Stochastic-Deterministic Multipath Model for Time-Delay Estimation

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I. Introduction

Global Navigation Satellite Systems (GNSS) are used in a wide field of applications, whether for positioning or time synchronization. In these applications the estimation accuracy of the line-of-sight (LOS) signal time-delay estimate directly influences the quality of the service. Multipath, i.e. superimposed replicas of the LOS signal, can severely degrade the LOS signal time-delay estimation [1].

In the past different multipath mitigation techniques have been studied. The maximum-likelihood (ML) estimator which estimates the channel parameters of each multipath together with the LOS parameters is the optimum approach for solving the multipath problem [2]. However, the optimum ML estimator requires an exact model order estimate and depending on the number of multipath signals a high computational effort. In order to avoid these problems advanced tracking loops [3] and multi-correlator bank based approaches [4] have been proposed for single antenna receivers. If an array of multiple antennas is used, the spatial diversity can be exploited for multipath suppression. However, for multi-antenna receivers the computational complexity of the ML estimator increases due to the additional spatial dimension. Therefore ,reduced complexity methods like the space alternating generalized expectation maximization (SAGE) algorithm [5] have been developed. While offering a significant reduction of computational effort in comparison to the exact ML estimator the SAGE algorithm and its extensions [6], [7] still require for a model order estimation. This can be avoided if a statistical multipath model [8], [9] is employed.

All of the methods mentioned above perform best if LOS and multipath signals are temporally and spatially uncorrelated, i.e. sufficiently separated in time and space. Dual-polarization antenna arrays, i.e. antenna arrays with right-hand-circularly polarized (RHCP) and left-hand-circularly polarized (LHCP) outputs can offer an additional degree of freedom to identify and separate spatially and temporally highly correlated multipath signals from the LOS signal. In [10] a multipath mitigation approach based on dual-polarization arrays has been proposed. Additionally the problem of model order estimation has been tackled by introducing the correlated path (CP) model, which divides the multipath signal into a signal correlated with the LOS signal and multipath interference. To achieve a simple ML estimator the multipath interference is modeled as temporally white Gaussian noise in [10]. This white noise assumption is inaccurate when considering the properties of the GNSS signals. Therefore, we show how to solve the ML estimator for the CP model for temporally coloured multipath interference in this work. Additionally, we derive the temporal multipath interference covariance matrix which is exact for temporally highly correlated LOS and multipath. In order to further reduce the computational complexity the signal is compressed with the help of a multi-correlator bank. The performance of the improved CP model is shown in an example of a dual-polarization global position system (GPS) receiver.

II. Multipath Signal Model

We consider a GNSS multipath scenario. One LOS signal with time delay $\tau_0 \in \mathbb{R}$ and L multipath signals with time delays $\tau_l \in \mathbb{R}$ are impinging on a dual polarization antenna array composed of M antenna elements. The unstructured base-band representation of the signal is

$$\boldsymbol{y}(t) = \boldsymbol{b}_0 c(t-\tau_0) + \sum_{l=1}^{L} \boldsymbol{b}_l c(t-\tau_l) + \boldsymbol{\eta}(t)\,, \qquad (1)$$

where $c(t) \in \mathbb{R}$ is the GNSS transmit signal with single-sided bandwidth $B \in \mathbb{R}$ and $\boldsymbol{b}_l \in \mathbb{C}^{2M}$ denotes the signal's spatial and polarization signature. A spatially structured model for $\boldsymbol{b}_l$ is given in [10]. In the following $\boldsymbol{\eta}(t) \in \mathbb{C}^{2M}$ is assumed as temporally and spatially white Gaussian noise, i.e. $\boldsymbol{\eta}(t) \sim \mathcal{CN}\big(0, \sigma_{\boldsymbol{\eta}}^2 \boldsymbol{I}_{2M}\big)$. After collecting N time samples of (1) at sampling rate $f_\mathrm{s} = 2B$ the discrete time representation is

$$\boldsymbol{Y} = \boldsymbol{b}_0 \boldsymbol{c}(\tau_0)^\mathrm{T} + \sum_{l=1}^{L} \boldsymbol{b}_l \boldsymbol{c}(\tau_l)^\mathrm{T} + \boldsymbol{E}, \qquad (2)$$

