Activity and Channel Estimation in Multi-User Wireless Sensor Networks

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***Abstract*—Machine-type communications is quite often of very low data rate and of sporadic nature. In a multi-user wireless sensor network, this sporadic transmission activity can be favourably exploited to facilitate a joint activity and data detection on the physical layer as it has been shown in previous works. This abolishes the need for shared channel access signaling on higher network layers and increases bandwidth efficiency. However, each data source transmits over a user-specific channel which makes channel estimation mandatory for phase-coherent reception and channel equalisation. Since a totally blind estimation of channels, transmit activities and user data – all at the same time – is practically infeasible, we propose a joint activity and channel estimation scheme based on pilots and matching pursuit algorithms. We show that Zadoff-Chu sequences lead to a better user separation and estimation performance than random Gaussian codes. And since we do not make any assumptions about the user's data payload, our results are generally valid to any frame structure.**

I. Introduction

One of the big emerging fields for future communication systems is machine-type communications. This term describes data traffic between two autonomous entities without human interaction. Nowadays high data rate systems, such as LTE, were designed for human-driven, high data rate traffic without machine-type communications in mind [1]. However, many applications arise, e.g. in the industrial automation context, where lots of sensor nodes communicate status information to a common base station (sink node). This is quite often of very low data rate and of sporadic nature.

In such a wireless uplink scenario, where sensor nodes sporadically transmit data to a central aggregation node, activity signaling to access the shared wireless medium generates a lot of overhead and makes communication inefficient compared to the small amount of payload data. Previous works in [2]–[4] therefore proposed a novel joint activity and data detection at the multi-user receiver, or data aggregation node, on the physical network layer. The cited works altogether assume perfect channel state information (CSI) at the receiver. But in a practical setup, CSI must be estimated for channel equalisation and phase-coherent reception of data symbols taken from a digital modulation alphabet, e.g. Phase-Shift Keying.

It is practically impossible to reliably estimate user-specific channels, activity and data symbols all at once. Hence, the partitioning of this problem into a joint multi-user *activity and channel estimation* followed by a classical non-sparse data detection, instead of a combined activity and data detection with given CSI, is a viable solution. This idea was first brought forward in [5] and it was shown that good results can be achieved with known data symbols serving as pilots.

We will take a similar approach and utilise user-specific code sequences to form a multi-user pilot signal. Additionally to random Gaussian sequences we will also investigate Zadoff-Chu (ZC) sequences [6] which promise good performance regarding channel equalisation and multi-user interference cancellation. User activities and channel responses can mathematically be written as a vector which is either sparse or block-sparse depending on whether there are frequency-flat or frequency-selective fading channels, respectively. However, detection algorithms may not be arbitrarily complex in respect of a potential implementation in physical layer modem hardware. For this reason, we will examine the detection performance of relatively simple Orthogonal Matching Pursuit (OMP), of which VLSI designs have already been reported, e.g. in [7], and its variant for block-sparse signals, Block Orthogonal Matching Pursuit (BOMP).

II. System Model

The multi-user wireless uplink communications system we consider in this work is depicted schematically in Figure 1. The N sensor nodes, shown on the left, transmit their data frames to a central aggregation node as receiver for further processing. Each transmitter only is sporadically active with probability p_a on a frame-to-frame basis. Each node transmits with the same probability p_a or, in other words, just a random subset of $N_a = p_a N$ nodes transmits data at a specific time instance, as indicated. We assume that the detector has probabilistic but not instantaneous knowledge about the activities, i.e. p_a is known by the receiver.

A data frame consists of a preamble and payload data. The preamble contains a user-specific pilot code sequence of length N_p. While we do not make any assumptions about the payload, we assume that all active users begin transmission synchronously at the same time instance. This basically is the system model of previous works extended by a preamble to facilitate pilot-based channel estimation [2], [3]. The user-specific pilot sequences become superimposed due to the concurrent channel access, similar to a direct-sequence code-division multiple access (CDMA) system. In the following, we restrict ourselves to the mathematical system description of this preamble only.

