Downlink Precoder and Equalizer Designs for Multi-User MIMO FBMC/OQAM

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***Abstract*—In this contribution we propose two new iterative precoder and equalizer designs for the *Downlink* (DL) scenario of *Multi-User* (MU)-MIMO systems which employ *Filter Bank based Multi-Carrier* (FBMC) with *Offset-Quadrature Amplitude Modulation* (O-QAM). In a MU-MIMO DL scenario, we must design our per-subcarrier filters to compensate the *inter-symbol* and *inter-carrier interference* (ISI and ICI) present in an FBMC/OQAM system in addition to *multi-user interference* (MUI). The first method presented takes advantage of the *Mean Squared Error* (MSE)-duality to design *Minimum MSE* (MMSE)-based precoders and equalizers. The second method looks at maximizing the *Signal-to-Leakage Ratio* (SLR) in the transmitter and the *Signal-to-Interference plus Noise Ratio* (SINR) in the receiver. Through numerical simulations we will evaluate the performance of these methods and compare them to recent approaches found in the literature.**

I. Introduction

In recent years, FBMC systems have received attention as a promising alternative to *Orthogonal Frequency Division Multiplexing* with *Cyclic-Prefix* (CP-OFDM) for the physical layer of the new 5-th generation mobile communication systems (5G). CP-OFDM is already a widely employed multi-carrier solution due to the simple equalization given the CP and an efficient implementation using the *Fast Fourier Transform* (FFT). However, this comes at the price of a loss in spectral efficiency due to the CP, which is extremely long in the presence of highly frequently selective channels. Furthermore, CP-OFDM comes with difficult synchronization requirements in the *Base Station* (BS) and the *User Equipment* (UE).

Due to the spectrally well designed *Synthesis* and *Analysis Filter Bank*s, (SFB) and (AFB), at the transmitters and the receivers [1], FBMC systems have a much lower out-of-band radiation compared with CP-OFDM [2]. By introducing the O-QAM, FBMC/OQAM systems do not require a CP and thus have an improved spectral efficiency. Using an appropriate design of the pulse shaping filters limits the ICI whilst contributing to more ISI within each sub-carrier. Furthermore, FBMC/OQAM systems are more efficient in the presence of highly frequency selective channels compared with CP-OFDM. This comes at the price of slightly higher computational complexity [3], [4].

To take advantage of the MU-MIMO DL scenario with *Space Division Multiple Access* (SDMA), we must introduce a multi-tap, fractionally spaced, complex valued, finite impulse response filters in the transmitters and/or receivers. These should be designed to mitigate the ISI, ICI and MUI.

In [5], a non-linear *spatial Tomlinson Harashima precoder* (STHP) design was introduced which showed promising results compared with CP-OFDM. However, this design was limited to a MU-MISO with a flat channel frequency response. The authors additionally looked at a block diagonalization design in [6] to mitigate the MUI and use a zero forcing based design to remove the remaining interference. In [7], the non-linear STHP design from [5] was generalized and a further iterative precoder and equalizer design was introduced to accommodate a multi-stream MU-MIMO scenario. However, this design was again limited to a flat channel frequency response. Furthermore, in [8] the authors look into splitting the computational complexity between the transmitter and receiver. They used two linear designs based on a maximization of the SLNR and SINR in the transmitter and receiver, respectively.

In [9], an iterative design for a *quasi* MMSE-based precoder filters and MMSE-based equalizer filters was introduced for the MU-MISO DL scenario. This design was extended in [10] to the MU-MIMO DL scenario and compared with an SLR-based precoder design. However, in both designs only a single tap, real valued equalizer with a *Maximal-Ratio Combining* (MRC) design was used at the receivers.

In this contribution we propose two new iterative designs for the single stream MU-MIMO DL scenario. The first takes advantage of the MSE-duality, [11], [12], between UL and DL scenarios, such that we only need to design MMSE-based MIMO equalizers and transform them into precoders. In the second method an iterative design will extend the SLR design in [10] to accommodate complex valued multi-tap equalizers at the receiver that maximize the SINR.

This paper is organized as followed; in Section II we briefly describe the MU-MIMO FBMC/OQAM model we investigated. In Section III and Section IV the two proposed precoders and equalizers designs will be discussed. Finally, in Section V and Section VI we will discuss the simulation results and draw our conclusions.

II. FBMC System Model

In a MIMO FBMC/OQAM system, the SFB in each transmitter antenna combines the M, complex valued QAM input signals $d_k^s[m]$ generated at a rate of $1/T_s$, into a single, complex valued signal generated at a higher rate of M/T_s. The signal is transmitted across a highly frequency selective additive white Gaussian noise channel to the receiver. In our system, M corresponds to the total number of sub-channels and M_u to the number of sub-carriers we transmit across. k corresponds to the sub-carrier index and s to the user index. The AFB separates the received signal back into its M_u components at a low rate per sub-carrier.

The first operation in the SFB is the O-QAM staggering $\mathcal{O}_k$ of the input symbols $d_k^s[m]$. The input symbol $d_k^s[m]$ is split into its real and imaginary parts, up-sampled by a factor of 2, then depending on which sub-carrier we observe, either the $\Re\{d_k^s[m]\}$ or $\mathrm{j}\,\Im\{d_k^s[m]\}$ symbol is delayed by $T_s/2$ and finally these components are added together. When the sub-carrier index k is even, the $\Re\{d_k^s[m]\}$ symbol is delayed and when the sub-carrier index is odd, the $\mathrm{j}\,\Im\{d_k^s[m]\}$ symbol is delayed. Therefore, the symbol $x_k^s[n]$ at the output of our $\mathcal{O}_k$ operation has an O-QAM structure, i.e., each symbol is either purely real or purely imaginary at a symbol rate $[n]$, which is double the symbol rate of the input signals $d_k^s[m]$. Due to this characteristic of the O-QAM symbols, there is a phase change of $\pi/2$ between immediately adjacent sub-carriers, ensuring orthogonality between each sub-carrier. At the receiver, the AFB applies O-QAM de-staggering to reconstruct the complex QAM $\hat{d}_k^s[m]$ symbols at the original symbol rate from the equalized $\hat{x}_k^s[n]$ symbols.

After the O-QAM staggering the signals $x_k^s[n]$ are filtered by the multi-tap precoders, upsampled by $M/2$ and pulse-shaped by narrowband filters that allow a good spectral containment of each sub-carrier. At the AFB similar filters are applied, a downsampling by $M/2$ and filtering by the equalizers are performed to the signals before the O-QAM de-staggering.

Efficient realization of the FBMC system can be achieved by taking advantage of exponentially modulated filters on both SFB and AFB given by

$$h_k[r] = h_\mathrm{p}[r]\,\exp\left(\mathrm{j}\,\frac{2\pi}{M}\,k\,\left(r-\frac{L_\mathrm{p}-1}{2}\right)\right),\ r=0,\dots,L_\mathrm{p}-1,$$

where $h_\mathrm{p}[r]$ is a lowpass narrowband prototype filter, here a *Root Raised Cosine* (RRC), with length $L_\mathrm{p}=KM+1$, with K representing the overlapping factor of the symbols in the time domain. K should be kept as small as possible not only to limit the complexity but also to reduce the time-domain spreading of the symbols and the transmission latency. Furthermore, by taking advantage of the polyphase decomposition of $h_\mathrm{p}[r]$ all the filtering can be performed at a rate of only $2/T_s$. The complex modulation is perfomed by a FFT.

