On MSE Balancing in the MIMO Broadcast Channel with Unequal Targets

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***Abstract*—Average *mean square error* (MSE) balancing in a *multiple-input multiple-output* (MIMO) scenario was studied for multiple streams per-user and even for various power limitations that bound the achievable performance, but usually with equal targets. In contrast, we focus on the case where the user MSE targets are different. In this situation, only a subset of the receivers may be served at the solution. At low transmit power, users with low target values are strictly prioritized in that transmission to users with large MSE targets can even be switched off. That way, MSE balancing distinguishes from *signal-to-interference-plus-noise* (SINR) and rate balancing optimizations, where transmission to all users is always active. The transmission to users dependent on the balancing level leads to several practical and theoretical questions that we address here.**

I. Introduction

Downlink balancing optimizations have been studied under perfect and imperfect *channel state information* (CSI) and different power restrictions [1]–[5]. The assumption of imperfect CSI at the transmitter is reasonable in practical systems over fading channels [6]. Moreover, considering generalized power constraints, e.g., per-user or per-antenna constraints instead of a sum-power constraint is convenient in real setups [7].

Several metrics can be employed for balancing optimizations, viz., the users' MSEs or data rates. The close connection between MSE and rate has been pointed out in several works for perfect and imperfect CSI (see e.g., [8], [9]). We exploit the fact that the MSE provides a lower bound for the rate. The rate is the most widely used metric for the *quality of service* (QoS) but is difficult to handle if only imperfect CSI is available at the transmitter. On the contrary, the problem becomes (quasi)convex if viewed isolated with regard to the transmit or receive filters when using the MSE metric. Thus, an *alternate optimization* (AO) is typically employed for both perfect and imperfect CSI, where the well known *broadcast channel* (BC) to *multiple access channel* (MAC) duality is employed for the precoder update steps [6], [10]–[12].

We focus on the case of unequal MSE targets which allows a flexible prioritization of users compared to the scenario where the targets (and the number of streams) for all the users are the same. In the latter scenario with equal targets, the MSEs for all the users are equal at the optimum. A similar behavior is observed when the figures of merit are the SINR or the data rate, where the trivial solution corresponds to the zero value for the balancing level. However, as soon as the transmit power is larger than zero, transmission to all the users is active. Remarkably, this behaviour is true for SINR and rate balancing even with unequal targets (see e.g. [1], [13], [14]), and in sharp constrast to balancing with different MSE targets.

If the MSE is the metric for the balancing problem and users have unequal targets, only a subset of users may actively be served at the optimum. Consider a subset of users with very low targets, large targets for the remaining users, and strictly limited transmit power. Then, the MSEs are balanced only for the subset of users with low targets. If the other users' MSEs would be balanced as well, their achieved values would lie above the trivial upper bound. In other words, transmission is activated only for the user with high performance demands, while transmission to users with low demands is switched off.

We study this interesting effect of (soft) switching on (and off) the transmission to users dependent on the achievable MSEs in the remainder of this work. Note that this behavior may even be exploited for scheduling in higher layers. The prioritization of users allows to distinguish between primary users, i.e., those with low MSE targets, and secondary users with weak targets. Only if the primary served users achieve a certain threshold, which is defined by the ratios between the target MSEs, the transmission to secondary users is activated.

In particular, we address the following questions that arise because of the transmission deactivation of users with large target values for low transmit power within min-max MSE optimization for downlink transmission:

- How similar do we have to choose the MSE targets such that all users are served for limited transmit power? Conversely, what is the minimum required transmit power for an active transmission to all users?
- What is the impact of switching off users in the MSE domain and how does the MSE balancing curve look in the rate domain? In other words, how to choose the MSE targets when we actually aim at rate balancing?
- What is the influence of the multiple power constraints compared to a single sum power constraint within the MSE and rate region?

We answer these questions by simulations and theoretical considerations. For the simulations, we adopt the AO method in [4], [5] to account for unequal MSE target values. Since this solution approach consists of an iterative process, it is necessary to enable each step of the AO to activate and deactivate transmissions to users while minimizing the MSE balancing factor.

In the remainder of this extended abstract, we provide the system model and problem statement in detail and sketch the proposed algorithmic solution for handling unequal MSE targets. Moreover, we briefly discuss the difference of MSE balancing with unequal target MSE values compared to rate balancing with different targets and MSE balancing with equal targets for a two-user *multiple-input single-output* (MISO) BC. The details for the algorithmic solutions of the substeps within the AO and a concise discussion for answering above questions will be provided in the full version of the paper.

