# Low-Cost and Accurate Broadband Beamforming Based on Narrowband Sub-Arrays

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***Abstract*—Simplified broadband beamformers can be constructed by sharing a single tapped-delay-line within a narrowband subbarray. This paper discusses the use of fractional delay filters to a steering in the digital domain. For the narrowband subarrays, an optimisation approach is proposed to maintain an off-broadside look direction constraint as best as possible across a given frequency range. We demonstrate the advantage that this approach has for generating beamformers with accurate off-broadside look direction compared to a benchmark.**

## I. Introduction

While a theoretical broadband beamforming requires each array element to be followed by a tap-delay-line implementing frequency-selective filters [1], state-of-the-art broadband radar hardware reaches a compromise. Complex multipliers follow the sensor element, which are then grouped into subarrays followed by hardware time delay units in order to reach an acceptable performance across the operating bandwidth [2].

The architecture of narrowband subarrays followed by a time delay is referred to as a subarray structure, and has been addressed e.g. in [3], [4], [5], [6]. The general problem that has been researched is the tiling of the subarrays in order to minimise quantisation sidelobes [3], [4], [5]. Sometimes also the narrowband beamforming weights are optimised in order to suppress sidelobes in the beamformer's broadband response [3], [7].

This paper explores a digital implementation of time delay using fractional delay filters, and instead of optimising sidelobe levels, in first instance we are concerned with minimising the deviation in the beamformer's gain in look-direction. We demonstrate that the combination of fractional delay filters and optimisation of narrowband weight can provide an acceptable performance.

## II. Array Configuration

For simplicity, our analysis relies on a uniform linear array as shown in Fig. 1. A total of $KM$ sensor elements is organised into $M$ sub-arrays comprising $K$ elements each. The arrangement operates similar to a delay-and-sum beamformer, whereby the filter $v_m[n]$, $m = 1 \ldots M$ implement delays such that the wavefront arriving from an angle $\vartheta_0$ is aligned w.r.t. the phase centres of the subarrays. This contribution evaluates fractional delay filters for this purpose.

Given the coarse alignment of the subarray centre points, the narrowband beamformer then provide a finer steering, which due to the subarray's narrowband nature can only be accurate at a given point in frequency. A second aim of this paper is to minimise the resulting error over the operating frequency range.

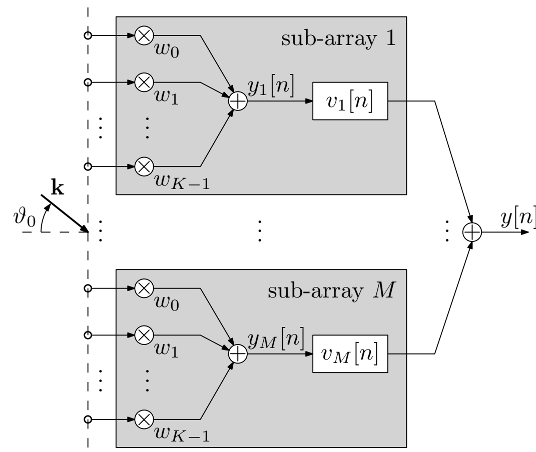


Fig. 1. Uniform linear array divided into $M$ narrowband sub-arrays of $K$ sensors each, which are then combined via $M$ filters $v_m[n]$, $m = 1 \ldots M$. The angle of arrival $\vartheta_0$ of an incoming farfield waveform with slowness vector $\mathbf{k}$ is measured against broadside.

## III. Broadband Beamformer Design

### A. Fractional Delay Filters

To approximate a fractional delay, a number of different filter implementations have been proposed [8]. While the optimum fractional delay is a sinc function of infinite support, finite causal version require a trunction with a rectangular window $p_N[n] = \sum_{\nu=-N}^{N} \delta[n-\nu]$ and a time shift. Such filters generally are inaccurate particularly close for frequencies close to half of the sampling rate, but performance can be enhanced by tapered windows [9], [10]. Using e.g. a von Hann window

$$w[n] = \cos^2(\frac{\pi n}{2N}) p_N[n]$$

a filter implementating a fractional delay $\tau_m$ can be constructed as

$$v_m[n] = \mathrm{sinc}[n - N - \tau_m] \cdot w[n - N - \tau_m]$$

where $\mathrm{sinc}[n]$ is the period sinc function and $\tau_m = \mathbf{k}^{\mathrm{T}} \mathbf{r}_m$, with $\mathbf{k}$ the slowness vector of the incoming waveform and $\mathbf{r}_m$ the centre point of the $m$th subarray.

