Semi-Blind Channel Estimation for FDD Massive MIMO Systems based on Correlatively Coded Analog Feedback

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***Abstract*—We propose a joint uplink/downlink channel estimation scheme for frequency-division duplex (FDD) massive MIMO systems that is based on the analog feedback of the downlink channel state information (CSI) and a blind estimation of the uplink channels. The latter operation is enabled by the correlative coding of the uplink signals. Our scheme reduces the resource overhead that is needed to acquire full CSI for FDD Massive MIMO systems. The proposed technique requires a minimum of $2M$ channel uses in contrast to the start-of-the-art algorithm that requires $2M+K$, where M denotes the number of antennas at the base station, and K denotes the number of single-antenna users. Moreover, the performance of the uplink channel estimation can be strongly improved for low SNR scenarios because M uplink channel uses are utilized (in contrast to K channel uses of the start-of-the-art algorithm Echo-MIMO).**

I. Introduction

Closed-loop CSI estimation in FDD massive MIMO systems is critical for harvesting the potential gains offered by the very large number of antennas. However, the required training overhead grows with the number of antennas at the base station as well as with the number of terminals, which makes the accurate acquisition of high dimensional CSI very costly. To overcome this problem, one typically assumes the availability of side information such as the knowledge of the channel statistics [1], or channel sparsity in the time, frequency or angular domain [2], [3]. In both cases, one of the main objectives is to reduce the feedback overhead that is needed to transfer the downlink CSI back to the base station. Though, if no a-priori information on the channel structure is available then the full CSI needs to be fed back in order to exploit the full dimension of the channel. For this case, the analog CSI feed back has been proposed in e.g., [4], [5], which employs two training phases; one for a dedicated uplink channel training and one for the downlink training and analog feedback in the uplink. In [6], [7] it is shown for the i.i.d. Rayleigh fading case, that the analog feedback is optimal the sense of mean square error of the downlink CSI if the number of feedback symbols equals the number of feedback channel uses. The practicability of this feedback scheme is demonstrated in [8].

Our goal is to combine the uplink and downlink training phases. Instead of using a dedicated uplink training phase, we use the feedback signals of the downlink CSI in order to estimate the uplink channels. Since the feedback signals are unknown, we have to resort to a blind channel estimation scheme. Though, blind techniques rely on specific characteristics of the received signals, such as cyclostationarity [9], higher order cumulants [10] or second-order statistics [11]. We adopt the latter case by utilizing correlative filters at the terminals in order to shape the feedback signals, similar to [12]. One should note that such an approach provides estimates of the uplink and downlink channel subspaces rather than the exact channel coefficients; that is, the channels are known only up to an unknown complex scalar scaling factor. However, for typical precoding and equalization methods such as zero-forcing or maximum ratio transmission/combining, the knowledge of the channel subspaces is completely sufficient.

After introducing the underlying system model in Section II, we describe in Section III the training scheme and the channel estimation procedure. In Section IV, we conduct numerical experiments in order to characterize the achievable CSI accurary in terms of root mean square subspace error[1] (RMSE), and we illustrate our scheme's sensible operation range in combination with maximum ratio transmission/combining. Finally, we give a short outlook on the final version of the paper in Section V.

Notation: Vectors and matrices are given in lowercase and uppercase boldface letters, respectively. $(\cdot)^H$ denotes the Hermitian transpose. The symbol E denotes the expectation operator. $\boldsymbol{I}_M$ denotes the $M\times M$ identity matrix. The Grasmannian $G_M(\mathbb{C}^\tau)$ denotes the set of M dimensional subspaces in $\mathbb{C}^\tau$. The superscript $\#$ denotes the Moore-Penrose pseudoinverse.