To minimize the complexity in the calculations of the equalizer and precoder filters, we set $K=4$ and the roll-off factor of our RRC filter equal to one. Thus, the frequency response of the filter $\mathbf{h}_k$ only significantly overlaps with the two adjacent filters.

In our MU-MIMO FBMC/OQAM DL system, we have assumed the BS to have a total of N_t transmitter antennas, each with an SFB and each UE to have a total of $N_{\mathrm{r}_\mathrm{s}}$ receiver antennas. In the MU-MIMO UL system we assume that the BS has $N_\mathrm{r}=N_\mathrm{t}$ receiver antennas and each UE has $N_{\mathrm{t}_\mathrm{s}}=N_{\mathrm{r}_\mathrm{s}}$ transmitter antennas. The total number of users is U.

To simplify the system model we define the following notation, $h_{l,j,r}^s[n]=\left(h_l * h_{\mathrm{ch},j,r}^s * h_k\right)[r]\,|_{r=nM/2}$. This represents the interference from the BS antenna j in sub-carrier l into the UE antenna r of user s in sub-carrier k. Where $l\in\{k-1,k,k+1\}$, $k\in\{1,\dots,M_\mathrm{u}\}$, $s\in\{1,\dots,U\}$, $j\in\{1,\dots,N_\mathrm{t}\}$ and $r\in\{1,\dots,N_{\mathrm{r}_\mathrm{s}}\}$. To simplify notation we do not include the sub-script index of the receiver sub-carrier since the interference is always relative to k. Furthermore, in the following derivations we will stack or sum the vectors of equivalent channels over the antennas to further simplify the notation. The resulting filter has the length

$$Q=\left\lceil\frac{2(L_\mathrm{p}-1)+L_\mathrm{ch}}{M/2}\right\rceil,$$

with the prototype filter length and channel impulse response length, L_p and L_ch, respectively.

After the O-QAM staggering operation, the sequences of input symbols, $\mathbf{x}_k^s[n]$, have the structure

$$\mathbf{x}_k^s[n]=\begin{cases}\begin{bmatrix}\alpha_k^s[m] & \mathrm{j}\beta_k^s[m] & \alpha_k^s[m-1] & \cdots\end{bmatrix}^T, & k \text{ is odd},\\ \begin{bmatrix}\mathrm{j}\beta_k^s[m] & \alpha_k^s[m] & \mathrm{j}\beta_k^s[m-1] & \cdots\end{bmatrix}^T, & k \text{ is even},\end{cases}$$

where $\alpha_k^s[m]$ and $\beta_k^s[m]$ represent the real and imaginary part of complex modulated QAM input symbol. In the following sections we work with a purely real notation and therefore define a purely real input sequence as $\mathbf{x}_k^s[n]=\mathbf{J}_k\tilde{\mathbf{x}}_k^s[n]$, i.e., $\tilde{\mathbf{x}}_k^s[n]\in\mathbb{R}^{B+Q-1}$ with

$$\mathbf{J}_k=\begin{cases}\mathrm{diag}\begin{bmatrix}1 & \mathrm{j} & 1 & \mathrm{j} & \cdots\end{bmatrix}, & k \text{ is odd},\\ \mathrm{diag}\begin{bmatrix}\mathrm{j} & 1 & \mathrm{j} & 1 & \cdots\end{bmatrix}, & k \text{ is even}.\end{cases}$$

The matrix $\mathbf{J}_k$ extracts the imaginary j 's from the input signal. In the following derivations, we will multiply the transposed convolution matrices of the equivalent channels with $\mathbf{J}_k$ and work with purely real notation. It can be shown, [9] and [13], that calculating the precoder or equalizer filters with either the real or imaginary part of the input symbol both result in the same filters.

III. MSE-Duality based Precoder and Equalizer Design

In this section we discuss an iterative algorithm to design joint MMSE-based precoder and equalizer filter for the MU-MIMO DL scenario. However, it should be noted that we only design receiver filters from both the BS and the UE perspective, i.e., we design the receiver filters from the MU-MIMO UL scenario and use the MSE-duality to transform these into transmitter filters in the DL scenario. In the following sections we will use the notation $(\check{\bullet})$ and $(\hat{\bullet})$ to indicate the DL scenario and the UL scenario, respectively. In our Algorithm 1, each step only depends on variables from the same iteration, i.e., (i) and thus to simplify notation, we will exclude the iteration index in the following derivations.

Our algorithm starts by initializing the equalizer filters in the DL scenario as a simple delay, i.e., the unit vector $\mathbf{e}$ with a 1 at the position $b/2$. The initial DL to UL (DL/UL) MSE-duality transformation in Step 3 sets the scaling factor $\check{\gamma}$ in the (0)th iteration equal to 1 which means the precoder filter in the UL scenario is also a delay.

As already mentioned we only need to design MMSE-based equalizers for the UL and DL scenarios as seen in Step 6 and Step 9. In Step 7, we use the UL/DL MSE-duality transformation to calculate the DL precoder filter in iteration (i). In Step 10, we see that we have to transform the DL equalizer filter into the UL precoder filter in the next iteration, i.e., for iteration $(i+1)$. Finally, our algorithm ends after a predefined number of iterations, n, have been executed.

Algorithm 1 can be applied to all of the MSE-duality transformation, in step 7 and 10 we observe that our designs keeps the UL/DL and DL/UL MSE-duality transformations the same for all iterations. In this example we have used the *System-Wide Sum-MSE* for the UL/DL transformation and the *User-Wise Sum-MSE* for the DL/UL transformation.

Algorithm 1 Joint MMSE-based Precoder and Equalizer Design using the MSE-duality Transformations

1: **Initialization:**
2: $\check{\mathbf{w}}^v_{k,(0)} = \mathbf{e}_{\mathrm{B}/2} \quad \forall v, k$
3: $\check{\gamma}_{(0)} = 1 \Rightarrow \hat{\mathbf{b}}^v_{k,(1)} = \mathbf{e}_{\mathrm{B}/2} \quad \forall v, k$
4: $i = 1$
5: **repeat**
6: $\quad \hat{\mathbf{w}}^v_{k,(i)} = \arg\min \mathrm{E}\left[\left|\hat{\alpha}^v_{k,(i)}[n] - \alpha^v_{k,(i)}[n-\nu]\right|_2^2\right]$
7: $\quad \hat{\gamma}_{(i)} \leftarrow$ UL/DL MSE-duality transformation
8: $\quad \check{\mathbf{b}}^v_{k,(i)} = \hat{\gamma}_{(i)} \hat{\mathbf{w}}^v_{k,(i)}$
9: $\quad \check{\mathbf{w}}^v_{k,(i)} = \arg\min \mathrm{E}\left[\left|\check{\alpha}^v_{k,(i)}[n] - \alpha^v_{k,(i)}[n-\nu]\right|_2^2\right]$
10: $\quad \check{\gamma}^s_{(i)} \leftarrow$ DL/UL MSE-duality transformation
11: $\quad \hat{\mathbf{b}}^v_{k,(i+1)} = \check{\gamma}^v_{(i)} \check{\mathbf{w}}^v_{k,(i)}$
12: $\quad i = i + 1$
13: **until** $i = n$

