A Versatile PAPR Reduction Algorithm for 5G Waveforms with Guaranteed Performance

For Special Session on Multiple Antenna Concepts for New 5G Air Interface Proposals

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I. Introduction

The PAPR problem refers to frequent occurrence of peaks in instantaneous power of the transmit signal that are considerably higher than the average power. The PAPR problem reduces power efficiency in transmitter as nonlinear distortion needs to be controlled by back-off in power amplifier. This problem is particularly a major technological bottleneck in uplink, i.e. handheld devices, due to limited battery life. As the research on waveforms in context of 5G [1] is actively pursued, the PAPR problem has regained attention. A category of PAPR reduction methods is based on invertible modifications to the signal, to which the proposed method belongs.

Although the described problem is commonly referred to as the PAPR problem, other metrics have as well been proposed to measure it. For instance, Cubic Metric (CM) has been reported to be superior to PAPR in determining the required back-off. Different metrics can behave differently in terms of reduction performance or mathematical tractability, which in turn might enhance algorithm design.

The PAPR problem exhibits itself as a more challenging problem when multiple antennas are used in the transmitter. The distortion caused by high PAPR of multicarrier signals causes two problems: in-band distortion and out-of-band distortion. The latter is often the bottleneck and in multiple transmit antennas, the worst level of out-of-band radiation dominates. In other words, the PAPR problem of n_t antennas transmitting independent data streams in parallel can be characterized by the worst PAPR in each signaling interval.

There has been considerable work done on PAPR reduction for OFDM signal, but relatively little on MIMO and more advanced waveforms. In this work we are interested in developing a flexible and high-performance algorithm for PAPR reduction and investigate its application to spatial diversity MIMO schemes using FBMC with offset QAM, a well-known and attractive model, and FBMC-QAM, a recently proposed waveform to improve on the former.

II. Contribution

a) Proposed method: The algorithm proposed in this work provides PAPR reduction by choosing signs of complex data symbols sequentially. By each sign decision, the goal is to reduce the expected value of the PAPR random variable conditioned on the already fixed signs. A sign selection approach has been previously proposed [2], in which limited information is extracted from the search space by essentially fixing all the undecided sign variables to 0. In other words, sign decision is made without considering contribution of data symbols with undecided signs. In contrast, the proposed method exploits the available information by considering expected values; an approach similar to mathematical geometric programming. As further explained in the following, the method allows for a performance analysis which provides substantially better upper bounds on reduction, in contrast to the deterministic worst-case bound presented in [2] and similar works.

The process can be described by considering the probability measure of PAPR, which is concentrated around its expected value [3]. Each step of the algorithm shifts the probability measure of the resulting PAPR to left, which has less randomness due to fixed signs. By the last step, the PAPR is no more random and is equal to the last expected value.

As a brief formalization, consider function $f(\mathbf{C},\mathbf{X})$ where $\mathbf{C}$ is the vector of complex data symbols and $\mathbf{X}$ is the corresponding vector of sign variables. The objective is to reduce PAPR by choosing sign $x(j)$ such that

$$\mathrm{E}[f(\mathbf{X},\mathbf{C})|\hat{x}(0\!:\!j),\mathbf{c}]=\min_{x(j)}\mathrm{E}[f(\mathbf{X},\mathbf{C})|\hat{x}(0\!:\!j\!-\!1),x(j),\mathbf{c}], \quad (1)$$

where $\hat{}$ denotes the previously decided sign variables. The order of signs is determined as follows: Consider the sequence $J=\{i\}_1^n$ and a permutation of it denoted as $J'=\{J'_i\}_1^n$. The sign $x(j)$ refers to element x_l of X such that $l=J'_j$. The notation $x(m:n)$ refers to $x(m),x(m+1),\ldots,x(n)$.

Clearly, calculation of conditional expectations is a key element of the algorithm. In particular, with PAPR as the choice for f, analytic calculation of the conditional expectations is not available and estimation is required. It will be shown in the following that estimation can be elegantly controlled by the analytic tool provided by concentration inequalities.

An important aspect of such distortionless PAPR reduction methods is undoing the modifications in the receiver. Two points of view can be discussed. 1) A reliable transmission of the required information, i.e. feed-forwarding of the modifications to the receiver. 2) Discarding the signs of data symbols used in the algorithm and presenting a rate loss to account for it. Both viewpoints shall be formally investigated.

b) Analysis: Such general trend of decisions on signs results in the sequence of conditional expectations

$$z_0=\mathrm{E}_{X(0:n-1)}[f(\mathbf{X},\mathbf{C})|\mathbf{c}]$$
$$z_j=\mathrm{E}_{X(j:n-1)}[f(\mathbf{X},\mathbf{C})|\hat{x}(0\!:\!j\!-\!1),\mathbf{c}]$$

