found [5], [1] by

$$\operatorname{CRB}\left(\boldsymbol{\theta}_m\right) = \frac{\sigma^2}{2}\begin{bmatrix} \operatorname{Re}\{\mathbf{X}^{\mathrm{H}}\mathbf{X}\} & -\operatorname{Im}\{\mathbf{X}^{\mathrm{H}}\mathbf{X}\} \\ \operatorname{Im}\{\mathbf{X}^{\mathrm{H}}\mathbf{X}\} & \operatorname{Re}\{\mathbf{X}^{\mathrm{H}}\mathbf{X}\} \end{bmatrix}^{-1}. \tag{8}$$

B. Frequency Offset Impaired

Using the form

$$\mathbf{y}_m = \mathbf{X}_{m,\omega}\mathbf{h}_{m,\phi} + \mathbf{n}_m \tag{9}$$

the new parameter vector of interest is $\boldsymbol{\theta}_m = \left[\operatorname{Re}\{\mathbf{h}_{m,\phi}^T\} \quad \operatorname{Im}\{\mathbf{h}_{m,\phi}^T\} \quad \boldsymbol{\omega}_m^T\right]^T$ and the CRB is found [5] as (10) at the bottom of the page, where $\mathbf{D}_m = \operatorname{diag}\left(1, \ldots, P\right) \cdot \mathbf{X}_{m,\omega} \cdot \operatorname{diag}\left(\mathbf{h}_{m,\phi}\right)$.

IV. LOS MIMO

As visible in (4), the channel coefficients in the LOS MIMO case are determined by the distances between transmit and receive antennas r_{mn}. We write the channel coefficients including the phase shifts

$$h_{mn,\Delta\phi} = \exp\left(-j2\pi\frac{r_{mn}}{\lambda_n}\right) \cdot \exp\left(j\Delta\phi_{mn}\right) \tag{11}$$

which can be jointly estimated as one term, c.f. (5). The introduced phase shifts at transmitter and receiver, which do not vary in time (practically this is not true), correspond to row and column operations on the initial $\mathbf{H} = \left[\mathbf{h}_1, \ldots, \mathbf{h}_M\right]^T$ which does not change the conditioning of the matrix.

Due to geometrical structure of the channel, the channel coefficients from one receive antenna to another typically do not vary randomly as is the case for Rayleigh scattering. For example, for all of the optimal ULA designs, see [4], the matrix $\mathbf{H}$ will have a Toeplitz structure, i.e., $\mathbf{h}_1, \ldots, \mathbf{h}_M$ are circularly shifted versions of each other.

A. Estimation in Frequency Offset free Case

In general, any training matrix $\mathbf{X}$ having orthogonal rows under transmit power constraint is optimal in the sense that it minimizes the CRB [6], i.e., $\mathbf{X}^{\mathrm{H}}\mathbf{X} = \mathbf{I}_N$. Note that this requires a pilot sequence of at least length $P = N$.

Now let us look specifically at the ULA mentioned above that generates an $\mathbf{H}$ with Toeplitz structure and consider a common oscillator at transmitter and receiver, as in (2). In that case the full matrix $\mathbf{H}_\phi$ will also be of Toeplitz character and we can use that a-priori information, to infer the full matrix from just one channel estimate. The accuracy of the estimation will, however, depend on the noise power, but could be improved by taking more (but still less than N) estimates and averaging over them. For the case of the individual phase offsets per transmit/receive antenna pair one needs a longer pilot sequence, since there are $M \cdot N$ unknown phase shifts that need to be estimated.

In Fig. 1 we show some results for the case of a common oscillator setup and a perfectly designed LOS MIMO system. The performance exploiting the Toeplitz structure is as least as good as the least-squares (LS) standard solution [6] (which does not use the similarity between matrix columns), but requires a shorter pilot sequence.