A. Base station Perspective

In this sub-section we investigate the MU-MIMO UL scenario. We define a multi-tap, fractionally spaced equalizer $\hat{\mathbf{w}} \in \mathbb{C}^{L_{\mathrm{eq}}}$ per user, sub-carrier and BS receiver antenna and a multi-tap, fractionally spaced precoder $\hat{\mathbf{b}} \in \mathbb{C}^{B}$ per user, sub-carrier and UE transmitter antenna. In our system we have U decentralized users, each with N_{r_s} transmitter antennas. Each user transmits sequences $\mathbf{x}^1_k, \cdots, \mathbf{x}^U_k$ of *independent and identically distributed* (i.i.d.) and Gaussian distributed input signals in every sub-carrier $k \in \{1, \ldots, M_{\mathrm{u}}\}$ to the N_{r} centralized BS receiver antennas. Furthermore, we assume the O-QAM input symbols to be have half the variance of the QAM input symbols σ^2_d, i.e.,

$$\mathrm{E}\left[\tilde{\mathbf{x}}^s_k[n] \tilde{\mathbf{x}}^{s,T}_k[n]\right] = (\sigma^2_{\mathrm{d}}/(2U))\mathbf{I} = \sigma^2_{\mathrm{M}}\mathbf{I}.$$

In the UL scenario, the real part of our receive signal for user v in sub-carrier k is defined as

$$\hat{\alpha}^v_k[n] = \hat{\mathbf{w}}^{v,\mathrm{T}}_k \left(\sum_{s=1}^{U} \sum_{l=k-1}^{k+1} \hat{\mathbf{P}}^s_l \tilde{\mathbf{x}}^s_l[n] + \hat{\boldsymbol{\Xi}}_k\right), \tag{1}$$

where $\hat{\mathbf{P}}^s_l$ is the transposed convolution matrix of the equivalent UL channel. We have added the UL precoder filter into the total transmission channel, i.e., $\hat{p}^s_{l,j,r} = \hat{b}^s_{l,r} * h^s_{l,j,r}$ where s, l, j, r represent the user index, sub-carrier index, BS receiver antenna index and UE transmitter antenna index, respectively. Furthermore, $\hat{\boldsymbol{\Xi}}_k \in \mathbb{R}^{2BN_{\mathrm{t}} \times 1}$ contains the stacked real and imaginary parts of $\boldsymbol{\Gamma}_k \eta_j$ with $\boldsymbol{\Gamma}_k$ as an $M/2$ downsampled, transposed convolution matrix of $\mathbf{h}_k$ which filters the noise $\boldsymbol{\eta}_j$. We assume the additive noise is Gaussian distributed with $\boldsymbol{\eta}_j[n] \sim \mathcal{N}_{\mathbb{C}}\left(\mathbf{0}, \sigma^2_\eta \mathbf{I}\right)$ at each BS receiver antenna.

Again, it should be noted that we have dropped the iteration (i) from these derivations, thus, the received signal, equalizer filter and the equivalent UL transmission chain described here depend on (i) and changes every iteration.

The optimization problem we wish to minimize is expressed with respect to the UL MSE $\hat{\epsilon}^v_k$ as

$$\begin{aligned}\hat{\mathbf{w}}^v_k &= \underset{\hat{\mathbf{w}}^v_k}{\arg\min}\, \mathrm{E}\left[\left|\hat{\alpha}^v_k[n] - \alpha^v_k[n-\nu]\right|^2\right], \\ &= \underset{\hat{\mathbf{w}}^v_k}{\arg\min}\, \hat{\epsilon}^v_k,\end{aligned} \tag{2}$$

where we define ν as the transmission latency in our system. This optimization of the MMSE-based equalizer filter can be seen in Step 6 of Algorithm 1. We solve the optimization problem in (29) similar to [13], arriving at an MMSE-based equalizer filter for all receiver antennas

$$\hat{\mathbf{w}}^v_k = \left(\sum_{s=1}^{U} \sum_{l=k-1}^{k+1} \sigma^2_{\mathrm{M}} \hat{\mathbf{P}}^s_l \hat{\mathbf{P}}^{s,\mathrm{T}}_l + \hat{\mathbf{R}}_\eta\right)^{-1} \sigma^2_{\mathrm{M}} \hat{\mathbf{P}}^v_k \mathbf{e}_\nu. \tag{3}$$

Given the MMSE-based equalizer we are left with a simplified, closed form expression for the UL MSE per user and per sub-carrier defined as

$$\hat{\epsilon}^v_k(\hat{\mathbf{w}}^v_k) = \sigma^2_{\mathrm{M}}\left(1 - \mathbf{e}^{\mathrm{T}}_\nu \hat{\mathbf{P}}^{v,\mathrm{T}}_k \hat{\mathbf{w}}^v_k\right), \tag{4}$$

where we define the stacking matrices as

$$\hat{\mathbf{w}}^{v,\mathrm{T}}_k = \left[\begin{array}{ccc} \bar{\mathbf{w}}^{v,\mathrm{T}}_{k,1} & \cdots & \bar{\mathbf{w}}^{v,\mathrm{T}}_{k,N_{\mathrm{r}}} \end{array}\right] \in \mathbb{R}^{1 \times 2BN_{\mathrm{r}}}, \tag{5}$$

$$\hat{\mathbf{P}}^s_l = \sum_{r=1}^{N_{\mathrm{t}_s}} \tilde{\mathbf{P}}^s_{l,r} \tag{6}$$

$$\tilde{\mathbf{P}}^s_{l,r} = \left[\begin{array}{ccc} \bar{\mathbf{P}}^s_{l,r,1} & \cdots & \bar{\mathbf{P}}^s_{l,r,N_{\mathrm{r}}} \end{array}\right]^{\mathrm{T}} \in \mathbb{R}^{2BN_{\mathrm{r}} \times (B+Q-1)}, \tag{7}$$

$$\mathbf{P}^s_{l,r,j} = \mathbf{J}_l \mathbf{P}^s_{l,r,j} \in \mathbb{C}^{B \times (B+L_{\mathrm{eq}}+Q-2)}, \tag{8}$$

$$\hat{\mathbf{R}}_\eta = \text{blockdiag}\left[\begin{array}{ccc} \bar{\mathbf{R}}_{\eta_1} & \cdots & \bar{\mathbf{R}}_{\eta_{N_{\mathrm{r}}}} \end{array}\right] \in \mathbb{R}^{2BN_{\mathrm{r}} \times 2BN_{\mathrm{r}}}, \tag{9}$$

$$\bar{\mathbf{R}}_{\eta,k} = \begin{bmatrix} \mathbf{R}_{\eta,k,1} & \mathbf{R}_{\eta,k,2} \\ -\mathbf{R}_{\eta,k,2} & \mathbf{R}_{\eta,k,1} \end{bmatrix} \in \mathbb{R}^{2B \times 2B}, \tag{10}$$

$$\text{with } \mathbf{R}_{\eta,k,1} = \frac{\sigma^2_\eta}{2}\left(\boldsymbol{\Gamma}^{(R)}_k \boldsymbol{\Gamma}^{(R),T}_k + \boldsymbol{\Gamma}^{(I)}_k \boldsymbol{\Gamma}^{(I),T}_k\right) \in \mathbb{R}^{B \times B}, \tag{11}$$

$$\mathbf{R}_{\eta,k,2} = \frac{\sigma^2_\eta}{2}\left(\boldsymbol{\Gamma}^{(R)}_k \boldsymbol{\Gamma}^{(I),T}_k - \boldsymbol{\Gamma}^{(I)}_k \boldsymbol{\Gamma}^{(R),T}_k\right) \in \mathbb{R}^{B \times B} \tag{12}$$

We use the notation $(\bar{\bullet})$ to indicate taking the real and imaginary part of a vector and stacking them on top of each other, i.e., $\bar{\mathbf{x}} = \left[\Re\{\mathbf{x}\}, \quad \Im\{\mathbf{x}\}\right]^{\mathrm{T}}$.