where with $T_\mathrm{s} = 1/f_\mathrm{s}$ and

$$\boldsymbol{Y} = [\boldsymbol{y}[T_\mathrm{s}]\ \boldsymbol{y}[2T_\mathrm{s}]\ \ldots\ \boldsymbol{y}[NT_\mathrm{s}]] \in \mathbb{C}^{2M\times N} \qquad (3)$$

$$\boldsymbol{c}(\tau_l) = [c[T_\mathrm{s}-\tau_l]\ c[2T_\mathrm{s}-\tau_l]\ \ldots\ c[NT_\mathrm{s}-\tau_l]]^\mathrm{T} \in \mathbb{C}^{N} \qquad (4)$$

$$\boldsymbol{E} = [\boldsymbol{\eta}[T_\mathrm{s}]\ \boldsymbol{\eta}[2T_\mathrm{s}]\ \ldots\ \boldsymbol{\eta}[NT_\mathrm{s}]] \in \mathbb{C}^{2M\times N}. \qquad (5)$$

The noise covariance is characterized by

$$\mathrm{E}\Big[\mathrm{vec}(\boldsymbol{E})\,\mathrm{vec}(\boldsymbol{E})^\mathrm{H}\Big] = \sigma_{\boldsymbol{\eta}}^2 \boldsymbol{I}_N \otimes \boldsymbol{I}_{2M}, \qquad (6)$$

where $\mathrm{vec}(\bullet)$ vectorizes a matrix by stacking its columns and $\mathrm{E}[\bullet]$ denotes the expected value.

A. Compression with a Multi-Correlator Bank

Since the number of samples N is often large, the received signal $\boldsymbol{Y}$ is compressed to a signal $\boldsymbol{Z} \in \mathbb{R}^{2M \times Q}$ with lower temporal dimension $Q < N$ using a multi-correlator bank. The compression can be represented by a multiplication of $\boldsymbol{Y}$ with the compression matrix $\boldsymbol{Q} \in \mathbb{R}^{N \times Q}$ from the right hand side

$$\boldsymbol{Z} = \boldsymbol{b}_0 \boldsymbol{c}(\tau_0)^{\mathrm{T}} \boldsymbol{Q} + \sum_{l=1}^{L} \boldsymbol{b}_l \boldsymbol{c}(\tau_l)^{\mathrm{T}} \boldsymbol{Q} + \boldsymbol{E}\boldsymbol{Q} \tag{7}$$

$$= \boldsymbol{b}_0 \boldsymbol{q}(\tau_0)^{\mathrm{T}} + \sum_{l=1}^{L} \boldsymbol{b}_l \boldsymbol{q}(\tau_l)^{\mathrm{T}} + \boldsymbol{E}_{\boldsymbol{Q}}. \tag{8}$$

(8) can be parameterized by

$$\boldsymbol{\xi} = \left[\tau_0, \ldots, \tau_L, \boldsymbol{b}_0^{\mathrm{T}}, \ldots, \boldsymbol{b}_L^{\mathrm{T}}, \sigma_{\boldsymbol{\eta}}^2\right]^{\mathrm{T}} \in \mathbb{C}^{(L+1)(2M+1)+1}. \tag{9}$$

For compression we employ the left singular vectors $\boldsymbol{U}$ of

$$\hat{\boldsymbol{Q}} = \left[\boldsymbol{c}(\kappa_1), \boldsymbol{c}(\kappa_2), \ldots, \boldsymbol{c}(\kappa_Q)\right]^{\mathrm{T}} = \boldsymbol{U}\boldsymbol{\Sigma}\boldsymbol{V}^{\mathrm{H}}, \tag{10}$$

i.e.