II. System Model

We consider a time-invariant, frequency-flat channel. A base station, equipped with M antennas, serves K single-antenna terminals. In the downlink, the base station array transmits and the terminals receive. At integer time t an $M\times 1$ complex vector $\boldsymbol{s}(t)=[s_1(t)\cdots s_M(t)]^T$ is transmitted

[1]Rather than estimating a vector $\boldsymbol{\xi}\in\mathbb{C}^M$ itself, we resort to the estimation of the subspace $\mathcal{R}(\boldsymbol{\xi})$ spanned by $\boldsymbol{\xi}$. Thus, the estimation takes place in the Grassmann manifold $G_1(\mathbb{C}^M)$. The accuracy of a subspace estimator is quantified by using the natural metric on $G_1(\mathbb{C}^M)$ [13]: The squared distance between two subspaces is given by the sum of the squared principle angles between these spaces. For two one-dimensional subspaces $\mathcal{R}(\boldsymbol{\xi})$ and $\mathcal{R}(\hat{\boldsymbol{\xi}})$, their distance is

$$d(\boldsymbol{\xi},\hat{\boldsymbol{\xi}})=\arccos\left(\frac{|\boldsymbol{\xi}^H\hat{\boldsymbol{\xi}}|}{\|\boldsymbol{\xi}\|\left\|\hat{\boldsymbol{\xi}}\right\|}\right). \tag{1}$$

The performance of a subspace estimator can be quantified by the root mean square subspace error (RMSE), which is defined as

$$\epsilon=\mathrm{E}\left[d(\boldsymbol{\xi},\hat{\boldsymbol{\xi}})^2\right]^{\frac{1}{2}} \tag{2}$$

and the K terminals collectively receive a $K \times 1$ vector $\boldsymbol{x}(t) = [x_1(t) \cdots x_K(t)]^T$

$$\boldsymbol{x}(t) = \sqrt{\rho_{\mathrm{D}}/M}\boldsymbol{H}\boldsymbol{s}(t) + \boldsymbol{w}(t) \tag{3}$$

where ρ_{D} denotes the downlink signal-to-noise ratio[2] (SNR), $\boldsymbol{H}$ is the $K \times M$ downlink propagation matrix and $\boldsymbol{w}(t)$ is a $K \times 1$ vector comprising both receiver noise and interference. The components of $\boldsymbol{s}(t)$ and $\boldsymbol{w}(t)$ are i.i.d. $\mathcal{CN}(0,1)$. The downlink propagation matrix is partitioned into K i.i.d. row vectors

$$\boldsymbol{H}^H = [\boldsymbol{h}_1, \ldots, \boldsymbol{h}_K] \tag{4}$$

where $\boldsymbol{h}_k$ is the $M \times 1$ propagation vector from the array to the k-th terminal. The components of $\boldsymbol{h}_k$ are i.i.d. $\mathcal{CN}(0,1)$.

In the uplink, the terminals transmit scalar symbols and the base station array receives their transmissions collectively. We assume the K terminals are synchronous and collectively transmit a $K \times 1$ complex vector $\boldsymbol{t}(t)$ and the array receives a $M \times 1$ vector $\boldsymbol{y}(t)$

$$\boldsymbol{y}(t) = \sqrt{\rho_{\mathrm{U}}}\boldsymbol{G}\boldsymbol{t}(t) + \boldsymbol{n}(t) \tag{5}$$

where ρ_{U} denotes the uplink SNR, $\boldsymbol{G}$ is the $M \times K$ uplink propagation matrix and $\boldsymbol{n}(t)$ is a $M \times 1$ vector comprising both receiver noise and interference. The components of $\boldsymbol{t}(t)$ and $\boldsymbol{n}(t)$ are i.i.d. $\mathcal{CN}(0,1)$. The uplink propagation matrix is partitioned into K i.i.d. column vectors

$$\boldsymbol{G} = [\boldsymbol{g}_1, \ldots, \boldsymbol{g}_K] \tag{6}$$

where $\boldsymbol{g}_k$ is the $M \times 1$ propagation vector from the k-th terminal to the array. The components of $\boldsymbol{g}_k$ are i.i.d. $\mathcal{CN}(0,1)$.

III. Joint Uplink/Downlink Channel Estimation

A. Pilot Transmission

The downlink propagation channel is learned through known pilots that are transmitted by the base station array. The required number of training symbols is proportional to M and independent of K. It is most efficient for the base station antennas to transmit power-scaled signals of duration τ that are mutually orthonormal. We may choose τ as large as needed to give the desired quality of the channel estimate.