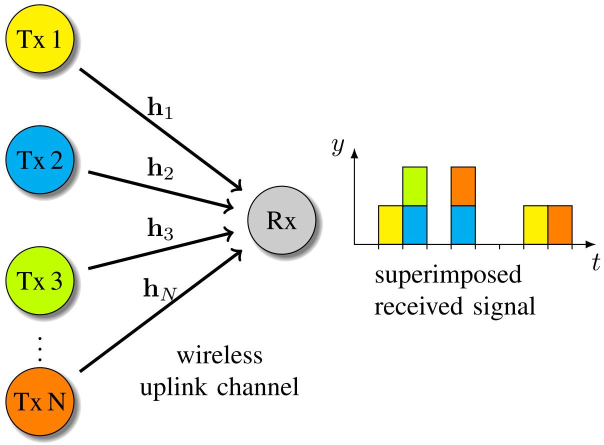

Fig. 1: Uplink wireless sensor network with sporadic activity and per-user channels $\mathbf{h}_n$ in a star topology. The frame-synchronous and superimposed received signal is shown schematically.

The system model in symbol clock can well be summarised by

$$\mathbf{y} = \mathbf{T}\mathbf{a} + \mathbf{w} = \mathbf{S}\mathbf{h} + \mathbf{w}\,, \tag{1}$$

where $\mathbf{y}$ is the received signal vector and $\mathbf{w}$ denotes additive white Gaussian noise with zero mean and variance σ_n^2. The multi-user vector $\mathbf{a} \in \{0,1\}^N$ defines the activity pattern of the sensor nodes during a frame and its nth entry, a_n, corresponds to the activity of node n. A transmitter is modelled as inactive if $a_n = 0$ or active if $a_n = 1$, i.e. a user-specific pilot code sequence is transmitted. Hence, $\Pr(a_n = 0) = 1 - p_a$ and $\Pr(a_n = 1) = p_a$. If p_a is sufficiently small, $\mathbf{a}$ is a sparse vector containing a considerable number of zero symbols. The system matrix $\mathbf{T}$ can be partitioned into two matrices,

$$\mathbf{T} = \mathbf{S}\mathbf{H} = \begin{bmatrix}\mathbf{S}_1 & \cdots & \mathbf{S}_N\end{bmatrix}\begin{bmatrix}\mathbf{h}_1 & & 0\\ & \ddots & \\ 0 & & \mathbf{h}_N\end{bmatrix}, \tag{2}$$

where $\mathbf{S}$ is the horizontal or row-wise concatenation of the convolution matrices $\mathbf{S}_n$ and $\mathbf{H}$ is a block-diagonal matrix of the channels $\mathbf{h}_n$, with $n = 1, ..., N$. Each $\mathbf{h}_n$ is a user-specific channel impulse response, a column vector of length L. The transmission of the user-specific pilot code sequences $\mathbf{s}_n \in \mathbb{C}^{N_p}$ corresponds to a convolution with the channel impulse responses, or

$$\mathbf{T} = \begin{bmatrix}\mathbf{s}_1 * \mathbf{h}_1 & \cdots & \mathbf{s}_N * \mathbf{h}_N\end{bmatrix}. \tag{3}$$

In (2) these convolution operations are expressed with the help of a convolution matrix such that $\mathbf{s}_n * \mathbf{h}_n = \mathbf{S}_n\mathbf{h}_n$, $\mathbf{S}_n \in \mathbb{C}^{N_p+L-1\times L}$.

In high data rate CDMA systems the channel impulse responses are sparse such that multipath propagation delays are distributed over a couple of echoes, usually appearing in clusters [8]. However, as stated above, machine-type communications are assumed to be of low data rate. Hence, it can be justified to model frequency-selective channels with only a few, non-sparse channel coefficients, all i.i.d. Rayleigh. If the transmission is of very low data rate, occupying only a narrow bandwidth, the fading is frequency-flat, i.e. $L = 1$. In that special case, each $\mathbf{h}_n$ simply is a single Rayleigh-distributed complex channel coefficient h_n. Then, $\mathbf{S} = \begin{bmatrix}\mathbf{s}_1 & \cdots & \mathbf{s}_N\end{bmatrix}$ contains column-wise arranged all user-specific pilot sequences and $\mathbf{H} = \mathrm{diag}\begin{bmatrix}h_1 & \cdots & h_N\end{bmatrix}$ is a diagonal matrix of the user's channel coefficients.