$$\boldsymbol{Q} = \boldsymbol{U}. \tag{11}$$

(10) realizes the canonical component (CC) method [11]. The CC is based on correlating the sampled received signal $\boldsymbol{Y}$ with Q replicas of $\boldsymbol{c}(\tau)$ with different delays κ_q and minimizes the Fisher information loss due to compression [11], as well as maintaining the multiple access properties of the direct sequence code division multiple access (DS-CDMA) system used in GNSS. Using the left matrix $\boldsymbol{U}$ from the singular value decomposition (SVD) ensures that the noise $\boldsymbol{E}_Q$ after correlation is still white Gaussian noise

$$\mathrm{E}\Big[\mathrm{vec}(\boldsymbol{E}_{\boldsymbol{Q}})\,\mathrm{vec}(\boldsymbol{E}_{\boldsymbol{Q}})^{\mathrm{H}}\Big] = \mathrm{Cov}[\mathrm{vec}(\boldsymbol{E}_{\boldsymbol{Q}})] \tag{12}$$

$$= \sigma_{\boldsymbol{\eta}}^2 \boldsymbol{I}_Q \otimes \boldsymbol{I}_{2M}, \tag{13}$$

where $\mathrm{Cov}[\bullet]$ denotes the covariance matrix operator.

III. Correlated Path Model

The optimum estimator for τ_0 in (8) is the ML estimator which estimates all parameters in (9) [2]. However, this estimator requires a model order estimation and must determine the actual LOS delay from all other multipath delays. Moreover, this estimator has to cope with a number of nuisance parameters. To avoid these problems we employ the CP model proposed in [10]. Let

$$\rho_l = \boldsymbol{q}(\tau_0)^{\mathrm{T}} \boldsymbol{q}(\tau_l) \tag{14}$$

denote the temporal correlation between LOS and l-th multipath signal. For $L = 1$ the multipath signal can than be decomposed

$$\boldsymbol{b}_1 \boldsymbol{q}(\tau_1)^{\mathrm{T}} = \rho_1 \boldsymbol{b}_1 \boldsymbol{q}(\tau_0)^{\mathrm{T}} + \sqrt{1-\rho_1^2}\,\boldsymbol{b}_1 \boldsymbol{u}^{\mathrm{T}}, \tag{15}$$

where the multipath interference $\boldsymbol{u} \in \mathbb{R}^Q$ is uncorrelated with the LOS signal $\boldsymbol{q}(\tau_0)$, i.e.

$$\mathrm{E}\big[\boldsymbol{u}^{\mathrm{H}} \boldsymbol{q}(\tau_0)\big] = 0 \tag{16}$$

and has a temporal covariance matrix

$$\mathrm{Cov}[\boldsymbol{u}] = \boldsymbol{Q}_{\boldsymbol{u}} \in \mathbb{R}^{Q \times Q}. \tag{17}$$

In [10] it is assumed that $\boldsymbol{u}$ is temporally white Gaussian noise and therefore $\boldsymbol{Q}_{\boldsymbol{u}}$ is an identity matrix. Due to the properties of the signal $\boldsymbol{q}(\tau_l)$ it can be shown that this is not the case in general. However, for highly correlated LOS and multipath signals it holds

$$\mathrm{Cov}[\boldsymbol{q}(\tau_1)] \approx \mathrm{Cov}[\boldsymbol{q}(\tau_0)] \approx \boldsymbol{Q}_{\boldsymbol{q}} = \boldsymbol{Q}_{\boldsymbol{u}} \tag{18}$$

which leads to a better multipath suppression in the case of temporally and spatially highly correlated multipath signals in comparison to [10]. Moreover it can be shown that for a single multipath

$$\boldsymbol{Q}_{\boldsymbol{u}} = \frac{\mathrm{Cov}[\boldsymbol{q}(\tau_1)] - \rho_1^2 \mathrm{Cov}[\boldsymbol{q}(\tau_0)]}{1-\rho_1^2}. \tag{19}$$