Now we move onto the MU-MIMO DL scenario where, from the BS perspective, we can define the real part of our receive signal for user v in sub-carrier k as

$$\breve{\alpha}_k^v[n] = \sum_{s=1}^{U} \sum_{l=k-1}^{k+1} \breve{\mathbf{b}}_l^{s,\mathrm{T}} \breve{\mathbf{Q}}_{kl}^v \tilde{\mathbf{x}}_l^s[n] + \Re\left\{ \breve{\mathbf{w}}_k^{v,\mathrm{T}} \mathbf{\Gamma}_k \boldsymbol{\eta}^v \right\}, \quad (13)$$

where $\breve{\mathbf{b}}_l^s$ is defined as the dual stacking vector to $\hat{\mathbf{w}}_l^s$ with $N_\mathrm{t} = N_\mathrm{r}$. Furthermore, $\breve{\mathbf{Q}}_{kl}^v = \sum_{r=1}^{N_{\mathrm{r}_s}} \tilde{\mathbf{Q}}_{kl,r}^v$ is the equivalent DL channel to move the DL equalizer into the total transmission chain, i.e., $\breve{q}_{kl,j,r}^v = \breve{w}_{k,r}^v * h_{l,j,r}^v$. Here, the first sub-carrier index refers to the equalizer filter $\breve{w}_{k,r}^v$ and the second index refers to the transmission channel $h_{l,j,r}^v$. Again we assume that the input signals $\tilde{\mathbf{x}}_k^s$ are i.i.d. and Gaussian distributed with an equivalent distribution to that defined in Section III-A. Furthermore, we assume the additive noise is Gaussian distributed with $\boldsymbol{\eta}^s[n] \sim \mathcal{N}_\mathbb{C}\left(\mathbf{0}, \sigma_\eta^2 \mathbf{I}\right)$ at each user.

The optimization problem in the DL scenario we would require to minimize is defined as

$$\begin{aligned}\breve{\mathbf{b}}_k^v &= \underset{\breve{\mathbf{b}}_k^v}{\arg\min}\, \mathrm{E}\left[\left| \breve{\alpha}_k^v[n] - \alpha_k^v[n-\nu] \right|^2 \right], \\ &= \underset{\breve{\mathbf{b}}_k^v}{\arg\min}\, \breve{\epsilon}_k^v \quad \text{s. t.} \sum_{v=1}^{U} \sum_{k=1}^{M_\mathrm{u}} \left\| \breve{\mathbf{b}}_k^v \right\|_2^2 \leq M_\mathrm{u} U.\end{aligned}$$

By plugging (13) into the argument of our optimization problem, we arrive at a close formed expression for the DL MSE from the BS perspective as

$$\begin{aligned}\breve{\epsilon}_k^v = \sigma_\mathrm{M}^2 &\left(\sum_{s=1}^{U} \sum_{l=k-1}^{k+1} \breve{\mathbf{b}}_l^{s,\mathrm{T}} \breve{\mathbf{Q}}_{kl}^v \breve{\mathbf{Q}}_{kl}^{v,\mathrm{T}} \breve{\mathbf{b}}_l^s - 2\mathbf{e}_\nu^\mathrm{T} \breve{\mathbf{Q}}_k^{v,\mathrm{T}} \breve{\mathbf{b}}_k^v + 1 \right) \\ &+ \breve{\mathbf{w}}_k^{v,\mathrm{T}} \breve{\mathbf{R}}_\eta^v \breve{\mathbf{w}}_k^v.\end{aligned} \quad (14)$$

We observe in (14) that the DL MSE expression for user v in sub-carrier k not only depends on the precoder filter $\breve{\mathbf{b}}_k^v$, but it additionally depends on the precoder filter of the neighboring sub-carriers as well as the precoder filter from all other users in the system. This interdependency between precoder filters makes the minimization of the MSE more difficult than in the MU-MIMO system.

B. BS-Side MSE-duality Transformations

In this sub-section we investigate the four different methods, from the BS perspective, of transforming our UL MIMO system into an equivalent DL MIMO system using the duality principle as introduced in [12] and [11]. In our iterative Algorithm 1, we are now at the UL/DL MSE-duality transformation in Step 7. In all the MSE-duality transformations, the total power is preserved [11], [12], i.e., $\sum_{v=1}^{U} \sum_{k=1}^{M_\mathrm{u}} \left\| \hat{\mathbf{b}}_k^v \right\|_2^2 \leq M_\mathrm{u} U$. A more detailed explanation as to what each MSE-duality transformation actually does in the MU-MIMO FBMC/OQAM system can be found in [14].

1) UL/DL System-Wide Sum-MSE: First, we define a relation between the DL and UL filters with a single, real-valued scaling factor such that

$$\breve{\mathbf{b}}_l^s = \hat{\gamma} \hat{\mathbf{w}}_l^s \quad \text{and} \quad \breve{\mathbf{w}}_k^v = \hat{\gamma}^{-1} \hat{\mathbf{b}}_k^v \quad (15)$$

with $\hat{\gamma} \in \mathbb{R}_+$. In the next step we set the system-wide sum-MSE equal between the UL and the DL scenarios. Thus, we sum over all users and all sub-carriers and set these MSE values to be equal, i.e., $\sum_{v=1}^{U} \sum_{k=1}^{M_\mathrm{u}} \hat{\epsilon}_k^v \overset{!}{=} \sum_{v=1}^{U} \sum_{k=1}^{M_\mathrm{u}} \breve{\epsilon}_k^v$, where the relation $\overset{!}{=}$ implies both sides of the equation must be equal. By solving this equation we can calculate a single scaling factor $\hat{\gamma}$ for all users, sub-carriers and transmitter antennas

$$\hat{\gamma}^2 = \frac{\sum_{v=1}^{U} \sum_{k=1}^{M_\mathrm{u}} \hat{\mathbf{b}}_k^{v,\mathrm{T}} \breve{\mathbf{R}}_\eta^v \hat{\mathbf{b}}_k^v}{\sum_{v=1}^{U} \sum_{k=1}^{M_\mathrm{u}} \sigma_\mathrm{M}^2 \left(\mathbf{e}_\nu^\mathrm{T} \hat{\mathbf{P}}_k^{v,\mathrm{T}} \hat{\mathbf{w}}_k^v - \sum_{s=1}^{U} \sum_{l=k-1}^{k+1} \hat{\mathbf{w}}_l^{s,\mathrm{T}} \hat{\mathbf{P}}_{kl}^v \hat{\mathbf{P}}_{kl}^{v,\mathrm{T}} \hat{\mathbf{w}}_l^s \right)}. \quad (16)$$