B. Estimation in Frequency Offset corrupted Case

It has been discussed in the literature that maximum likelihood estimators can be used to achieve the CRBs in a Rayleigh channel case [1] for such a setup, but requiring a high computational complexity. Those estimators are again

$$\operatorname{CRB}\left(\boldsymbol{\theta}_m\right) = \frac{\sigma^2}{2}\begin{bmatrix} \operatorname{Re}\{\mathbf{X}_{m,\omega}^{\mathrm{H}}\mathbf{X}_{m,\omega}\} & -\operatorname{Im}\{\mathbf{X}_{m,\omega}^{\mathrm{H}}\mathbf{X}_{m,\omega}\} & \operatorname{Re}\{\mathbf{X}_{m,\omega}^{\mathrm{H}}\mathbf{D}_m\} \\ \operatorname{Im}\{\mathbf{X}_{m,\omega}^{\mathrm{H}}\mathbf{X}_{m,\omega}\} & \operatorname{Re}\{\mathbf{X}_{m,\omega}^{\mathrm{H}}\mathbf{X}_{m,\omega}\} & \operatorname{Im}\{\mathbf{X}_{m,\omega}^{\mathrm{H}}\mathbf{D}_m\} \\ \operatorname{Re}\{\mathbf{D}_m^{\mathrm{H}}\mathbf{X}_{m,\omega}\} & -\operatorname{Im}\{\mathbf{D}_m^{\mathrm{H}}\mathbf{X}_{m,\omega}\} & \operatorname{Re}\{\mathbf{D}_m^{\mathrm{H}}\mathbf{D}_m\} \end{bmatrix}^{-1} \tag{10}$$

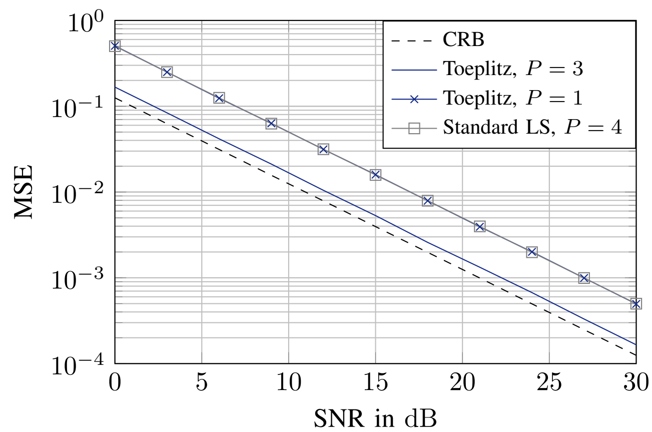

Fig. 1. MSE for different channel estimators, common oscillator setup, $N = M = 4$

based on the idea that $M \cdot N$ random channel coefficients and frequency offsets have to be estimated.

In the LOS MIMO case we can exploit the non-randomness of the channel coefficients. For the example of a shared oscillator as discussed above, this can be used to reduce complexity. The estimate from time instant to time instant can be directly used to infer the frequency offsets, as the channel coefficients for each of the receiving antennas are just shifted versions of each other. Since there is only one frequency offset to be estimated, we can then remove the channel coefficients from, e.g., two received vectors and we end up with estimates of the frequency offset. As an example, consider $M = N = 3$ and a training matrix of

$$\mathbf{X} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix},$$

the received vectors would be

$$\mathbf{y}_1 = \begin{bmatrix} e^{j\Delta\omega} h_{11,\Delta\phi} \\ e^{j2\Delta\omega} h_{21,\Delta\phi} \end{bmatrix}, \mathbf{y}_2 = \begin{bmatrix} e^{j\Delta\omega} h_{21,\Delta\phi} \\ e^{j2\Delta\omega} h_{11,\Delta\phi} \end{bmatrix},$$

$$\mathbf{y}_3 = \begin{bmatrix} e^{j\Delta\omega} h_{31,\Delta\phi} \\ e^{j2\Delta\omega} h_{21,\Delta\phi} \end{bmatrix},$$

where we have again used the possible Toeplitz structure of the channel. It should be visible that those vectors are sufficient to gain estimates of the desired parameters.

V. Outlook

In the full paper we will discuss the structure of the channel in more detail, e.g., the matrix is not Toeplitz for URAs but there is also a structure that can be exploited, and how suitable pilot sequences for different cases can be found. Furthermore, it is of practical interest how the estimators behave when the matrix is not perfect, i.e., there are offsets from the ideal antenna positions. Also in such cases there will still be some structure left in the matrix due to the general geometrical setup of the system. Finally we will investigate how a joint estimation of all parameters in the individual oscillator setup can be made with fewer pilots/higher accuracy than the commonly known methods for Rayleigh fading channels by taking the structure of the LOS channel into account.

References

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