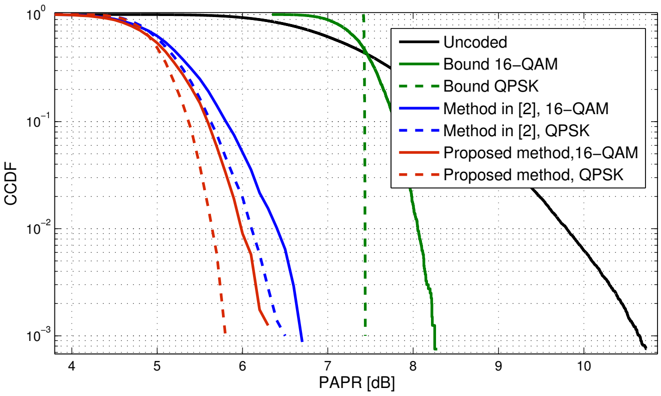

Fig. 1. Preliminary result for performance of the algorithm in single transmit antenna settings for $n = 64$. The upper bounds on PAPR reduction are as well shown for 16QAM and QPSK, where the latter is remarkably sharp.

$$z_n = \mathrm{E}_{\emptyset}[f(\mathbf{X}, \mathbf{C})|\hat{\mathbf{x}}, \mathbf{c}] = f(\hat{\mathbf{x}}, \mathbf{c}). \tag{2}$$

which gives z_0 as the upper bound on z_n, which is the reduced value of f. This inequality can provide two general methods for analytically investigating the performance of the algorithm: Analysis of distribution of z_0 by concentration inequalities and estimation of the distribution of z_0. A preliminary observation in Fig. 1 shows simulation result for a single stream of data with OFDM model and $n = 64$ subcarriers. Performance of the method in [2] is included for comparison.

As mentioned before, estimation is required in calculation of conditional expectations. While standard analysis of the estimator for max operator in PAPR provides limited information, concentration inequalities can be shown to complete the algorithm in this regard. Consider the estimator

$$g(\mathbf{C}, \mathbf{X}_1^{(k)}, \ldots, \mathbf{X}_q^{(k)}) = \frac{1}{q}\sum_{i=1}^{q} f(\mathbf{X}_i^{(k)}, \mathbf{C}), \tag{3}$$

where the first k signs are fixed. It will be shown that the following concentration equality holds

$$\mathbb{P}(|g(\mathbf{C},\mathbf{X}_1^{(k)}, \ldots, \mathbf{X}_q^{(k)}) - \bar{g}(\mathbf{C})| \geq \alpha) \leq 2\exp(-\frac{\alpha^2 p_a}{d^2}\frac{nq}{n-k}), \tag{4}$$

where $\bar{g}(\mathbf{C})$ is expected value of g with respect to $\mathbf{X}_1^{(k)}, \ldots, \mathbf{X}_q^{(k)}$, d is the maximum distance between constellation points and p_a is the average power. Together with some further analysis, it can be shown that this bound determines the required number of shots q. The derived bound supports the intuition that fewer shots are required for higher k.

Considering a spatial diversity MIMO scheme, one important characteristic of this method needs to be mentioned. The sign selections, in whatever order, must finally get to the last sign variable because the trajectory of f itself is quite different from $\mathrm{E}[f]$. The formalization above can be considered for each stream. Then a strategy needs to be decided on how to proceed with parallel streams. It is convenient to see sign variables as resources. Streams with lower uncoded PAPR can be treated with fewer signs. This is somewhat similar to directed SLM [4] approach suggested by for OFDM. But the main difference and challenge is that the number of signs must be chosen beforehand. In dSLM, the worst case PAPR receives the next iteration of the algorithm and the number of signs for each branch changes somewhat randomly. As a preliminary approach, we shall adopt this strategy and define $J'^{(i)} = \{J_i'\}_1^{\sigma_i}$ where σ_i is the number of sign variables allocated to i^{th} branch such that $\sigma = \sum_i \sigma_i$ is the total allocation and can be pre-determined.

All algorithm details, measure concentration and performance bounds will be derived in the full paper.

c) Application to waveforms: One major candidate for 5G waveforms is FBMC with MIMO. The most important component is that only purely real data symbols are allowed, but by a spectral efficiency of 2. Therefore, QAM symbols are broken into two pieces which constitutes what is referred to as offset QAM (OQAM). Desirable localization of pulses in frequency domain dictates some spreading in time-domain, which leads to overlapping among the waveform components pertaining to subsequent blocks of n data symbols. As opposed to OFDM, the overlapping complicates the PAPR reduction algorithm. Considering a joint PAPR reduction for several n-blocks is a natural approach. Such approach might be already compatible to frame-based transmission schemes and the degradation in the boundaries can be neglected. The FBMC approach and related performance bounds will be also described in detail in the full paper.

The major difficulty in using FBMC from the equalization and detection point of view is the so-called intrinsic interference [5], [6]. A recently proposed scheme for 5G called FBMC-QAM [7] aims to eliminate the intrinsic interference issue of FBMC-OQAM by using two prototype pulse shapes for even-numbered and odd-numbered subcarriers. This facilitates application of spatial multiplexing and diversity techniques while maintaining high spectral efficiency, i.e. no CP is required. Application of the proposed PAPR reduction method to the FBMC-QAM signal model is expected to exhibit similar behaviour. We derive algorithms and performance bounds for this new method and finally compare all proposed waveforms.

References

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