2) UL/DL User-Wise Sum-MSE: Next, we define a relation between the DL and UL filters with a real-valued scaling factor per user such that

$$\breve{\mathbf{b}}_l^s = \hat{\gamma}^s \hat{\mathbf{w}}_l^s \quad \text{and} \quad \breve{\mathbf{w}}_k^v = \left(\hat{\gamma}^v\right)^{-1} \hat{\mathbf{b}}_k^v \quad (17)$$

with $\hat{\gamma}^s \in \mathbb{R}_+$. Following this, we set the user-wise sum-MSE equal between the UL and the DL system. Thus, we sum over the MSE in all sub-carriers and set them equal per user, i.e., $\sum_{k=1}^{M_\mathrm{u}} \hat{\epsilon}_k^v \overset{!}{=} \sum_{k=1}^{M_\mathrm{u}} \breve{\epsilon}_k^v, \quad \forall v \in \{1, 2, \ldots, U\}$. We end up with a system of linear equations to solve for U scaling factors $\hat{\gamma}^s$,

$$\mathbf{A}^s \begin{bmatrix} \left(\hat{\gamma}^1\right)^2 \\ \vdots \\ \left(\hat{\gamma}^U\right)^2 \end{bmatrix} = \underbrace{\begin{bmatrix} \sum_{k=1}^{M_\mathrm{u}} \hat{\mathbf{b}}_k^{1,\mathrm{T}} \breve{\mathbf{R}}_\eta^1 \hat{\mathbf{b}}_k^1 \\ \vdots \\ \sum_{k=1}^{M_\mathrm{u}} \hat{\mathbf{b}}_k^{U,\mathrm{T}} \breve{\mathbf{R}}_\eta^U \hat{\mathbf{b}}_k^U \end{bmatrix}}_{\mathbf{y}^s}, \quad (18)$$

where the matrix $\mathbf{A}^s \in \mathbb{R}^{U \times U}$ has strictly positive main diagonal elements. The matrix $\mathbf{A}^s$ is defined as

$$[\mathbf{A}^s]_{v,y} = \begin{cases} \sum_{k=1}^{M_\mathrm{u}} \sum_{l=k-1}^{k+1} \sigma_\mathrm{M}^2 \left(\mathbf{e}_\nu^\mathrm{T} \hat{\mathbf{P}}_k^{v,\mathrm{T}} \hat{\mathbf{w}}_k^v \right. \\ \qquad \left. - \hat{\mathbf{w}}_l^{v,\mathrm{T}} \hat{\mathbf{P}}_{kl}^v \hat{\mathbf{P}}_{kl}^{v,\mathrm{T}} \hat{\mathbf{w}}_l^v \right), & \text{if } v = y, \\ -\sum_{k=1}^{M_\mathrm{u}} \sum_{l=k-1}^{k+1} \sigma_\mathrm{M}^2 \left(\hat{\mathbf{w}}_l^{y,\mathrm{T}} \hat{\mathbf{P}}_{kl}^v \hat{\mathbf{P}}_{kl}^{v,\mathrm{T}} \hat{\mathbf{w}}_l^y \right), & \text{if } v \neq y. \end{cases} \quad (19)$$

3) UL/DL Sub-Carrier-Wise Sum-MSE: Next, we define a relation between the DL and UL filters with a real-valued scaling factor per sub-carrier such that

$$\breve{\mathbf{b}}_l^s = \hat{\gamma}_l \hat{\mathbf{w}}_l^s \quad \text{and} \quad \breve{\mathbf{w}}_k^v = \hat{\gamma}_k^{-1} \hat{\mathbf{b}}_k^v \quad (20)$$

with $\hat{\gamma}_k \in \mathbb{R}_+$. Next we set the sub-carrier-wise sum MSE equal between the UL and the DL system, i.e., we sum over all users and set the sum-MSE values equal per sub-carrier $\sum_{v=1}^{U} \hat{\epsilon}_k^v \overset{!}{=} \sum_{v=1}^{U} \breve{\epsilon}_k^v, \quad \forall k \in \{1, 2, \ldots, M_\mathrm{u}\}$. We end up with a system of linear equations to solve for M_u scaling factors $\hat{\gamma}_k$

$$\mathbf{A}^k \begin{bmatrix} \hat{\gamma}_1^2 \\ \vdots \\ \hat{\gamma}_{M_\mathrm{u}}^2 \end{bmatrix} = \begin{bmatrix} \sum_{v=1}^{U} \hat{\mathbf{b}}_1^{v,\mathrm{T}} \breve{\mathbf{R}}_\eta^v \hat{\mathbf{b}}_1^v \\ \vdots \\ \sum_{v=1}^{U} \hat{\mathbf{b}}_{M_\mathrm{u}}^{v,\mathrm{T}} \breve{\mathbf{R}}_\eta^v \hat{\mathbf{b}}_{M_\mathrm{u}}^v \end{bmatrix}, \quad (21)$$

where the tri-diagonal matrix $\mathbf{A}^k \in \mathbb{R}^{M_\mathrm{u}\times M_\mathrm{u}}$ has strictly positive elements on the main diagonal and strictly negative off-diagonal elements. This matrix is defined as

$$\left[\mathbf{A}^k\right]_{k,m} = \begin{cases} \sum_{v,s=1}^{U} \sigma_\mathrm{M}^2 \left(\mathbf{e}_\nu^\mathrm{T}\hat{\mathbf{P}}_k^{v,\mathrm{T}}\hat{\mathbf{w}}_k^v \right. \\ \qquad \left. -\hat{\mathbf{w}}_k^{s,\mathrm{T}}\hat{\mathbf{P}}_k^v\hat{\mathbf{P}}_k^{v,\mathrm{T}}\hat{\mathbf{w}}_k^s\right), & \text{if } k=m, \\ -\sum_{v,s=1}^{U} \sigma_\mathrm{M}^2 \left(\hat{\mathbf{w}}_m^{s,\mathrm{T}}\hat{\mathbf{P}}_{km}^v\hat{\mathbf{P}}_{km}^{v,\mathrm{T}}\hat{\mathbf{w}}_m^s\right), & \text{if } |k-m|=1, \\ 0 & \text{else.} \end{cases} \tag{22}$$

4) UL/DL User and Sub-Carrier-Wise MSE: Finally, we define a relation between the DL and UL filters with a real-valued scaling factor per user and per sub-carrier such that

$$\check{\mathbf{b}}_l^s = \hat{\gamma}_l^s \hat{\mathbf{w}}_l^s \quad \text{and} \quad \check{\mathbf{w}}_k^v = \left(\hat{\gamma}_k^v\right)^{-1}\hat{\mathbf{b}}_k^v \tag{23}$$

with $\hat{\gamma}_k^s \in \mathbb{R}_+$. We then set the user and sub-carrier-wise sum-MSE equal between the UL and the DL system, i.e., we set the individual MSE expressions per user and per sub-carrier equal such that $\check{\epsilon}_k^v \overset{!}{=} \hat{\epsilon}_k^v, \quad \forall v \in \{1,2,\ldots,U\}$ and $\forall k \in \{1,2,\ldots,M_\mathrm{u}\}$. We end up with a system of linear equations to solve for $U \times M_\mathrm{u}$ scaling factors $\hat{\gamma}_k^s$

$$\begin{bmatrix} \mathbf{A}_{1,1} & \mathbf{A}_{1,2} & \mathbf{0}_U & \cdots & \mathbf{0}_U \\ \mathbf{A}_{2,1} & \mathbf{A}_{2,2} & \mathbf{A}_{2,3} & \ddots & \vdots \\ \mathbf{0}_U & \ddots & \ddots & \ddots & \mathbf{0}_U \\ \vdots & \ddots & \ddots & \ddots & \mathbf{A}_{M_\mathrm{u}-1,M_\mathrm{u}} \\ \mathbf{0}_U & \cdots & \mathbf{0}_U & \mathbf{A}_{M_\mathrm{u},M_\mathrm{u}-1} & \mathbf{A}_{M_\mathrm{u},M_\mathrm{u}} \end{bmatrix} \begin{bmatrix} \hat{\boldsymbol{\gamma}}_1 \\ \hat{\boldsymbol{\gamma}}_2 \\ \hat{\boldsymbol{\gamma}}_3 \\ \vdots \\ \vdots \\ \hat{\boldsymbol{\gamma}}_{M_\mathrm{u}} \end{bmatrix} = \begin{bmatrix} \boldsymbol{\kappa}_1 \\ \boldsymbol{\kappa}_2 \\ \boldsymbol{\kappa}_3 \\ \vdots \\ \vdots \\ \boldsymbol{\kappa}_{M_\mathrm{u}} \end{bmatrix}, \tag{24}$$