Let $\boldsymbol{S}^H = [\boldsymbol{s}(1) \ldots \boldsymbol{s}(\tau)]$ denote the $M \times \tau$ matrix of collective training signals. Then $\boldsymbol{S}$ may be written

$$\boldsymbol{S}^H = \sqrt{\rho_{\mathrm{D}}\tau/M}\boldsymbol{\Phi}^H \tag{7}$$

where $\boldsymbol{\Phi}$ is a $\tau \times M$ unitary matrix (i.e., $\boldsymbol{\Phi}^H\boldsymbol{\Phi} = \boldsymbol{I}_M$). Then, the $1 \times \tau$ signal vector received at the k-th terminal is

$$\boldsymbol{x}_k^H = \sqrt{\rho_{\mathrm{D}}\tau/M}\boldsymbol{h}_k^H\boldsymbol{\Phi}^H + \boldsymbol{w}_k^H \tag{8}$$

where $\boldsymbol{x}_k^H = [x_k(1) \ldots x_k(\tau)]$ and $\boldsymbol{w}_k^H = [w_k(1) \ldots w_k(\tau)]$.

The uplink channel is learned through the analog feedback of the downlink signals in the uplink. To do so, we impose specific spectral properties on the individual uplink signals, and utilize a blind source separation technique known as the second-order blind identification (SOBI) algorithm [11]. To guarantee the separability of the superimposed signals at the base station, the transmit signals must satisfy the second-order identifiability condition [14, Theorem 2, (A2)]; that is, they must have different normalized spectra. Therefor, each terminal employs a distinct correlative transmit filter with the impulse response $\boldsymbol{c}_k$ of length τ. Each filter realizes an AR model of order 1 with coefficient $a_k = \alpha_k \exp(j\theta_k)$. The angles are equidistantly distributed in the interval $[0, 2\pi]$, e.g., $\theta_k = k2\pi/K$. The modulus $\alpha_k \in (0,1)$ of the AR coefficient can be used to tweak the channel estimation performance. A typical value is $\alpha_k = 0.9$. Given the AR model, the corresponding filter impulse response is given by

$$\boldsymbol{c}_k = \left[1, a_k, a_k^2, \ldots, a_k^{\tau-1}\right]^T \tag{9}$$

The k-th transmit signal, scaled by the power control coefficient $\sqrt{\beta_k}$ to ensure $\mathrm{E}[\boldsymbol{t}_k^2] = 1$, is

$$\boldsymbol{t}_k = \sqrt{\beta_k}(\boldsymbol{c}_k * \boldsymbol{x}_k)(t) = \sqrt{\beta_k}\boldsymbol{C}_k\boldsymbol{x}_k \tag{10}$$

where $\boldsymbol{C}_k$ is the Toeplitz matrix specified by $\boldsymbol{c}_k$. Note that the performance of the SOBI algorithm can be improved by replacing the linear convolution by a circular[3] one.

The CSI-bearing signal, received at the base station, is

$$\boldsymbol{Y} = \sqrt{\rho_{\mathrm{D}}}\boldsymbol{G}\left[\boldsymbol{t}_1, \ldots, \boldsymbol{t}_K\right]^H + \boldsymbol{N} \tag{11}$$

where the $\boldsymbol{N} = [\boldsymbol{n}(1) \ldots \boldsymbol{n}(\tau)]$.

B. Uplink Channel Estimation

For the uplink channel estimation, we utilize the SOBI algorithm, which consists of the following steps [11, III.D]:

1) Estimate the sample covariance matrix $\hat{\boldsymbol{R}}(0)$ for time lag $\delta = 0$; that is,

$$\hat{\boldsymbol{R}}(0) = \tau^{-1}\boldsymbol{Y}\boldsymbol{Y}^H \tag{12}$$

Denote by $\lambda_1, \ldots, \lambda_K$ the K largest eigenvalues and $\boldsymbol{v}_1, \ldots, \boldsymbol{v}_K$ the corresponding eigenvectors of $\hat{\boldsymbol{R}}(0)$.

