Tensor-Based Approach for Time-Delay Estimation

Bilal Hammoud[1], Felix Antreich[1], Josef A. Nossek[2], João Paulo C. L. da Costa[3], and André L. F. de Almeida[4]

[1]German Aerospace Center (DLR), Institute for Communications and Navigation, Wessling, Germany
[2]Institute for Circuit Theory and Signal Processing, Technical University Munich, Munich, Germany
[3]Department of Electrical Engineering, University of Brasília (UnB), Brasília, Distrito Federal, Brazil
[4]Department of Teleinformatics Engineering, Federal University of Ceará (UFC), Fortaleza, Brazil

I. Introduction

The quality of the ranging data provided by a Global Navigation Satellite Systems (GNSS) receiver largely depends on the synchronization error, that is, on the accuracy of the propagation time-delay estimation of the line-of-sight (LOS) with respect to each satellite. In case the LOS signal is corrupted by several superimposed delayed replicas (reflective, diffractive, or refractive multipath), the estimation of the propagation time-delay and thus the position can be severely degraded using state-of-the-art GNSS receivers [1], [2], [3]. Especially, for high precision and safety-critical applications, *e.g.* aviation, maritime, rail, precision farming, surveying or automotive applications, multipath mitigation is very important in order to enable robust and reliable positioning.

Several techniques have been proposed in the literature for solving the multipath problem in GNSS using one antenna, see *e.g.* [4], [5], [6]. When using antenna arrays high resolution parameter estimation algorithms provide high accurate results [7], [8], [9], but they entail rather high complexity in the parameter estimation as multi-dimensional nonlinear problems have to be solved. Furthermore, they also require the use of accurate model order estimation algorithms [7].

In this work, we present an approach for which no multi-dimensional nonlinear problem needs to be solved and also no model order estimation is required. We derive a tensor-based filtering approach using an antenna array and a compression method based on canonical components (CC) with a bank of signal-matched correlators [9] in order to mitigate multipath and to estimate the time-delay of the LOS signal. First, we resort to multi-dimensional filtering based on the principal singular vectors of the received data tensor. In order to separate highly correlated signal components in the multi-dimensional signal subspace methods like forward-backward averaging (FBA) [10], spatial smoothing (SPS) [11], and the recently developed expanded spatial smoothing (SPS-EXP) [12] are applied. Afterwards, time-delay estimation of the LOS signal is performed with a simple interpolation based on the multi-dimensional filtered cross-correlation values of the bank of correlators.

The proposed pre-processing schemes require that the antenna array response is left centro-hermitian. In case the array response is not left centro-hermitian, signal adaptive array interpolation methods can be applied to transform the array response to a centro-hermitian array response [13].

The proposed approach is capable of separating highly correlated and even coherent signals and is approaching the respective Cramer-Rao lower bound (CRLB) for time-delay estimation in the compressed time domain.

II. Signal Model

In the following, we define the pre- and post-correlation signal model for a multi-antenna GNSS receiver and we introduce a compression method based on a bank of signal matched correlators.

A. Pre-correlation Signal Model

The complex baseband signal of one GNSS satellite with bandwidth B that is received by an antenna array with M sensor elements can be given as

$$\mathbf{x}(t) = \mathbf{s}(t) + \mathbf{n}(t) = \sum_{\ell=1}^{L} \mathbf{s}_\ell(t) + \mathbf{n}(t) \tag{1}$$

where $\mathbf{s}(t) \in \mathbb{C}^{M\times 1}$ denotes the superimposed signal replicas

$$\mathbf{s}_\ell(t) = \mathbf{a}\left(\phi_\ell\right)\ \gamma_\ell\, c(t-\tau_\ell). \tag{2}$$