where $\mathbf{A}_{k,m} \in \mathbb{R}^{U\times U}$ and $\hat{\boldsymbol{\gamma}}_k \in \mathbb{R}_+^U,\ k \in \{1,\ldots,M_\mathrm{u}\}$. The elements on the *Right-Hand-Side* (RHS) are defined as

$$\boldsymbol{\kappa}_k = \left[\hat{\mathbf{b}}_k^{1,\mathrm{T}}\check{\mathbf{R}}_\eta^1\hat{\mathbf{b}}_k^1, \quad \cdots, \quad \hat{\mathbf{b}}_k^{U,\mathrm{T}}\check{\mathbf{R}}_\eta^U\hat{\mathbf{b}}_k^U\right]^T. \tag{25}$$

We define the matrices $\mathbf{A}_{k,k}$ and $\mathbf{A}_{k,m}$ for $m \neq k$ as follows

$$\left[\mathbf{A}_{k,k}\right]_{v,s} = \begin{cases} \sigma_\mathrm{M}^2\left(\mathbf{e}_\nu^\mathrm{T}\hat{\mathbf{P}}_k^{v,\mathrm{T}}\hat{\mathbf{w}}_k^v \right. \\ \qquad \left. -\hat{\mathbf{w}}_k^{v,\mathrm{T}}\hat{\mathbf{P}}_k^v\hat{\mathbf{P}}_k^{v,\mathrm{T}}\hat{\mathbf{w}}_k^v\right), & \text{if } v=s, \\ -\sigma_\mathrm{M}^2\hat{\mathbf{w}}_k^{s,\mathrm{T}}\hat{\mathbf{P}}_k^v\hat{\mathbf{P}}_k^{v,\mathrm{T}}\hat{\mathbf{w}}_k^s, & \text{if } v\neq s, \end{cases} \tag{26}$$

$$\left[\mathbf{A}_{k,m}\right]_{v,s} = \left\{-\sigma_\mathrm{M}^2\hat{\mathbf{w}}_m^{s,\mathrm{T}}\hat{\mathbf{P}}_m^v\hat{\mathbf{P}}_m^{v,\mathrm{T}}\hat{\mathbf{w}}_m^s, \quad \text{if } |k-m|=1\right. \tag{27}$$

C. User Equipment Perspective

In this sub-section we move on to look at the design of the DL MIMO MMSE-based equalizer filter from the perspective of the UE. First we must calculate the DL and the UL UE-side received signals and find a closed form expression for the MSE to transform these scenarios. The following derivations represent Steps 9 and 10 in our iterative Algorithm 1.

From the UE perspective in the MU-MIMO DL scenario, the real part of the received signal for user v in sub-carrier k can be defined as

$$\check{\alpha}_k^v[n] = \check{\mathbf{w}}_k^{v,\mathrm{T}}\left(\sum_{s=1}^{U}\sum_{l=k-1}^{k+1}\check{\mathbf{P}}_l^{sv}\tilde{\mathbf{x}}_l^s[n] + \Re\left\{\mathbf{\Gamma}_k\boldsymbol{\eta}^v\right\}\right), \tag{28}$$

where we define $\check{\mathbf{w}}_k^v$ as the multi-tap, fractionally spaced UE equalizer. Again, we assume i.i.d. input symbols $\mathbf{x}_k^s$ and AWGN noise. The transposed convolution matrix $\check{\mathbf{P}}_l^{sv}$ is the equivalent DL channel to move the DL precoder into the total transmission chain, i.e., $\check{p}_{l,j}^{sv} = \check{b}_l^s * h_{l,j}^v$. We should note that the super-script sv represents the convolution of user s' BS precoding filter and the transmission channel from the BS to user v.

The optimization problem we wish to minimize on the UE-side, is expressed with respect to the DL MSE $\check{\epsilon}_k^v$ as

$$\begin{aligned} \check{\mathbf{w}}_k^v &= \underset{\check{\mathbf{w}}_k^v}{\arg\min}\,\mathrm{E}\left[\left|\check{\alpha}_k^v[n] - \alpha_k^v[n-\nu]\right|^2\right], \\ &= \underset{\check{\mathbf{w}}_k^v}{\arg\min}\,\check{\epsilon}_k^v, \end{aligned} \tag{29}$$

To this end we can calculate the MMSE-based equalizer filter per user, and sub-carrier in the DL system as

$$\check{\mathbf{w}}_k^v = \left(\sum_{s=1}^{U}\sum_{l=k-1}^{k+1}\sigma_\mathrm{M}^2\check{\mathbf{P}}_l^{sv}\check{\mathbf{P}}_l^{sv,\mathrm{T}} + \check{\mathbf{R}}_\eta^v\right)^{-1}\sigma_\mathrm{M}^2\check{\mathbf{P}}_k^v\mathbf{e}_\nu. \tag{30}$$

Given the MMSE-based DL equalizer we are left with a simplified, closed form expression for the UE-side DL MSE per user and per sub-carrier defined as

$$\check{\epsilon}_k^v\left(\check{\mathbf{w}}_k^v\right) = \sigma_\mathrm{M}^2\left(1 - \mathbf{e}_\nu^\mathrm{T}\check{\mathbf{P}}_k^{v,\mathrm{T}}\check{\mathbf{w}}_k^v\right), \tag{31}$$

Now we move on to the MU-MIMO UL scenario from the UE perspective. The real part of our receive signal for user v in sub-carrier k is defined as

$$\hat{\alpha}_k^v[n] = \sum_{s=1}^{U}\sum_{l=k-1}^{k+1}\hat{\mathbf{b}}_l^{s,\mathrm{T}}\hat{\mathbf{Q}}_{kl}^{vs}\tilde{\mathbf{x}}_l^s[n] + \hat{\mathbf{w}}_k^{v,\mathrm{T}}\Re\left\{\hat{\mathbf{\Gamma}}_k\hat{\boldsymbol{\eta}}\right\}, \tag{32}$$

where we define $\hat{\mathbf{b}}_k^v$ as the multi-tap, fractionally spaced precoder and $\hat{\mathbf{w}}_k^v$ is the UL MMSE-based equalizer designed in (3). The transposed convolution matrix $\hat{\mathbf{Q}}_l^{sv}$ is the equivalent UL channel to move the UL equalizer into the total transmission chain, i.e., $\hat{q}_{kl,j}^{vs} = \hat{w}_k^v * h_{l,j}^s$. Using (32) we can reformulate the UL MSE as follows

$$\begin{aligned} \hat{\epsilon}_k^v = &\sigma_\mathrm{M}^2\left(\sum_{s=1}^{U}\sum_{l=k-1}^{k+1}\hat{\mathbf{b}}_l^{s,\mathrm{T}}\hat{\mathbf{Q}}_{kl}^{vs}\hat{\mathbf{Q}}_{kl}^{vs,\mathrm{T}}\hat{\mathbf{b}}_l^s - 2\sigma_\mathrm{M}^2\hat{\mathbf{b}}_k^{v,\mathrm{T}}\hat{\mathbf{Q}}_k^v\mathbf{e}_\nu + 1\right) \\ &+ \hat{\mathbf{w}}_k^{v,\mathrm{T}}\hat{\mathbf{R}}_\eta\hat{\mathbf{w}}_k^v \end{aligned} \tag{33}$$

D. User Equipment-Side MSE-duality Transformations

In this sub-section we investigate the two possible methods, from the UE perspective, of transforming our DL equalizer filters into equivalent UL precoder filters. These are similar to the transformations introduced in Sub-Section III-B, however, since we have decentralized users, we concluded that spreading the transmit power over the users was not meaningful. Therefore, we end up with only two forms of DL/UL MSE-duality transformations, i.e., the *DL/UL User-Wise Sum-MSE* and the *DL/UL User and Sub-carrier-Wise MSE* transformation.