Using (19) increases the performance of the LOS delay estimation when the CP model is applied. Even though the CP model is based on the assumption of $L = 1$ multipath signals, simulation results show that it also performs well in the case of more than one multipath signals, if these are temporally or spatially highly correlated [10]. In this case ρ_1 and $\boldsymbol{Q}_{\boldsymbol{u}}$ reflect the overall correlation between LOS and multipath and temporal multipath interference covariance matrix while $\boldsymbol{b}_1$ reflects the overall multipath spatial signature. To emphasize these properties we denote the correlation between LOS and multipath with ρ while $\boldsymbol{b}_{\mathrm{CP}}$ denotes the overall multipath spatial signature. The CP model is finally given by

$$\boldsymbol{Z} = \left(\boldsymbol{b}_0 + \rho \boldsymbol{b}_{\mathrm{CP}}\right) \boldsymbol{q}(\tau_0)^{\mathrm{T}} + \sqrt{1-\rho^2}\,\boldsymbol{b}_{\mathrm{CP}} \boldsymbol{u}^{\mathrm{T}} + \boldsymbol{E}_{\boldsymbol{Q}} \tag{20}$$

with parametrization

$$\boldsymbol{\xi}_{\mathrm{CP}} = \left[\tau_0, \boldsymbol{b}_0^{\mathrm{T}}, \boldsymbol{b}_{\mathrm{CP}}^{\mathrm{T}}, \rho, \sigma_{\boldsymbol{\eta}}^2\right] \in \mathbb{C}^{2M+3}. \tag{21}$$

A. Parameter Estimation

In order to estimate (21) the singular value decomposition approach presented in [10] can be extended to the ML estimator of a spatio-temporal model. Assuming Gaussian noise, the probability density function is

$$\mathrm{p}(\boldsymbol{Y}|\boldsymbol{\xi_{CP}}) = \frac{1}{\pi^{MN} \det(\boldsymbol{Q}(\boldsymbol{\xi}_{\mathrm{CP}}))} \cdot \exp\Big(-\mathrm{vec}(\boldsymbol{M}(\boldsymbol{\xi}_{\mathrm{CP}}))^{\mathrm{H}} \boldsymbol{Q}(\boldsymbol{\xi}_{\mathrm{CP}})^{-1} \mathrm{vec}(\boldsymbol{M}(\boldsymbol{\xi}_{\mathrm{CP}}))\Big) \tag{22}$$

with

$$\boldsymbol{M}(\boldsymbol{\xi}_{\mathrm{CP}}) = \boldsymbol{Y} - \left(\boldsymbol{b}_0 + \rho \boldsymbol{b}_{\mathrm{CP}}\right) \boldsymbol{q}(\tau_0)^{\mathrm{T}} \tag{23}$$

$$\boldsymbol{Q}(\boldsymbol{\xi}_{\mathrm{CP}}) = \mathrm{Cov}\Big[\sqrt{1-\rho^2}\,\boldsymbol{b}_{\mathrm{CP}} \boldsymbol{u}^{\mathrm{T}} + \boldsymbol{E}_Q\Big]$$

$$= \boldsymbol{Q}_{\boldsymbol{u}} \otimes \left(1-\rho^2\right) \boldsymbol{b}_{\mathrm{CP}} \boldsymbol{b}_{\mathrm{CP}} + \sigma_{\boldsymbol{\eta}}^2 \boldsymbol{I}_Q \otimes \boldsymbol{I}_{2M}. \tag{24}$$

The optimum estimator for $\boldsymbol{\xi}_{\mathrm{CP}}$ in (15) is given by the ML estimate

$$\hat{\boldsymbol{\xi}}_{\mathrm{CP}} = \arg\max_{\boldsymbol{\xi}_{\mathrm{CP}}} \mathrm{p}(\boldsymbol{Y}|\boldsymbol{\xi_{CP}}). \tag{25}$$

It can be shown that the optimization problem (25) has a closed form solution for all parameters except τ_0 if $\boldsymbol{Q}_{\boldsymbol{u}}$ is assumed to be known.