$\mathbf{a}\left(\phi_\ell\right) \in \mathbb{C}^{M\times 1}$ defines the steering vector of an antenna array with azimuth angle ϕ_ℓ, $c(t-\tau_\ell)$ denotes a periodically repeated pseudo random (PR) sequence $c(t)$ with time-delay τ_ℓ, chip duration T_c, and period $T = N_c T_c$ with $N_c \in \mathbb{N}$. γ_ℓ is the complex amplitude. Additionally, we assume temporally and spatially white complex Gaussian noise $\mathbf{n}(t) \in \mathbb{C}^{M\times 1}$. In the following the parameters of the line-of-sight (LOS) signal are indicated with $\ell = 1$ and the parameters of the non-LOS (NLOS) signals (multipath) with $\ell = 2, \ldots, L$. We define the signal parameter vectors

$$\boldsymbol{\eta} = [\mathrm{Re}\{\boldsymbol{\gamma}\}^{\mathrm{T}}, \mathrm{Im}\{\boldsymbol{\gamma}\}^{\mathrm{T}}, \boldsymbol{\phi}^{\mathrm{T}}, \boldsymbol{\tau}^{\mathrm{T}}]^{\mathrm{T}} \tag{3}$$

$$\boldsymbol{\eta}_\ell = [\mathrm{Re}\{\gamma_\ell\}, \mathrm{Im}\{\gamma_\ell\}, \phi_\ell, \tau_\ell]^{\mathrm{T}} \tag{4}$$

with $\boldsymbol{\gamma} = [\gamma_1, \ldots, \gamma_L]^{\mathrm{T}}$, $\boldsymbol{\phi} = [\phi_1, \ldots, \phi_L]^{\mathrm{T}}$ and $\boldsymbol{\tau} = [\tau_1, \ldots, \tau_L]^{\mathrm{T}}$. The spatial observations are collected in K periods of the PR sequence of N time instances, thus $\mathbf{x}[(k-1)N+n] = \mathbf{x}(((k-1)N+n)\,T_s)$ with $n = 1, \ldots, N$, $k = 1, \ldots, K$, and the sampling frequency $\frac{1}{T_s} = 2B$. The channel parameters are assumed constant at least during the

k-th period of the observation interval. Collecting the samples of the k-th period of the observation interval leads to

$$\mathbf{X}[k] = \left[\mathbf{x}[(k-1)N+1], \dots, \mathbf{x}[(k-1)N+N]\right] \in \mathbb{C}^{M\times N} \quad (5)$$

$$\mathbf{N}[k] = \left[\mathbf{n}[(k-1)N+1], \dots, \mathbf{n}[(k-1)N+N]\right] \in \mathbb{C}^{M\times N} \quad (6)$$

$$\mathbf{S}[k;\boldsymbol{\eta}] = \left[\mathbf{s}[(k-1)N+1], \dots, \mathbf{s}[(k-1)N+N]\right] \in \mathbb{C}^{M\times N} \quad (7)$$

$$\mathbf{S}_\ell[k;\boldsymbol{\eta}_\ell] = \left[\mathbf{s}_\ell[(k-1)N+1], \dots, \mathbf{s}_\ell[(k-1)N+N]\right] \in \mathbb{C}^{M\times N}. \quad (8)$$

Thus, the signal can be written in matrix notation as

$$\begin{aligned} \mathbf{X}[k] = \mathbf{S}[k;\boldsymbol{\eta}] + \mathbf{N}[k] &= \sum_{\ell=1}^{L} \mathbf{S}_\ell[k;\boldsymbol{\eta}_\ell] + \mathbf{N}[k] \\ &= \mathbf{A}[k]\,\boldsymbol{\Gamma}[k]\,\mathbf{C}[k] + \mathbf{N}[k] \quad (9) \end{aligned}$$

where

$$\mathbf{A}[k] = [\mathbf{a}(\phi_1), \dots, \mathbf{a}(\phi_\ell), \dots, \mathbf{a}(\phi_L)] \in \mathbb{C}^{M\times L} \quad (10)$$

denotes the steering matrix, while

$$\boldsymbol{\Gamma}[k] = \mathrm{diag}\{\boldsymbol{\gamma}\} \in \mathbb{C}^{L\times L} \quad (11)$$

is a diagonal matrix whose entries are the complex amplitudes of the signal replicas $\boldsymbol{\gamma} = [\gamma_1, \dots, \gamma_L]^{\mathrm{T}}$. Furthermore,