We should note that for the two UL/DL transformations where we summed over the sub-carriers i.e., III-B1 and III-B2, we used the *DL/UL User-Wise Sum-MSE* for the DL/UL transformation. For the UL/DL transformations where we summed over users or set the individual MSEs equal, i.e., III-B3 and III-B4, we used the *DL/UL User and Sub-carrier-Wise MSE* for the DL/UL transformation.

1) DL/UL User-Wise Sum-MSE: Again, we define a relation between the UL and DL filters with a real-valued scaling factor $\check{\gamma}_{(i)}^s \in \mathbb{R}_+$ per user such that

$$\hat{\mathbf{b}}_l^s = \check{\gamma}_{(i)}^s \check{\mathbf{w}}_l^s \quad \text{and} \quad \hat{\mathbf{w}}_k^v = \left(\check{\gamma}_{(i)}^v\right)^{-1} \check{\mathbf{b}}_k^v. \tag{34}$$

By summing over all sub-carriers as in Section III-B2 and setting this up for all users, we end up with a system of linear equations in the same form as (18). Now the RHS of the equation is defined as

$$\mathbf{y}^s = \left[\sum_{k=1}^{M_\mathrm{u}} \check{\mathbf{b}}_k^{1,\mathrm{T}} \hat{\mathbf{R}}_\eta^1 \check{\mathbf{b}}_k^1 \quad \cdots \quad \sum_{k=1}^{M_\mathrm{u}} \check{\mathbf{b}}_k^{U,\mathrm{T}} \hat{\mathbf{R}}_\eta^U \check{\mathbf{b}}_k^U\right]^\mathrm{T}, \tag{35}$$

and the matrix $\mathbf{A}^s$ is defined as

$$[\mathbf{A}^s]_{v,y} = \begin{cases} \sum_{k=1}^{M_\mathrm{u}} \sum_{l=k-1}^{k+1} \sigma_\mathrm{M}^2 \Big(\mathbf{e}_\nu^\mathrm{T} \check{\mathbf{P}}_k^{v,\mathrm{T}} \check{\mathbf{w}}_k^v \\ \qquad - \check{\mathbf{w}}_l^{v,\mathrm{T}} \check{\mathbf{P}}_{kl}^{vs} \check{\mathbf{P}}_{kl}^{vs,\mathrm{T}} \check{\mathbf{w}}_l^v \Big), & \text{if } v = y, \\ -\sum_{k=1}^{M_\mathrm{u}} \sum_{l=k-1}^{k+1} \sigma_\mathrm{M}^2 \left(\check{\mathbf{w}}_l^{y,\mathrm{T}} \check{\mathbf{P}}_{kl}^{vy} \check{\mathbf{P}}_{kl}^{vy,\mathrm{T}} \check{\mathbf{w}}_l^y \right), & \text{if } v \neq y. \end{cases} \tag{36}$$

2) DL/UL User and Sub-Carrier-Wise MSE: Finally, we define a relation between the UL and DL filters with a real-valued scaling factor $\check{\gamma}_k^s \in \mathbb{R}_+$ per user and per sub-carrier such that

$$\hat{\mathbf{b}}_l^s = \check{\gamma}_l^s \check{\mathbf{w}}_l^s \quad \text{and} \quad \hat{\mathbf{w}}_k^v = \left(\check{\gamma}_k^v\right)^{-1} \check{\mathbf{b}}_k^v. \tag{37}$$

We then set the UL and DL MSE equal for each user and sub-carrier similar to Section III-B4. Again, we end up with a system of linear equations similar to 24. Whereby, we now solve for the $M_\mathrm{u} \times U$ DL/UL scaling factors, i.e. $\check{\boldsymbol{\gamma}}_k \in \mathbb{R}_+^U$, $k \in \{1, \ldots, M_\mathrm{u}\}$ and the elements on the RHS of the system of equations are defined as

$$\boldsymbol{\kappa}_k = \left[\check{\mathbf{b}}_k^{1,\mathrm{T}} \hat{\mathbf{R}}_\eta^1 \check{\mathbf{b}}_k^1, \quad \ldots, \quad \check{\mathbf{b}}_k^{U,\mathrm{T}} \hat{\mathbf{R}}_\eta^U \check{\mathbf{b}}_k^U\right]^T. \tag{38}$$

We define the tri-diagonal matrices $\mathbf{A}^{v,v}$ and $\mathbf{A}^{v,s}$ for $s \neq v$ as follows

$$[\mathbf{A}_{k,k}]_{v,s} = \begin{cases} \sigma_\mathrm{M}^2 \Big(\mathbf{e}_\nu^\mathrm{T} \check{\mathbf{P}}_k^{v,\mathrm{T}} \check{\mathbf{w}}_k^v \\ \qquad - \check{\mathbf{w}}_k^{v,\mathrm{T}} \check{\mathbf{P}}_k^v \check{\mathbf{P}}_k^{v,\mathrm{T}} \check{\mathbf{w}}_k^v \Big), & \text{if } v = s, \\ -\sigma_\mathrm{M}^2 \check{\mathbf{w}}_k^{s,\mathrm{T}} \check{\mathbf{P}}_k^{vs} \check{\mathbf{P}}_k^{vs,\mathrm{T}} \check{\mathbf{w}}_k^s, & \text{if } v \neq s, \end{cases} \tag{39}$$

$$[\mathbf{A}_{k,m}]_{v,s} = \left\{ -\sigma_\mathrm{M}^2 \check{\mathbf{w}}_m^{s,\mathrm{T}} \check{\mathbf{P}}_{km}^{vs} \check{\mathbf{P}}_{km}^{vs,\mathrm{T}} \check{\mathbf{w}}_m^s, \quad \text{if } |k - m| = 1 \right. \tag{40}$$

IV. SLR-BASED PRECODER AND SINR-BASED EQUALIZER DESIGN

The second design method we consider here is based on the one introduced in [10]. The precoder is designed to maximize the SLR and the equalizer to maximize the SINR. We employ complex valued, multi-tap precoders and equalizers, and we also consider an iterative algorithm as shown in Algorithm 2. The expression to calculate the SLR is equation (26) of [10] and the SINR expression has a similar structure to the SLR expression, but with an additional noise component. In the final paper we will show both expressions and how to derive them.