2) Estimate the uplink noise variance σ_N^2 by averaging the $M - K$ smallest eigenvalues of $\hat{\boldsymbol{R}}(0)$. Then, perform a whitening of the received signal $\boldsymbol{Y}$ by left-multiplying it with the matrix

$$\hat{\boldsymbol{\Psi}} = \left[(\lambda_1 - \hat{\sigma}_N^2)^{-\frac{1}{2}}\boldsymbol{v}_1, \ldots, (\lambda_K - \hat{\sigma}_N^2)^{-\frac{1}{2}}\boldsymbol{v}_K\right]^H, \tag{13}$$

yielding the $K \times \tau$ matrix $\boldsymbol{Z} = \hat{\boldsymbol{\Psi}}\boldsymbol{Y}$.

3) Compute the sample covariance matrix

$$\underline{\hat{\boldsymbol{R}}}(\delta) = \frac{1}{\tau - \delta}\sum_{t=1}^{\tau-\delta}\boldsymbol{z}(t+\delta)\boldsymbol{z}^H(t) \tag{14}$$

[2]For simplicity, it is assumed the the downlink and uplink SNRs are equal for all terminals.

[3]The estimation of uplink channels by blind separation of received (superimposed) signals relies on the estimation of (spatial) covariance matrices for different time lags. However, due to the finite length of the uplink signals such estimation may suffer from a finite sample support. By employing a circular convolution it is possible to introduce correlation between *all* the samples of signal, while a linear correlation filter would exhibit some transient behavior at the beginning of the signal. As a result, the covariance matrix estimation accuracy can be improved for finite sample support.

for the time lag $\delta = 1$. Optionally, a set of non-zero time lags $\mathcal{D} = \{\delta_j | j = 1, \ldots, D\}$ can be used.

4) A unitary $\hat{\boldsymbol{U}}$ is then obtained as joint diagonalizer [15] of the set $\{\hat{\underline{\boldsymbol{R}}}(\delta_j) | j = 1, \ldots, D\}$. For the case $D = 1$, $\hat{\boldsymbol{U}}$ is directly given by the eigendecomposition $\hat{\underline{\boldsymbol{R}}}(\delta) = \hat{\boldsymbol{U}}\boldsymbol{\Sigma}\hat{\boldsymbol{U}}^H$.
5) The transmit signals are estimated (up to a permutation and scalar scaling factor) as $\left[\hat{\boldsymbol{t}}_1, \ldots, \hat{\boldsymbol{t}}_K\right]^H = \hat{\boldsymbol{U}}^H \hat{\boldsymbol{\Psi}} \boldsymbol{Y}$, and the uplink channel matrix $\boldsymbol{G}$ is estimated as $\hat{\boldsymbol{G}} = \hat{\boldsymbol{\Psi}}^{\#} \hat{\boldsymbol{U}}$

Remark 1. The SOBI algorithm provides estimates for the uplink channels up to a permutation of the terminal indicies. We assume that the permutation ambiguity can be resolved; e.g., by means of a subsequent uplink data transmission whereby the data includes terminal-specific information such as scrambled cyclic redundancy checks.

Remark 2. For the case $\tau = M$ (i.e., the pilot matrix $\boldsymbol{\Psi}$ is a square unitary matrix), the received sequences $\boldsymbol{x}_k$ are realizations of a complex circular white Gaussian process. They are indistinguishable from a statistical domain point of view because their power spectral densities exhibit a flat pattern. By filtering them with the described AR models of order 1, we assign distinct spectral patterns to transmit signals; that is, the spectral density $f_k(\lambda)$ of the k-th transmit signal becomes

$$f_k(\lambda) \propto \frac{1}{\left|1 - a_k e^{-j2\pi\lambda}\right|^2}. \tag{15}$$

This spectral shaping ensures that a set of time lags $\mathcal{D}$ exists such that the joint diagonalizer of the set of spatial covariance matrices is unique, up to irrelevant scaling and permutation of columns. Note that the SOBI algorithm estimates the spatial covariance matrices, and the corresponding estimation error vanishes as τ grows large; that is, the angle between the subspaces spanned by the estimated and the true uplink channel vectors will go to zero as τ goes to infinity.