$$\mathbf{C}[k] = [\mathbf{c}[k;\tau_1] \cdots \mathbf{c}[k;\tau_\ell] \cdots \mathbf{c}[k;\tau_L]]^{\mathrm{T}} \in \mathbb{R}^{L\times N} \quad (12)$$

contains the sampled and shifted $c(t)$ for each impinging wavefront

$$\mathbf{c}[k;\tau_\ell] = [c(((k-1)N+1)T_s - \tau_\ell), \dots, \quad (13)$$

$$\dots, c(((k-1)N+N)T_s - \tau_\ell)]^{\mathrm{T}}. \quad (14)$$

In general $||\mathbf{c}[k;\tau_\ell]||_2^2 \neq N$ for all τ_ℓ, however in many cases[1] we can assume that $||\mathbf{c}[k;\tau_\ell]||_2^2 \approx N, \forall\tau_\ell \quad \forall k$ and if additionally $N \geq N_c$ and $N/N_c \in \mathbb{N}$ we get $\mathbf{c}[k;\tau_\ell] = \mathbf{c}(\tau_\ell), \quad \forall k$. In the following we assume that the array response $\mathbf{A}[k]$ is left centro-hermitian with

$$\mathbf{A}[k] = \boldsymbol{\Pi}_M \mathbf{A}^*[k] \quad (15)$$

where

$$\boldsymbol{\Pi}_M = \begin{bmatrix} & & 1 \\ & \iddots & \\ 1 & & \end{bmatrix} \in \mathbb{R}^{M\times M}. \quad (16)$$

B. Post-correlation Signal Model

A Fisher Information preserving compression applying a bank of Q correlators at the output of each antenna is used. We follow a canonical component (CC) method where the information about the signal parameters are extracted from the received signal by correlating with several delayed replicas of the signal with relative delays associated to a regular grid [9]. Thus, the signal at the output of the q-th correlator of the bank of correlators at the output of each antenna element with $q = 1, \dots, Q$ can be written

$$\mathbf{y}_q[k] = \mathbf{X}[k](\mathbf{c}[k;\kappa_q])^* \in \mathbb{C}^{M\times 1} \quad (17)$$

[1] e.g. in case of GPS C/A PR sequences with bandwidth $B \geq 1.023$ MHz.

where κ_q denotes the time-delay for the correlator tap q. We can define the output signal of the bank of correlators by

$$\mathbf{Y}[k] = [\mathbf{y}_1[k], \dots, \mathbf{y}_q[k], \dots, \mathbf{y}_Q[k]] = \mathbf{X}[k]\mathbf{Q}[k] \in \mathbb{C}^{M\times Q} \quad (18)$$

and the compression matrix

$$\mathbf{Q}[k] = [\mathbf{c}[k;\kappa_1], \dots, \mathbf{c}[k;\kappa_q], \dots, \mathbf{c}[k;\kappa_Q]] \in \mathbb{R}^{N\times Q}. \quad (19)$$

Thus, we can write

$$\mathbf{Y}[k] = \mathbf{A}[k]\boldsymbol{\Gamma}[k]\mathbf{C}[k]\mathbf{Q}[k] + \mathbf{N}[k]\mathbf{Q}[k]. \quad (20)$$

The so-called thin singular value decomposition (SVD) or also called economy size SVD of $\mathbf{Q}[k]$ in case $Q \ll N$ is given by $\mathbf{Q}[k] = \mathbf{U}\boldsymbol{\Sigma}\mathbf{V}^{\mathrm{H}}$, where the columns of $\mathbf{U} \in \mathbb{C}^{N\times Q}$ and $\mathbf{V} \in \mathbb{C}^{Q\times Q}$ only refer to the non-zero singular values and thus all diagonal elements of $\boldsymbol{\Sigma} \in \mathbb{C}^{Q\times Q}$ are larger than zero. We define the compression matrix $\mathbf{Q}_\omega[k]$, with $\mathbf{Q}_\omega^{\mathrm{H}}[k]\mathbf{Q}_\omega[k] = \mathbf{I}_Q$, that preserves the input noise properties at the output of the bank of correlators using the thin SVD as follows:

$$\mathbf{Q}_\omega[k] = \mathbf{Q}[k](\boldsymbol{\Sigma}\mathbf{V}^{\mathrm{H}})^{-1} = \mathbf{U} \in \mathbb{C}^{N\times Q} \quad (21)$$

Here, $\mathbf{I}_Q$ denotes a $Q \times Q$ identity matrix.