Of special interest is the product

$$\mathbf{h} = \mathbf{H}\mathbf{a}\,, \tag{4}$$

with $\mathbf{h} \in \mathbb{C}^{LN}$ being the stacked vector of all user channel impulse responses multiplied with the activity of the corresponding users. Incorporating (4) into the system model leads to the right hand side of (1). $\mathbf{h}$ is like $\mathbf{a}$ a sparse vector, but in fact it is *block-sparse* with N_a blocks of non-zero elements of length L. Generally, a vector $\mathbf{x}$ of N blocks can be defined as

$$\mathbf{x} = \Big[\underbrace{x_1 \cdots x_d}_{\mathbf{x}^T[1]}\ \underbrace{x_{d+1} \cdots x_{2d}}_{\mathbf{x}^T[2]} \cdots \underbrace{x_{(N-1)d} \cdots x_{Nd}}_{\mathbf{x}^T[N]}\Big]^T, \tag{5}$$

whereby $\mathbf{x}[\ell]$ denotes or selects the ℓth block of length d. If only k blocks contain non-zero elements, this vector is said to be k-block-sparse, or mathematically

$$\|\mathbf{x}\|_{2,0} = \sum_{\ell=1}^{N} I(\|\mathbf{x}[\ell]\|_2 > 0) \le k\,, \tag{6}$$

with $I(\cdot)$ being the indicator function [9].

III. Joint Activity and Channel Estimation

The goal of this work is to estimate the multi-user activity and channel coefficent vector $\mathbf{h}$, eq. (4), given the measurements $\mathbf{y}$ according to (1). The support of $\mathbf{h}$, i.e. the positions of the non-zero entries, represents the activity pattern of the transmit nodes, and the values of the non-zero entries correspond to the channel response of the active users. Hence, the estimation of $\mathbf{h}$ constitutes a joint activity and channel estimation.

As discussed in the previous section, there can be frequency-selective fading (channel impulse responses of length L) or as a special case ($L = 1$) frequency-flat fading. Depending on this, the recovery problems and applicable algorithms differ somewhat.

A. Frequency-Flat Fading Channels

The sparsity-aware MAP (S-MAP) optimisation problem associated with (1) and $L = 1$ is

$$\hat{\mathbf{h}} = \arg\min_{\mathbf{h}\in\mathbb{C}^N} \|\mathbf{y} - \mathbf{S}\mathbf{h}\|_2^2 + \lambda\|\mathbf{h}\|_0\,, \tag{7}$$

where $\|\mathbf{x}\|_0 = \#\{i : x_i \neq 0\}$ is the ℓ_0-(pseudo) norm of a vector $\mathbf{x}$, i.e. the total number of non-zero elements or the cardinality of the support. The penalty term $\lambda = 2\sigma_n^2 \log((1-p_a)/p_a)$ reflects the a priori statistics of the activity vector and scales with the noise power σ_n^2, cp [10].

1: **function** OMP($\mathbf{y}, \mathbf{S}, k$)
2: $\quad \mathbf{r} \leftarrow \mathbf{y}$; $\mathbf{h} \leftarrow \mathbf{0}$; $\mathcal{I} \leftarrow \emptyset$ $\triangleright$ initialisation
3: $\quad$ **for** $1, ..., k$ **do**
4: $\quad\quad \mathbf{c} \leftarrow \mathbf{S}^H \mathbf{r}$ $\triangleright$ correlation
5: $\quad\quad \ell^* \leftarrow \arg\max_\ell |c_\ell|$ $\triangleright$ selection
6: $\quad\quad \mathcal{I} \leftarrow \mathcal{I} \cup \ell^*$ $\triangleright$ index set
7: $\quad\quad \mathbf{h}_\mathcal{I} \leftarrow \arg\min_{\tilde{\mathbf{h}}_\mathcal{I}} \left\|\mathbf{y} - \mathbf{S}_\mathcal{I}\tilde{\mathbf{h}}_\mathcal{I}\right\|_2$ $\triangleright$ least squares
8: $\quad\quad \mathbf{r} \leftarrow \mathbf{y} - \mathbf{S}\mathbf{h}$ $\triangleright$ residual
9: $\quad$ **end for**
10: $\quad$ **return** $\mathbf{h}$
11: **end function**

Fig. 2: Pseudocode of Orthogonal Matching Pursuit (OMP), cp. [11].