Algorithm 2 Joint SLR-based Precoder and SINR-based Equalizer Design

Initialization:
$\check{\mathbf{w}}_{k,(0)}^v = \mathbf{e}_{\mathrm{B}/2} \quad \forall v, k$
$i = 1$
repeat
  $\check{\mathbf{b}}_{k,(i)}^v = \arg\max \mathrm{SLR}_{k,(i)}^v$
  $\check{\mathbf{w}}_{k,(i)}^v = \arg\max \mathrm{SINR}_{k,(i)}^v$
  $i = i + 1$
until $i = n$

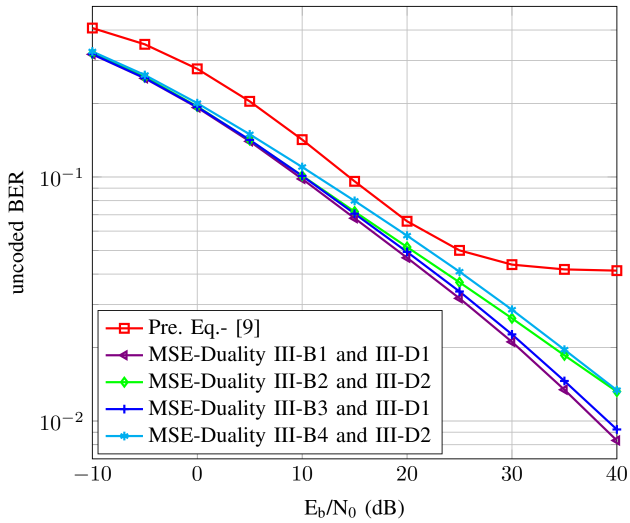

Figure 1. BER of the iterative design- [9] and the four MSE-duality based designs for $N_\mathrm{t} = 4$ and $U = 4$

It is important to note the differences to the scheme in [10], where only real valued single-tap were used in the UEs.

V. SIMULATION RESULTS

In this section we discuss the simulation results of the MSE-duality based precoder and equalizer filter designs for the DL MU-MISO scenario. This is merely a simplification of the MU-MIMO system where each UE only has a single receiver antenna, i.e., $N_{\mathrm{r_s}} = 1$. We use channel realizations from the *Wireless World Initiative New Radio* (WINNER II) project which is an extension to the *Spacial Channel Model* (SCM) [15] developed by the *3rd Generation Partnership Project* (3GPP).

We transmit data across $M_\mathrm{u} = 210$ of the available $M = 256$ sub-carriers per user and per transmitter antenna. We use a sampling rate of $f_s = 11.2$ MHz. We use randomly generated 16-QAM symbols and take a block length of 1000 symbols per sub-carrier. We have a channel impulse response of $L_\mathrm{ch} = 124$ taps. With these system configurations, especially due to $L_\mathrm{ch} = 124$ and the highly frequency selective channel, a CP-OFDM system would have required a CP with a minimum length of 123 taps [4], [3]. This limits the data-throughput of the CP-OFDM to almost 50%, therefore we do not include a direct comparison in the simulation results. We take the quantity of

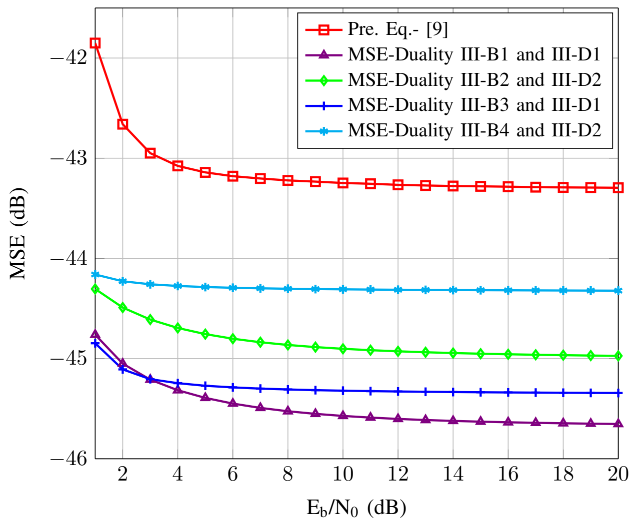

Figure 2. MSE Convergence of the iterative design- [9] and the four MSE-duality based designs for $N_{\mathrm{t}} = 4$ and $U = 4$

E_{b}/N_0 to be a pseudo-*Signal-to-Noise Ratio* (SNR) per user for the MU-MISO simulations. We take the uncoded *Bit Error Rate* (BER) and MSE as an average over all users. We average over 500 randomly generated channel realizations.

In our MU-MISO system we have a precoder length of $B = 3$ taps per user, sub-carrier and transmitter antenna, and we have an equalizer length of $L_{\mathrm{eq}} = 5$ taps per user and sub-carrier. In Fig. 1 we see the uncoded BER versus SNR for a system with 4 BS transmitter antennas and $U = 4$ users. We stopped our iterative algorithm after $n = 5$ iterations and observe that our MSE-duality based designs perform much better over the whole SNR range when compared with the joint precoder and equalizer design from [9]. Furthermore, we notice that the design in [9] saturates in the high SNR range. This can be attributed to the fact that they used a direct precoder design which did not take the noise covariance matrix into account and only minimizes the *quasi*-MSE. We notice that the *System-wide Sum-MSE* transformation performs the best over the whole SNR range, since this method allows the transmit power to be spread across the users and sub-carriers to compensate poor channels.

In Fig. 2 we see the convergence of the different MMSE-based precoder and equalizer designs. Here we notice that all four MSE-duality based designs outperform the design from [9] starting at the first iteration. Furthermore, we notice that the *System-Wide Sum-MSE* transformation converges to the smallest MSE after only a few iterations. After 5 iterations, the MSE of all the designs does not significantly improve anymore, this is why we used $n = 5$ for the uncoded BER simulations.

In Fig. 3 we see the uncoded BER results taken from [10]. The iterative results correspond to the precoder design method presented in [9] with a MRC receiver presented in [10].

VI. Conclusions

In the final paper we will present two design methods for precoder and equalizer designs for MU-MIMO DL FBMC/OQAM systems. In this first version we have shown the MSE-duality based design method and some preliminary results for the special MU-MISO DL case. The second method is an extension of one of the methods presented in [10].

Both schemes are iterative and the first one is an MMSE design based on the MSE duality and the second one maximizes the SLR and SINR.

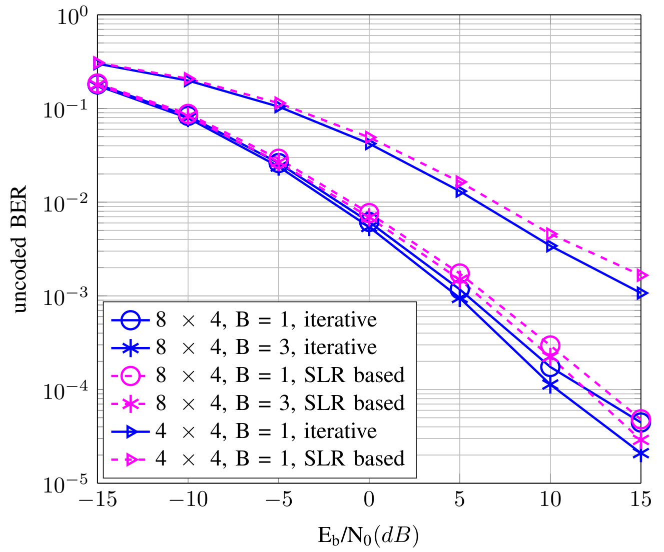

Figure 3. BER performance in different multi-user MIMO downlink settings where $U = 2$, $N_{\mathrm{R}_1} = N_{\mathrm{R}_2} = 2$, $N_{\mathrm{T}} = 8$, or 4, and $\alpha = 0.025$

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