In the following, we assume that the time-delays τ_ℓ , the azimuth angles ϕ_ℓ and thus also the compression matrix $\mathbf{Q}_\omega[k]$ are constant with respect to K periods. This is a reasonable assumption for GNSS *e.g.* for moving users in an urban city center where the average life span of echoes is approximately 1 m [14]. Life span refers to the motion distance across which a multipath signal is observable, i.e. active. In the latter case, for an observation time of 30 ms ($K = 30$ for a GPS C/A signal with $N = 2046$ and $B = 1.023$ MHz), a maximum velocity of 100 km/h, and a spatial resolution of $c/B \approx 293$ m, the multipath time-delays can be assumed constant, where c denotes the speed of light. The time-delay of the LOS signal τ_1 in general can be assumed constant for an even longer observation time. Also the azimuth angles of LOS and NLOS signals ϕ_ℓ can be assumed constant for such an observation time.

Thus, we can write

$$\begin{aligned} \bar{\mathbf{Y}}[k] &= \mathbf{A}\boldsymbol{\Gamma}[k]\mathbf{C}\mathbf{Q}_\omega + \mathbf{N}[k]\mathbf{Q}_\omega \quad (22) \\ &= \mathbf{A}\boldsymbol{\Gamma}[k]\mathbf{C}\mathbf{Q}_\omega + \mathbf{N}_\omega[k] \quad \in \mathbb{C}^{M\times Q}. \quad (23) \end{aligned}$$

Applying the vec-operator on matrix $\bar{\mathbf{Y}}[k]$, we get

$$\begin{aligned} \tilde{\mathbf{y}}[k] &= \mathrm{vec}\{\bar{\mathbf{Y}}[k]\} = \underbrace{\mathrm{vec}\{\mathbf{A}\boldsymbol{\Gamma}[k]\mathbf{C}\mathbf{Q}_\omega\}}_{=\tilde{\mathbf{s}}[k]} + \underbrace{\mathrm{vec}\{\mathbf{N}_\omega[k]\}}_{=\tilde{\mathbf{n}}[k]} \\ &= ((\mathbf{C}\mathbf{Q}_\omega)^{\mathrm{T}} \diamond \mathbf{A})\boldsymbol{\gamma}[k] + \tilde{\mathbf{n}}[k] \quad \in \mathbb{C}^{MQ\times 1} \quad (24) \end{aligned}$$

where $\diamond$ denotes the Khatri-Rao product. Collecting the data samples during K periods, we obtain

$$\tilde{\mathbf{Y}} = \underbrace{((\mathbf{C}\mathbf{Q}_\omega)^{\mathrm{T}} \diamond \mathbf{A})\tilde{\boldsymbol{\Gamma}}}_{=\tilde{\mathbf{S}}} + \tilde{\mathbf{N}} \quad \in \mathbb{C}^{MQ\times K} \quad (25)$$

with

$$\tilde{\mathbf{Y}} = [\tilde{\mathbf{y}}[1], \ldots, \tilde{\mathbf{y}}[k], \ldots, \tilde{\mathbf{y}}[K]] \tag{26}$$

$$= \begin{bmatrix} \mathbf{y}_1[1], \ldots, \mathbf{y}_1[k], \ldots, \mathbf{y}_1[K] \\ \vdots \\ \mathbf{y}_q[1], \ldots, \mathbf{y}_q[k], \ldots, \mathbf{y}_q[K] \\ \vdots \\ \mathbf{y}_Q[1], \ldots, \mathbf{y}_Q[k], \ldots, \mathbf{y}_Q[K] \end{bmatrix} \tag{27}$$

$$\tilde{\mathbf{S}} = [\tilde{\mathbf{s}}[1], \ldots, \tilde{\mathbf{s}}[k], \ldots, \tilde{\mathbf{s}}[K]] \in \mathbb{C}^{MQ \times K} \tag{28}$$

$$\tilde{\mathbf{N}} = [\tilde{\mathbf{n}}[1], \ldots, \tilde{\mathbf{n}}[k], \ldots, \tilde{\mathbf{n}}[K]] \in \mathbb{C}^{MQ \times K} \tag{29}$$

$$\tilde{\mathbf{\Gamma}} = [\boldsymbol{\gamma}[1], \ldots, \boldsymbol{\gamma}[k], \ldots, \boldsymbol{\gamma}[K]] \in \mathbb{C}^{L \times K}. \tag{30}$$