C. Downlink Channel Estimation

Given the estimate $\hat{\boldsymbol{t}}_k$ of the k-th terminal's signal, an estimator for the downlink channel $\boldsymbol{h}_k$ is by

$$\hat{\boldsymbol{h}}_k = \hat{\boldsymbol{t}}_k \boldsymbol{D}_k (\boldsymbol{D}_k^H \boldsymbol{D}_k)^{-1}, \tag{16}$$

where $\boldsymbol{D}_k = \boldsymbol{C}_k \Phi$.

IV. Numerical Experiments

We simulate a large scale antenna array with $M \in \{40, 80, 160\}$ antennas, that estimates the uplink and downlink channels simultaneously for $K = 8$ terminals. We assume a symmetric link budget with $\rho_\mathrm{D} = 25 \cdot \rho_\mathrm{U}$; that is, the total transmit power of the base station is 14dB higher than the transmit power of the terminals. The modulus parameters of the terminal's AR coefficients are set to $\alpha_k = 0.9, \forall k$. For the SOBI algorithm we use a single time lag $\delta = 1$ for the matrix diagonalization.

As a reference case we simulate the Echo-MIMO scheme [16]. This scheme does not require correlative filter at the terminals, but relies on a separate uplink training phase in order to learn the uplink channels. This channel knowledge is then used to separate the uplink and downlink channel matrices in the received signal $\boldsymbol{Y}$. For the dedicated training phase we assume the same uplink SNR ρ_U. Note that this training requires additional K transmission resources in the uplink.

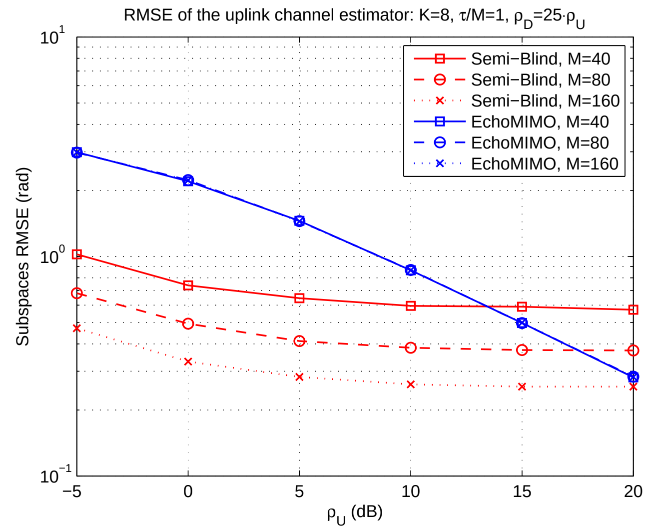

Fig. 1. RMSEs of the uplink channel estimators versus uplink SNR γ_U for $K = 8, T/M = 1, \gamma_\mathrm{D} = 25 \cdot \rho_\mathrm{U}$

Figures 1 and 2 show the achievable root mean square error of the channel estimators. The semi-blind approach achieves higher uplink channel accuracies because it employs M observations instead of K, as done by the Echo-MIMO scheme. However, its performance saturates for high SNR due to the limited sample support in the sample covariance matrix estimation. In addition, the accuracy of the downlink channel estimates strongly suffers from the correlative filtering, which needs to be reverted at the base station and which causes a noise amplication. With increasing training length τ, the error floor gets smaller.

In order to illustrate the impact of the CSI quality on the system performance, we conducted additional Monte Carlo simulations for the maximum ratio transmission in the downlink and the maximum ratio combining in the uplink. We assume the same uplink and downlink SNR for the training and data phase. The achievable average SINRs are shown in Figures 3 and 4. Interestingly, the proposed scheme outperforms the Echo-MIMO algorithm for low SNR regimes. Moreover, for moderate SNRs the performance loss stays relatively small and decreases with larger training lengths τ. Note that for zero-forcing precoding/equalization (not depicted here), the behavior is similar.

V. Outlook for Final Version

We will provide extensive simulation results for different channel models (i.e.; i.i.d. Rayleigh fading and the QUADRIGA channel model [17]), and different configurations for M, τ and α_k in order to clearly assess the advantages and drawbacks of the proposed scheme.