1: **function** BOMP($\mathbf{y}, \mathbf{S}, k, L$)
2: $\quad \mathbf{r} \leftarrow \mathbf{y}$; $\mathbf{h} \leftarrow \mathbf{0}$; $\mathcal{I} \leftarrow \emptyset$
3: $\quad$ **for** $1, ..., k$ **do**
4: $\quad\quad \mathbf{c} \leftarrow \mathbf{S}^H \mathbf{r}$
5: $\quad\quad \ell^* \leftarrow \arg\max_\ell \left\|\mathbf{c}_{[\ell]}\right\|_2$ $\triangleright$ block selection
6: $\quad\quad \mathcal{I} \leftarrow \mathcal{I} \cup \ell^*$
7: $\quad\quad \mathbf{h}_{[\mathcal{I}]} \leftarrow \arg\min_{\tilde{\mathbf{h}}_{[\mathcal{I}]}} \left\|\mathbf{y} - \mathbf{S}_{[\mathcal{I}]}\tilde{\mathbf{h}}_{[\mathcal{I}]}\right\|_2$
8: $\quad\quad \mathbf{r} \leftarrow \mathbf{y} - \mathbf{S}\mathbf{h}$
9: $\quad$ **end for**
10: $\quad$ **return** $\mathbf{h}$
11: **end function**

Fig. 3: Pseudocode of Block Orthogonal Matching Pursuit (BOMP), cp. [9].

An exhaustive search over all 2^N different support patterns (activity states of the multi-user network) is an NP-hard problem. Furthermore, $\mathbf{S}$ may well be a fat matrix in an overloaded communications system which renders common regularised least squares (LS) solutions ineffective as they are not capable to exploit the sparsity on hand, i.e. even if the system of equations is fully determined, LS would not be able to recover the truly sparse nature of $\mathbf{h}$ or $\mathbf{a}$. As an additional practical constraint, the estimation procedure has to take place on the physical network layer and must thus be implementable into a digital modem. This rules out computationally intensive convex optimisation algorithms.

Taking all these considerations into account, we propose OMP as a suitable algorithm [11], [12]. OMP solves eq. (7) approximately in a greedy fashion while it exploits the fact that $\mathbf{h}$ is maximally k-sparse, i.e. it contains maximally k non-zero entries. As stated above, we assume statistical knowledge of the activity ratio in our uplink sensor network, i.e. p_a is known and translates to k by $k = \lceil p_a N \rceil$.

The pseudocode of OMP is given in Fig. 2. It recovers a k-sparse approximation of a vector in k iterations (line 3). In every iteration, one column in $\mathbf{S}$ is selected that is most strongly correlated with the residual $\mathbf{r}$ (lines 4 and 5). The index of the chosen column identifies the non-zero location, also called atom, of the sparse result vector $\mathbf{x}$. Then, a least squares step is computed in line 7 on a reduced system of equations. $\mathcal{I}$ is the index set of all atoms that have been chosen so far (support set). Hence, $\mathbf{S}_\mathcal{I}$ denotes a matrix composed of all selected pilot code sequences, and $\mathbf{h}_\mathcal{I}$ denotes a short vector of all selected atoms. An equivalent formulation of line 7 would be $\mathbf{h}_\mathcal{I} = \mathbf{S}_\mathcal{I}^\dagger \mathbf{y}$, whereby $(\cdot)^\dagger$ denotes the Moore-Penrose pseudoinverse.

B. Frequency-Selective Fading Channels

Frequency-selective fading necessitates the estimation of a block-sparse vector $\mathbf{h}$ of size LN, as stated in eq. (4). The MAP optimisation program for block-sparsity-aware joint activity and channel estimation is

$$\hat{\mathbf{h}} = \arg\min_{\tilde{\mathbf{h}} \in \mathbb{C}^{LN}} \|\mathbf{y} - \mathbf{S}\mathbf{h}\|_2^2 + \lambda \|\mathbf{h}\|_{2,0} \,. \tag{8}$$

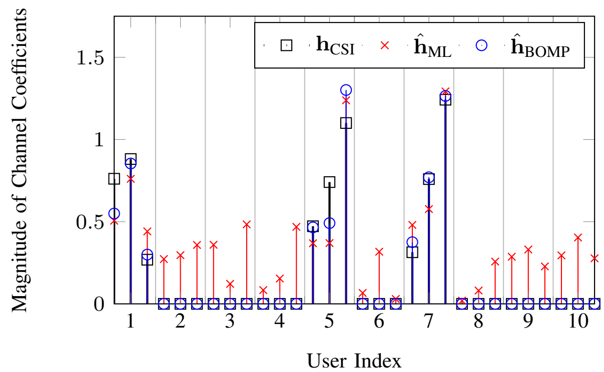

Fig. 4: Channel impulse responses estimated by BOMP compared to the ground truth and ML estimation.