We define the tensor $\boldsymbol{\mathcal{S}} \in \mathbb{C}^{K \times Q \times M}$ collecting the signal data and a tensor $\boldsymbol{\mathcal{N}} \in \mathbb{C}^{K \times Q \times M}$ collecting the white noise data, respectively. The three different matrix unfoldings of the tensor $\boldsymbol{\mathcal{S}}$ can be expressed as [15]

$$[\boldsymbol{\mathcal{S}}]_{(1)} = \tilde{\mathbf{\Gamma}}^{\mathrm{T}}((\mathbf{C}\mathbf{Q}_\omega)^{\mathrm{T}} \diamond \mathbf{A})^{\mathrm{T}} \in \mathbb{C}^{K \times QM} \tag{31}$$

$$[\boldsymbol{\mathcal{S}}]_{(2)} = (\mathbf{C}\mathbf{Q}_\omega)^{\mathrm{T}}(\mathbf{A} \diamond \tilde{\mathbf{\Gamma}}^{\mathrm{T}})^{\mathrm{T}} \in \mathbb{C}^{Q \times MK} \tag{32}$$

$$[\boldsymbol{\mathcal{S}}]_{(3)} = \mathbf{A}(\tilde{\mathbf{\Gamma}}^{\mathrm{T}} \diamond (\mathbf{C}\mathbf{Q}_\omega)^{\mathrm{T}})^{\mathrm{T}} \in \mathbb{C}^{M \times KQ}. \tag{33}$$

Finally, we can write the tensor signal model

$$\boldsymbol{\mathcal{Y}} = \boldsymbol{\mathcal{S}} + \boldsymbol{\mathcal{N}} \in \mathbb{C}^{K \times Q \times M}. \tag{34}$$

It is instructive to mention that the signal tensor $\boldsymbol{\mathcal{S}}$ follows a third-order Parallel Factors (PARAFAC) decomposition [15], [16] with matrix factors $\tilde{\mathbf{\Gamma}}^{\mathrm{T}}$, $(\mathbf{C}\mathbf{Q}_\omega)^{\mathrm{T}}$, and $\mathbf{A}$.

III. Proposed Tensor-Based Approach for Time-Delay Estimation

In this section, we present different algorithms that use multi-linear algebra in order to estimate the time-delay of the LOS signal while mitigating the effect of the NLOS signals (multipath). In the following, we assume that the receive power of the LOS signal is larger than those of the NLOS signals.

A. High Order Singular Value Decomposition (HOSVD)

Applying HOSVD on our signal model, we can write [15]

$$\boldsymbol{\mathcal{Y}} = \boldsymbol{\mathcal{R}} \times_1 \mathbf{U}^{(1)} \times_2 \mathbf{U}^{(2)} \times_3 \mathbf{U}^{(3)} \tag{35}$$

with the data tensor $\boldsymbol{\mathcal{Y}}$ and the core tensor $\boldsymbol{\mathcal{R}} \in \mathbb{C}^{K \times Q \times M}$, and the unitary matrices $\mathbf{U}^{(1)} \in \mathbb{C}^{K \times K}$, $\mathbf{U}^{(2)} \in \mathbb{C}^{Q \times Q}$, and $\mathbf{U}^{(3)} \in \mathbb{C}^{M \times M}$. Here, the operator $\times_n$ denotes the so-called n-mode product of a tensor by a matrix [15]. Based on the core tensor $\boldsymbol{\mathcal{R}}$ ordering properties, we find that the n-mode singular vectors $\mathbf{u}_i^{(n)}$ are ordered in the unitary matrices $\mathbf{U}^{(n)}$ in a decreasing order of the magnitude of its corresponding singular values. Therefore, we can now define the vector $\mathbf{q}$ as