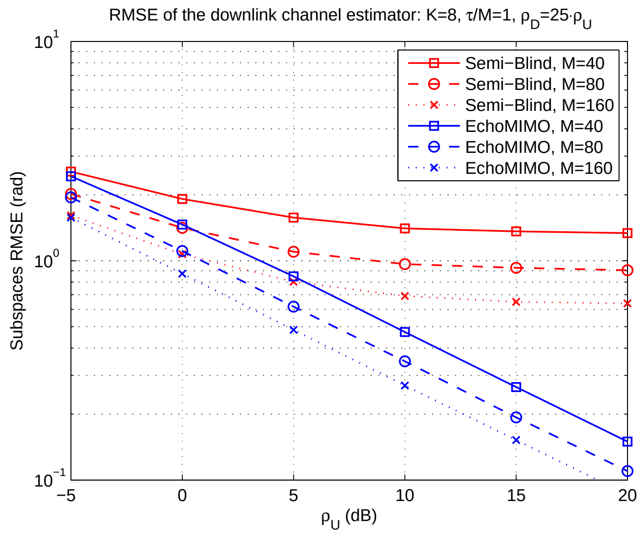

Fig. 2. RMSEs of the downlink channel estimators versus uplink SNR $\gamma_{\rm U}$ for $K = 8, T/M = 1, \gamma_{\rm D} = 25 \cdot \rho_{\rm U}$

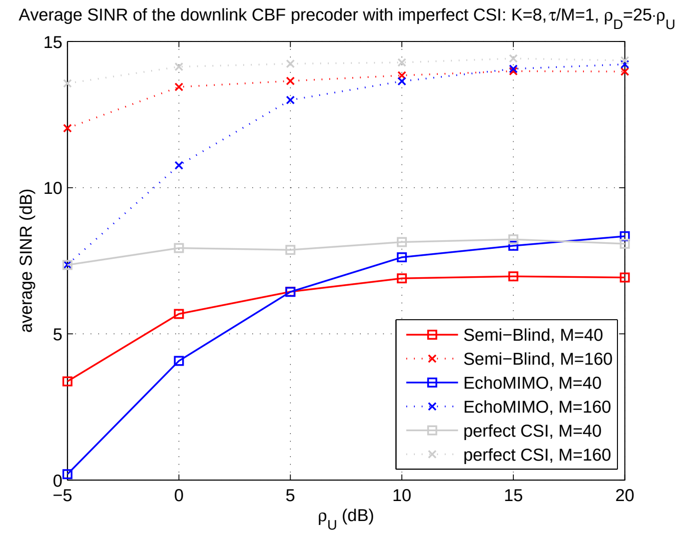

Fig. 4. Average SINR of the downlink CBF precoder (based on estimated CSI) versus uplink SNR $\gamma_{\rm U}$ for $K = 8, T/M = 1, \gamma_{\rm D} = 25 \cdot \rho_{\rm U}$

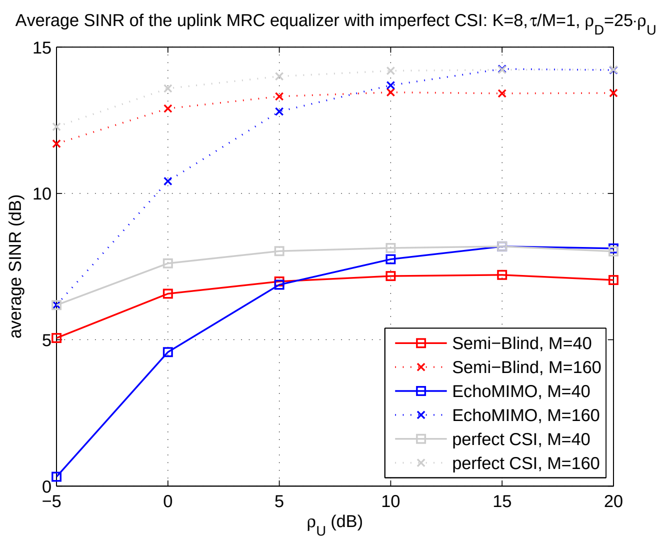

Fig. 3. Average SINR of the uplink MRC equalizer (based on estimated CSI) versus uplink SNR $\gamma_{\rm U}$ for $K = 8, T/M = 1, \gamma_{\rm D} = 25 \cdot \rho_{\rm U}$

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