Now, the cost function promotes block-sparsity with $\|\tilde{\mathbf{h}}\|_{2,0}$ according to eq. (6) and block length $d = L$, i.e. the length of the channel impulse responses. A modified OMP which recovers block-sparse signals should approximately solve it. This algorithm is called BOMP and was introduced in [9].

The pseudocode of BOMP is given in Fig. 3. The major difference to OMP can be found in line 5 where not a single atom is selected but the ℓ^*th *block* of atoms of length L. The selection is based on the maximal Euclidean norm of the correlation values of each block. Here $\mathcal{I}$ therefore denotes the set of selected block indices. In order to clarify notational issues, the index notation $\mathbf{c}_{[\ell]}$ returns the smaller vector $\mathbf{c}[\ell]$ of the ℓth block in $\mathbf{c}$ in analogy to how single elements of a vector are addressed, $c_i = \mathbf{c}(i)$. When a set is given as index, e.g. $\mathbf{h}_{[\mathcal{I}]}$ in line 7, all elements of all blocks in $\mathcal{I}$ are returned as a vector. Thus, the least squares step of the same line computes $L \cdot |\mathcal{I}|$ values. The result of BOMP is a block-sparse vector with k blocks of non-zero coefficients.

IV. Numerical Simulation Results

We investigated the system performance by numerical Monte-Carlo simulations. Fig. 4 exemplarily shows for illustration purposes one random channel realisation of a multi-user channel impulse response vector $\mathbf{h}_{\text{CSI}}$ according to (4). Here, the system consists of $N = 10$ users, whereby $N_a = 3$ users

Parameter	Symbol	Value
No. of Users	N	30
Activity Probability	p_a	$\{0.1, 0.2, 0.3\}$
No. of Active Users	N_a	$p_a N$
Channel Imp. Resp. Length	L	$\{1, 3\}$
Pilot Sequence Length	N_p	$\{LN, LN/2\}$
Pilot Sequence Type	—	Zadoff-Chu, Random Gaussian

TABLE I: Simulation parameters.

are active (no. 1, 5 and 7). The channel impulse response length is $L = 3$, thus $\mathbf{h}_{\text{CSI}}$ is of length $LN = 30$ and N_a-block-sparse with $LN_a = 9$ non-zero elements. The system of equations is fully determined since we chose $N_p = LN$. The BOMP algorithm correctly detects all active users and returns a truly sparse estimation $\hat{\mathbf{h}}_{\text{BOMP}}$. Other than that, simple ML estimation distributes the measured noise over all coefficients and an arbitrary threshold level to recover the support would obviously lead to an erroneous activity detection.

For the following results we used a standard set of simulation parameters, given in Table I. Each data point was averaged over at least 20000 random system model realisations. We employed two different types of pilot sequences to compare their performance. On the one hand, random zero-mean complex Gaussian sequences can be used as pilot codes. They facilitate a better user separation compared to traditional pseudo-noise sequences [13]. On the other hand, Zadoff-Chu (ZC) sequences promise a good performance as they are so-called CAZAC sequences, which means "constant amplitude and zero autocorrelation" [6]. This good autocorrelation behaviour is especially useful in frequency-selective fading environments. Additionally, there are bounds on the cross-correlation.

Figure 5 shows the average activity detection error rate (AER) over SNR. AER is defined to be $\bar{N}_f/N$, i.e. the average number of false detections (false active and false inactive) over the number of users in our system. In this case, the system is fully determined with $N_p = LN = 90$. Hence, only measurement noise degrades the detection performance. It becomes obvious that frequency-selective fading enhances the activity and channel detection process. This is mainly because of the lesser outage probability of Rayleigh fading channels with $L > 1$. ZC can benefit more from this effect though, especially when many users are active (better user separation). The plotted curves are for different levels of user activity ($p_a = 0.2$ not shown for the sake of clarity), where $p_a = 0.3$ corresponds to 9 active users, which is quite a lot compared to results given in other works [5]. In all cases, ZC sequences show a better performance than random Gaussian sequences. The quality of the channel estimation can also be expressed in terms of normalised mean square error (NMSE) which is defined as $\text{NMSE}(\mathbf{h}, \hat{\mathbf{h}}) = \|\mathbf{h} - \hat{\mathbf{h}}\|_2^2 / \|\mathbf{h}\|_2^2$. This is shown in Fig. 6 for $p_a = 0.2$. It can be observed that ZC sequences lead to estimations which are closer to the true channel. Frequency-selective channels improve the NMSE as well.