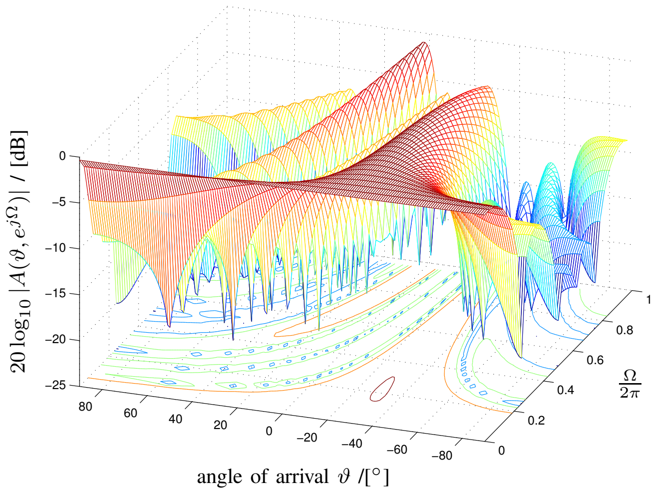


Fig. 2. Subarray architecture pointing towards $\vartheta_0 = -30°$ with narrowband beamformers selected w.r.t. centre frequency.

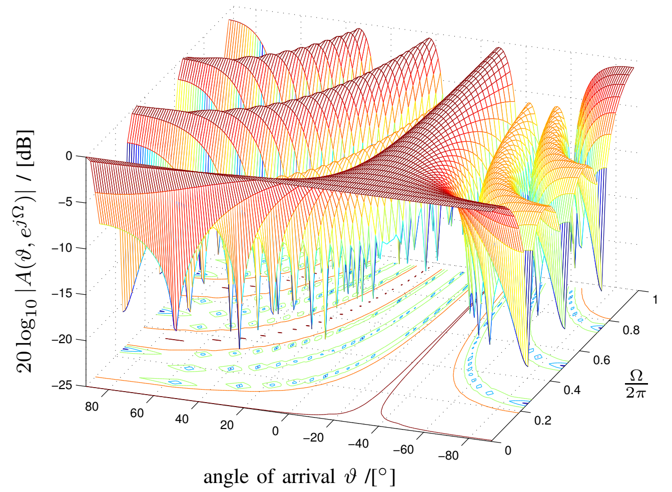


Fig. 3. Subarray architecture pointing towards $\vartheta_0 = -30°$ with narrowband beamformers optimised w.r.t. (1).

### *B. Narrowband Subarray Optimisation*

If $\mathbf{a}(\Omega, \vartheta_0)$ represents a steering vector in look direction $\vartheta_0$ at a normalised angular frequency $\Omega$, then the deviation from unit gain by a beamformer with weights $\mathbf{w}$ is measured by

$$e(\Omega) = \mathbf{a}^{\mathrm{H}}(\Omega, \vartheta_0)\mathbf{w} - 1 \; .$$

Evaluated over a range of frequencies $\Omega \in [\Omega_l; \Omega_u]$, the cost function for the optimisation of the narrowband beamforming weights $\mathbf{w}^{\mathrm{H}} = [w_0 \; w_1 \; \ldots \; w_{M-1}]$ is given by

$$\xi = \min_{\mathbf{w}} \int_{\Omega_l}^{\Omega_u} |e(\Omega)|^2 d\Omega \; . \tag{1}$$

This can be solved either by a Wiener-Hopf type solution, or by sampling the frequency range into $P$ bins, whereby a matrix $\mathbf{A} \in \mathbb{C}^{M \times P}$ contains stacked steering vectors $\mathbf{a}(\Omega_p, \vartheta_0)$, $p = 0 \ldots (P-1)$, such that $\Omega_p = \Omega_l + \frac{p}{P-1}(\Omega_u - \Omega_l)$. The cost function can be shown to be minimised in the least squares sense by

$$\mathbf{w}_{\mathrm{opt}} = \mathbf{A}^{\dagger}\underline{1} \; , \tag{2}$$

using the pseudo-inverse $\mathbf{A}^{\dagger}$ [11].

## IV. (Some) Simulation Results

Below, the architecture is simulated over one octave with $\Omega_l = \frac{\pi}{2}$ and $\Omega_i = \pi$. A total of 32 sensors are split into $M = 4$ subarrays of $K = 8$ elements each. The fractional delay filters are Hann-windowed sinc functions [9] of length $N = 25$. Noting that fractional delay filters are imperfect for $\Omega \longrightarrow \pi$, the performance at the upper limit of the frequency operating range cannot be expected to be highly accurate.

Fig. 2 shows the beamformer's directivity pattern (or gain response) $A(\vartheta, e^{j\Omega})$ for the case where the tapped delay line filters are designed appropriately as fractional delay filters for a waveform with AoA of $\vartheta_o = -30°$. As a benchmark, Fig. 2 uses a steering vector for $\vartheta_0$ and the centre frequency of the interval $[\Omega_l; \; \Omega_u]$. In contrast, Fig. 3 shows the array response for the case of narrowband filter design according to (2). Grating lobes have appeared, but the beam response in look direction $\vartheta_o = -30°$ better preserved than in the case of Fig. 2.