$$\mathbf{q} = \left(\left(\boldsymbol{\mathcal{Y}} \times_1 \left(\mathbf{u}_1^{(1)}\right)^{\mathrm{H}} \times_3 \left(\mathbf{u}_1^{(3)}\right)^{\mathrm{H}}\right) \mathbf{\Sigma}\mathbf{V}^{\mathrm{H}}\right)^{\mathrm{T}} \in \mathbb{C}^{Q \times 1} \tag{36}$$

where $\mathbf{q}$ represents the multi-dimensionally filtered cross-correlation values at each tap of the correlator bank.

Based on $\mathbf{q}$ and a cubic spline interpolation using the absolute value of its entries, we can derive the cost function $F(\tau)$ and then estimate the time-delay of the LOS signal by solving the problem

$$\hat{\tau}_1 = \arg\max_{\tau}\{F(\tau)\}. \tag{37}$$

B. Forward Backward Averaging (FBA)

In general, LOS and NLOS signals are highly correlated in case of GNSS. If a left centro-hermitian sensor array is assumed, the separation of LOS and NLOS signals by an adaptive multi-dimensional filtering as given in (36) can be improved using FBA [10].

The *extended* 3-mode unfolding of $\boldsymbol{\mathcal{Y}}$ can be given as

$$\mathbf{Z} = \begin{bmatrix} [\boldsymbol{\mathcal{Y}}]_{(3)} & \mathbf{\Pi}_M [\boldsymbol{\mathcal{Y}}]_{(3)}^* \mathbf{\Pi}_{KQ} \end{bmatrix} \in \mathbb{C}^{M \times 2KQ}. \tag{38}$$

Given the SVD

$$\mathbf{Z} = \mathbf{U}_{\mathrm{FBA}}^{(3)} \mathbf{\Sigma}_{\mathrm{FBA}}^{(3)} \left(\mathbf{V}_{\mathrm{FBA}}^{(3)}\right)^{\mathrm{H}} \tag{39}$$

we can select $\mathbf{u}_{1,\mathrm{FBA}}^{(3)} \in \mathbb{C}^{M \times 1}$ as the first column of $\mathbf{U}_{\mathrm{FBA}}^{(3)}$. Consequently, we can use $\mathbf{u}_{1,\mathrm{FBA}}^{(3)}$ instead of $\mathbf{u}_1^{(3)}$ in (36) in order to derive an improved space-time filtered vector of cross-correlations denoted as $\mathbf{q}_{\mathrm{FBA}}$.

C. Spatial Smoothing (SPS)

SPS [11] is another pre-processing scheme that can be used to *de-correlate* the impinging wavefronts in case of a left centro-hermitian or Vandermonde sensor array. To this end, a uniform linear array (ULA) with M sensors can be divided into L_s subarrays, each containing $M_s = M - L_s + 1$ sensor elements. The selection matrix corresponding to the ℓ_s-th subarray with $\ell_s = 1, \ldots, L_s$ can be defined as

$$\mathbf{J}_{\ell_s}^{(M)} = \begin{bmatrix} \mathbf{0}_{M_s \times \ell_s - 1} & \mathbf{I}_{M_s} & \mathbf{0}_{M_s \times L_s - \ell_s} \end{bmatrix} \in \mathbb{R}^{M_s \times M}. \tag{40}$$

Therefore, the spatially smoothed *extended* 3-mode unfolding of $\boldsymbol{\mathcal{Y}}$ is given by

$$\mathbf{W} = \begin{bmatrix} \mathbf{J}_1^{(M)}[\boldsymbol{\mathcal{Y}}]_{(3)} \cdots \mathbf{J}_{\ell_s}^{(M)}[\boldsymbol{\mathcal{Y}}]_{(3)} \cdots \mathbf{J}_{L_s}^{(M)}[\boldsymbol{\mathcal{Y}}]_{(3)} \end{bmatrix} \in \mathbb{C}^{M_s \times KQL_s}. \tag{41}$$

Given the SVD

$$\mathbf{W} = \mathbf{U}_{\mathrm{SPS}}^{(3)} \mathbf{\Sigma}_{\mathrm{SPS}}^{(3)} \left(\mathbf{V}_{\mathrm{SPS}}^{(3)}\right)^{\mathrm{H}} \tag{42}$$

we can get $\mathbf{u}_{1,\mathrm{SPS}}^{(3)} \in \mathbb{C}^{M_s \times 1}$ which is the singular vector corresponding to the strongest singular value.