IV. Preliminary Results

We assume a GPS C/A code with chip duration $T_\mathrm{c} = 997.52\,\mathrm{ns}$, bandwidth $B = 1.023\,\mathrm{MHz}$ and $N_\mathrm{d} = 1023$ chips per code period as transmit signal $c(t)$. The receive array is a 2 antenna dual-polarization uniform linear array (ULA) with $10\,\mathrm{dB}$ separation between RHCP and LHCP channels, and spatially white Gaussian noise. The RHCP channel signal-to-noise ratio (SNR) is

$$\mathrm{SNR} = \mathrm{C/N_0} - 10\log_{10}(2B) + 10\log_{10}(N_\mathrm{c}), \qquad (26)$$

with carrier-to-noise density $\mathrm{C/N_0} = 46\,\mathrm{dB\text{-}Hz}$ and number of observed code periods $N_\mathrm{c} = 4$. During the observation interval, channel parameters are assumed to be constant. The LOS azimuth angle-of-arrival is $\phi_0 = 70°$. In [10] it has been shown that the CP model can be applied to more than one multipath signal if LOS and multipath are temporally and spatially highly correlated. Therefore, we simulate $L = 6$ multipath signals with angles between $60°$ and $80°$. The multipath energy is equally divided into RHCP and LHCP power. The spatial signatures $\boldsymbol{b}_l$ are calculated with the dual-polarization multipath model introduced in [10]. Figure 1 shows the RMSE of the estimate $\hat{\tau}_0$ for different choices of $\boldsymbol{Q_u}$ over the mean delay difference $\varDelta\bar{\tau} = \frac{1}{L}\sum_{l=1}^{L}\tau_l - \tau_0$. The τ_l are evenly spread within an interval of $0.2 \cdot T_\mathrm{c}$. Especially for $\varDelta\bar{\tau}$ around $0.5 \cdot T_\mathrm{c}$ assuming a non-identity covariance for $\boldsymbol{u}$ yields a higher estimation performance for τ_0 than assuming $\boldsymbol{Q_u}$ as white noise as done in [10]. Using the single multipath approximation $\boldsymbol{Q_u} = \frac{\mathrm{Cov}[\boldsymbol{q}(\tau_1)] - \rho^2 \mathrm{Cov}[\boldsymbol{q}(\tau_0)]}{1-\rho^2}$ instead of $\boldsymbol{Q_u} = \boldsymbol{Q_q}$ also yields a better estimation performance in this $\varDelta\bar{\tau}$ range, even though $L = 6$ multipath signals are present. This is due to the fact that all multipath signal delays lie within in interval of $0.2 \cdot T_\mathrm{c}$ and are temporally highly correlated. Therefore, the temporal multipath interference covariance matrix given in (19) is still valid, even though it is calculated for only one multipath signal.

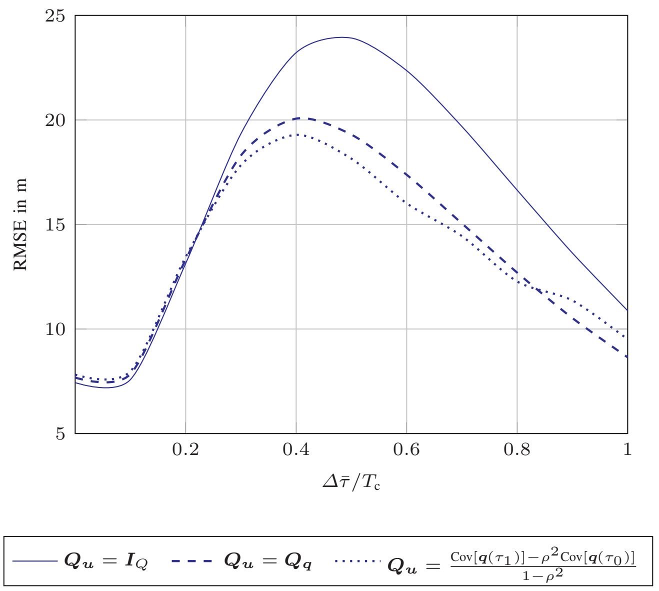

Figure 1. Estimation Performance of the CP Model for Different Choices of the Multipath Interference Temporal Covariance Matrix $\boldsymbol{Q_u}$

V. Content of the Full Paper

In the full paper we derive the temporal multipath noise covariance matrix $\boldsymbol{Q_u}$ and the closed form solution for the ML estimator (25). The performance of LOS time-delay estimation with the improved CP path model is assessed in comparison to the CP model with white noise assumption, a single path ML estimator [8] and the Cramer-Rao lower bound (CRLB). Additionally, the performance dependency on direction of arrival, number of antennas and LOS and multipath space-time correlation is investigated.

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