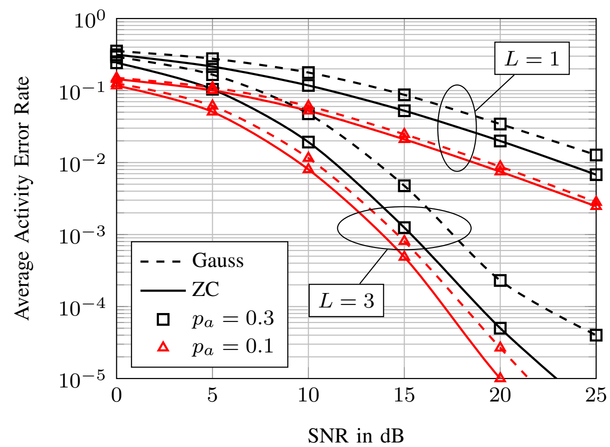

Fig. 5: Average activity error rate for random Gaussian (dashed) and Zadoff-Chu (solid) sequences. The system is fully determined.

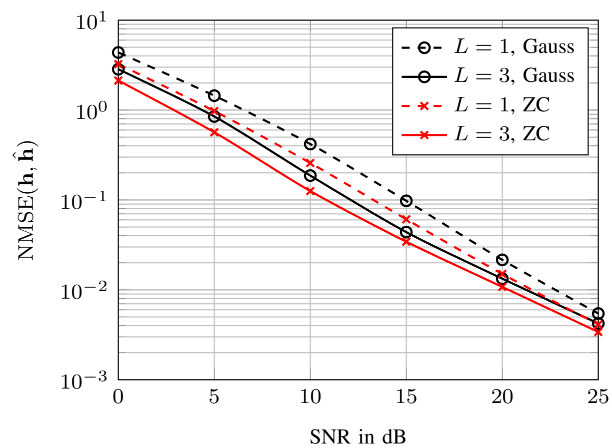

Fig. 6: Averaged normalised MSE of the estimated multi-user flat (dashed) and frequency-selective (solid) fading channel for Gauss and Zadoff-Chu pilot sequences.

In Fig. 7 the same setup is investigated with shorter pilots of length $N_p = LN/2$, i.e. a underdetermined system with factor $1/2$, for a better bandwidth efficiency. As ZC sequences do not exist for this case, we can only resort to random Gaussian sequences. These, however, perform fairly well and only little loss can be observed for $p_a = 0.1$ compared to the fully determined case. Detection is again improved by frequency-selective channels. Nonetheless, the spread of the curves is wider, which means that the underdetermined system is less tolerant to support many active users. For $p_a > 0.3$ reliable estimation is not possible.

V. Conclusion

We investigated the combined activity and channel estimation in a wireless sensor network based on user-specific pilot code sequences. This, was shown, can successfully be facilitated especially in frequency-selective fading environments with (Block) Orthogonal Matching Pursuit. Zadoff-Chu

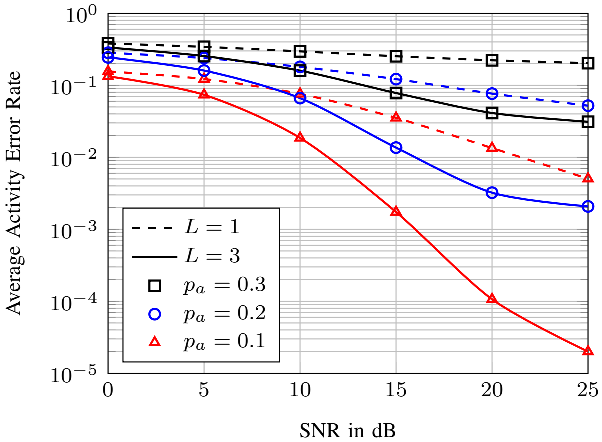

Fig. 7: Average activity error rate for flat (dashed) and frequency-selective (solid line) fading channels and random Gaussian sequences. The system is underdetermined with the factor $1/2$.

sequences outperform random Gaussian codes in detection performance (lesser activity error rates) and quality. However only the latter are able to support sequences shorter than the number of users, but only when there are few active users.

Since we did not make assumptions about the user's data payload, our results are generally valid. They are applicable to any possible frame structure and supply the data detector with user activity and channel state information.

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