Consequently, we can calculate a spatially smoothed multi-dimensionally filtered cross-correlation vector $\mathbf{q}_{\mathrm{SPS}} \in \mathbb{C}^{Q \times 1}$ using

$$\mathbf{q}_{\mathrm{SPS}} = \left(\left(\boldsymbol{\mathcal{Y}} \times_3 \mathbf{J}_1^{(M)} \times_1 \left(\mathbf{u}_1^{(1)}\right)^{\mathrm{H}} \times_3 \left(\mathbf{u}_{1,\mathrm{SPS}}^{(3)}\right)^{\mathrm{H}}\right) \mathbf{\Sigma}\mathbf{V}^{\mathrm{H}}\right)^{\mathrm{T}}. \tag{43}$$

Note that $\boldsymbol{\mathcal{Y}} \times_3 \mathbf{J}_1^{(M)}$ selects the first M_s rows of the 3-mode unfolding matrix $[\boldsymbol{\mathcal{Y}}]_{(3)}$. After cubic spline interpolation based on the absolute value of the entries of $\mathbf{q}_{\mathrm{SPS}}$, the estimation of the time-delay of the LOS signal is obtained by solving problem (37).

D. Expanded Spatial Smoothing (SPS-EXP)

The idea of expanded spatial smoothing (SPS-EXP) recently proposed in [12] is to use a fourth dimension for the subarrays instead of accumulating the spatially smoothed *extended* data in the time dimension.

We define the 4-th order tensor $\boldsymbol{\mathcal{Y}}_{\text{SPS-EXP}} \in \mathbb{C}^{K\times Q\times M_s\times L_s}$ using

$$[\boldsymbol{\mathcal{Y}}_{\text{SPS-EXP}}]_{(3)} = \left[\ \mathbf{J}_1^{(M)}[\boldsymbol{\mathcal{Y}}]_{(3)}\cdots\mathbf{J}_{\ell_s}^{(M)}[\boldsymbol{\mathcal{Y}}]_{(3)}\cdots\mathbf{J}_{L_s}^{(M)}[\boldsymbol{\mathcal{Y}}]_{(3)}\right]. \tag{44}$$

By applying HOSVD on $\boldsymbol{\mathcal{Y}}_{\text{SPS-EXP}}$, we can get the singular vectors corresponding to the strongest singular values of the SVDs of the unfoldings in K, M_s and L_s dimensions. Thus, we can calculate an extended spatially smoothed multi-dimensionally filtered cross-correlation vector as

$$\mathbf{q}_{\text{SPS-EXP}} = \left(\left(\boldsymbol{\mathcal{Y}}_{\text{SPS-EXP}} \times_1 \left(\mathbf{u}_{1,\text{SPS-EXP}}^{(1)}\right)^{\text{H}} \times_3 \left(\mathbf{u}_{1,\text{SPS-EXP}}^{(3)}\right)^{\text{H}} \times_4 \left(\mathbf{u}_{1,\text{SPS-EXP}}^{(4)}\right)^{\text{H}}\right)\boldsymbol{\Sigma}\mathbf{V}^{\text{H}}\right)^{\text{T}}. \tag{45}$$

After spline interpolation based on the absolute value of the entries of $\mathbf{q}_{\text{SPS-EXP}}$ estimation of the time-delay of the LOS signal can be performed as given in (37).