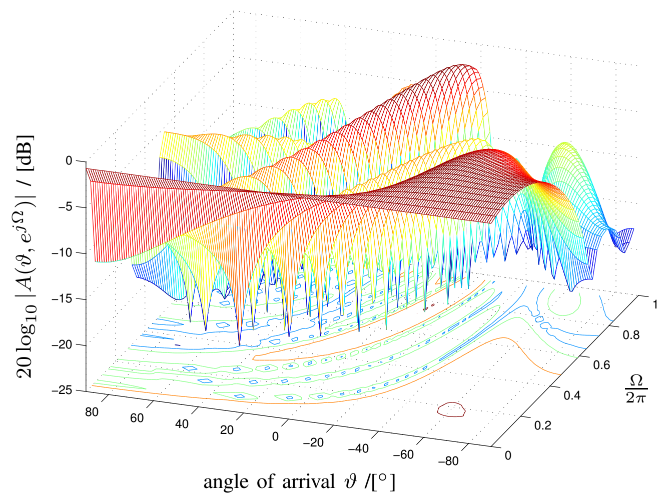


Fig. 4. Subarray architecture pointing towards $\vartheta_0 = -60°$ with narrowband beamformers selected w.r.t. centre frequency.

The same aray configuration is used to implement a look direction of $\vartheta_0 = -60°$. In this case, the beam squint or variation of the steering vector $\mathbf{a}(\Omega, \vartheta)$ over the operating frequency range is great than for the previous example, and the narrowband beamformers introduce a greater error compared to a broabband beamformer with a tapped delay line attached to every sensor element. The result for the subarray architecture and a narrowband design at the centre frequency of the interval $[\Omega_l; \; \Omega_u]$ is shown in Fig. 4. The introduced error is such that the desired unit gain in the look direction cannot be maintained.

For the proposed optimised design of the narrowband beam-

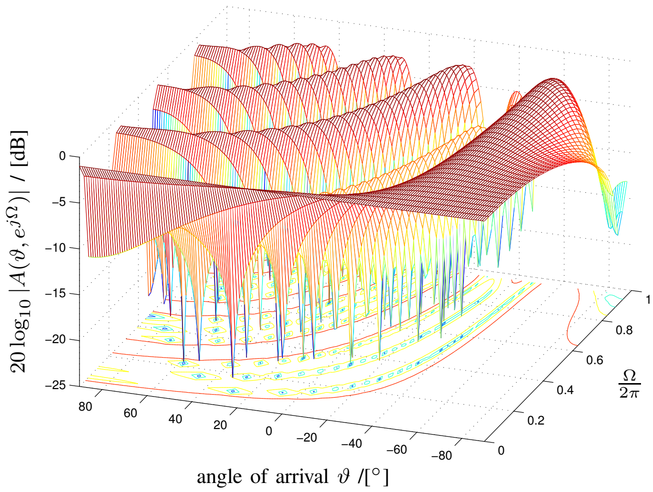


Fig. 5. Subarray architecture pointing towards $\vartheta_0 = -60°$ with narrowband beamformers optimised w.r.t. (1).

former, the resulting directivity patterns is shown in Fig. 5. There is a significant difference to the standard case in Fig. 4, as the unit gain in look direction is maintained. A small deviation towards $\Omega = \pi$ is due to the inaccuracies of the fractional delay filters.

As a drawback of the proposed design, Figs. 3 and 5 exhibit stronger grating lobes compared to the benchmark approach in Figs. 2 and 4. In parts, this can be bypassed by selecting non-uniform subarray configurations as discussed in [3], [4], [5]. This can be accommodated by designing, different from our architecture shown in Fig. 1, the narrowband beamforming coefficients for each subarray individually.

## V. Conclusions and Full Paper

This paper has proposed a subarray architecture where fractional delay filters coarsely align subarrays in time. The implementation here has been demonstrated by windowed sinc functions of moderate order. A finer tuning for every subarray is performed by narrowband weights. By definition, these narrowband weights can only provide an accurate answer at one given frequency, and are likely to generate an error in the look direction gain at other frequencies. Therefore, an overall error minimisation for this gain has been adopted in order to assign the subarray coefficients.

The full paper will provide more insight into the design. We will exhaustively assess the impact of varying angle and frequency range over which a look direction gain of unity should be maintained. Also, we will investigate how the narrowband design can be applied to individual, non-uniform subarrays, and the impact this has on the reduction of grating lobes.

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