IV. Simulations

We assume a left centro-hermitian ULA with $M = 8$ isotropic sensor elements with half-wavelength spacing ($\Delta = \lambda/2$). For the SPS and SPS-EXP, we assume that the ULA is divided into $L_s = 5$ subarrays, each containing $M_s = 4$ sensor elements. The received signal is a GPS C/A signal with bandwidth $B = 1.023$ MHz and carrier frequency $f_c = 1575.42$ MHz. We consider a two-path scenario with a LOS and one NLOS signal ($L = 2$). The number of samples taken within one observation period k is $N = 2046$. The number of observation periods $K = 30$ and we assume that all the channel parameters are constant over K. The azimuth angle difference between LOS and NLOS signal is $\Delta\phi = 60°$. The signal phase for LOS and NLOS signals, denoted by $\arg\{\gamma_1\}$ and $\arg\{\gamma_2\}$, are assumed independent and identically distributed (i.i.d.) for each Monte Carlo simulation and drawn from a uniform distribution $[0, 2\pi[$. We performed 2000 Monte Carlo simulations to derive the root mean square error of the time-delay of the LOS signal $\text{RMSE}(\tau_1)$. The number of correlators in the bank is $Q = 11$. The carrier to noise density ratio is $C/N_0 = 48$ dB-Hz. Thus, the pre-correlation SNR approximately is -15 dB, and the post-correlation SNR approximately is 15 dB. The signal to multipath ratio SMR $= 5$ dB. The expectation of the Cramer Rao Lower Bound (CRLB) of the time-delay of the LOS signal with respect to the random signal phases $\arg\{\gamma_1\}$ and $\arg\{\gamma_2\}$ denoted by $\text{E}[\sqrt{\text{CRLB}(\tau_1)}]$ is derived to be used as a lower bound for comparison of the performance of the proposed multi-dimensional filters and subsequent time-delay estimation. The time-delay difference between LOS and NLOS signal is normalized by T_c and is denoted by $\Delta\tau/T_c$. The $\text{RMSE}(\tau_1)$ for the different methods presented above, HOSVD, HOSVD with FBA, HOSVD with SPS, and HOSVD with SPS-EXP as well as the $\text{E}[\sqrt{\text{CRLB}(\tau_1)}]$ are presented in Figure 1. The HOSVD with SPS-EXP shows high resolution time-delay estimation of the LOS signal. An advantage of such an approach is that no multi-dimensional nonlinear problems need to be solved and also no model order estimation is required.

V. Content of Full Paper

The full paper will include more simulation results for different $\Delta\phi$ and different SMR. In addition, the performance of the algorithms will be assessed for different variations of channel parameters of the LOS and the NLOS signals. A simulation based comparison between the tensor approach and a 2-D matrix approach will be included in order to show the benefits of using a tensor-based approach. Finally, the computational complexity of each proposed algorithm will be studied in detail.

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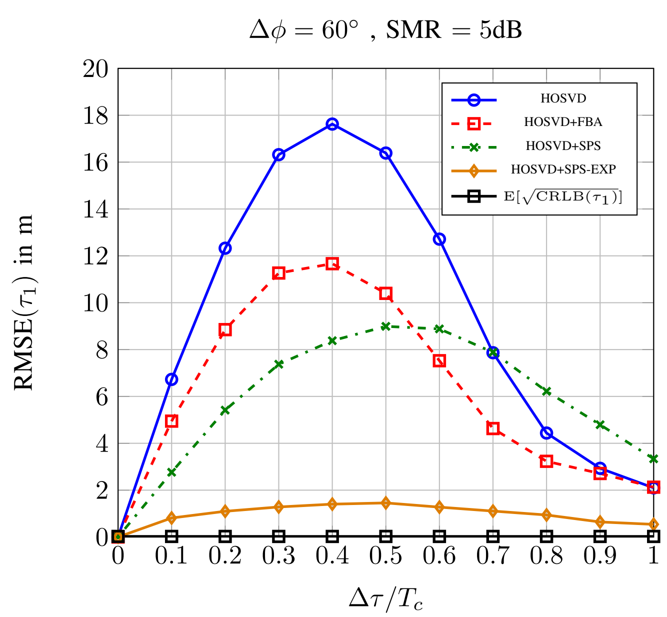

Figure 1. $\text{RMSE}(\tau_1)$ and $\text{E}[\sqrt{\text{CRLB}(\tau_1)}]$ of the estimate $\hat{\tau}_1$ for all implemented algorithms

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