II. System model

We consider a MIMO BC where an N-antenna BS sends M_k streams to user $k \in \{1, \ldots, K\}$, which is equipped with R_k antennas. The signal is sent over the channel $\boldsymbol{H}_k \in \mathbb{C}^{N \times R_k}$ and is perturbed by the AWGN $\boldsymbol{\eta}_k \in \mathbb{C}^{R_k}$. The estimated data signal at the k-th receiver reads as

$$\hat{\boldsymbol{s}}_k = \boldsymbol{F}_k^{\mathrm{H}} \boldsymbol{H}_k^{\mathrm{H}} \sum_{i=1}^{K} \boldsymbol{B}_i \boldsymbol{s}_i + \boldsymbol{F}_k^{\mathrm{H}} \boldsymbol{\eta}_k, \tag{1}$$

where $\boldsymbol{F}_k \in \mathbb{C}^{R_k \times M_k}$, $\boldsymbol{B}_k \in \mathbb{C}^{N \times M_k}$ and $\boldsymbol{s}_k \in \mathbb{C}^{M_k}$, are the equalizer, precoder and data vector for the k-th user, respectively. The MSE between the transmitted and estimated data vectors, i.e., $\mathrm{MSE}_k = \mathrm{E}[\|\boldsymbol{s}_k - \hat{\boldsymbol{s}}_k\|_2^2]$, is given by

$$\begin{aligned} \mathrm{MSE}_k = & M_k - 2\Re\left\{\mathrm{tr}\left(\boldsymbol{F}_k^{\mathrm{H}} \boldsymbol{H}_k^{\mathrm{H}} \boldsymbol{B}_k\right)\right\} \\ & + \sum_{i=1}^{K} \left\|\boldsymbol{F}_k^{\mathrm{H}} \boldsymbol{H}_k^{\mathrm{H}} \boldsymbol{B}_i\right\|_{\mathrm{F}}^2 + \mathrm{tr}\left(\boldsymbol{F}_k^{\mathrm{H}} \boldsymbol{C}_{\boldsymbol{\eta}_k} \boldsymbol{F}_k\right) \end{aligned} \tag{2}$$

with the noise covaraince matrix $\boldsymbol{C}_{\boldsymbol{\eta}_k}$.

Our assumption is that the users acquire full information about the channel and, on the contrary, the BS only knows statistical information about the channel, e.g., $\hat{\boldsymbol{H}} = \boldsymbol{H} + \boldsymbol{E}$, where $\boldsymbol{E}$ is the estimation error. Consequently, (2) is not appropriate and we consider the average MSE instead, $\overline{\mathrm{MSE}}_k^{\mathrm{DL}} = \mathrm{E}[\mathrm{MSE}_k]$. Let us now define v_k as a factor scaling the average receive power for the k-th user, $\boldsymbol{F}_k = v_k \tilde{\boldsymbol{F}}_k$. Hence, (2) is rewritten as follows

$$\begin{aligned} \overline{\mathrm{MSE}}_k^{\mathrm{DL}} = & M_k - 2\Re\left\{v_k^* \,\mathrm{tr}\left(\mathrm{E}\left[\tilde{\boldsymbol{F}}_k^{\mathrm{H}} \boldsymbol{H}_k^{\mathrm{H}}\right] \boldsymbol{B}_k\right)\right\} \\ & + |v_k|^2 \Big(\mathrm{E}\Big[\sum_{i=1}^{K} \left\|\tilde{\boldsymbol{F}}_k^{\mathrm{H}} \boldsymbol{H}_k^{\mathrm{H}} \boldsymbol{B}_i\right\|_{\mathrm{F}}^2 + \mathrm{tr}\left(\tilde{\boldsymbol{F}}_k^{\mathrm{H}} \boldsymbol{C}_{\boldsymbol{\eta}_k} \tilde{\boldsymbol{F}}_k\right)\Big]\Big). \end{aligned} \tag{3}$$

Since the CSI is perfect at the users, the receive filters are calculated minimizing the MSE, i.e.,

$$\boldsymbol{F}_k^{\mathrm{MMSE}} = \boldsymbol{W}_k^{-1} \boldsymbol{H}_k^{\mathrm{H}} \boldsymbol{B}_k, \tag{4}$$

with $\boldsymbol{W}_k = \boldsymbol{H}_k^{\mathrm{H}} \sum_{i=1}^{K} \boldsymbol{B}_i \boldsymbol{B}_i^{\mathrm{H}} \boldsymbol{H}_k + \boldsymbol{C}_{\boldsymbol{\eta}_k}$

At the transmitter, L power restrictions are imposed. Using the following expression, various limitations could be considered, e.g., sum power, per-beam or per-antenna (see [5])

$$\sum_{k=1}^{K} \mathrm{tr}\left(\boldsymbol{B}_k^{\mathrm{H}} \boldsymbol{A}_{k,l} \boldsymbol{B}_k\right) = \sum_{k=1}^{K} \left\|\boldsymbol{A}_{k,l}^{1/2} \boldsymbol{B}_k\right\|_2^2 \leq P_l, \tag{5}$$

with $l = 1, \ldots, L$, where L is the number of restrictions, $\boldsymbol{A}_{k,l} \in \mathbb{C}^{N \times N} \succeq 0$ and $\mathrm{rank}(\sum_{l=1}^{L} \boldsymbol{A}_{k,l}) = N$.

III. Problem Formulation

The main goal is to minimize the maximum ratio between the MSE and the target for each of the users, while fulfilling the L power restrictions, that is

$$\min_{\{\boldsymbol{F}_k, \boldsymbol{B}_k\}_{k=1}^{K}} \max_j \frac{\overline{\mathrm{MSE}}_j^{\mathrm{DL}}}{\varepsilon_j} \text{ s.t. } \sum_{k=1}^{K} \left\|\boldsymbol{B}_k^{\mathrm{H}} \boldsymbol{A}_{k,l}^{1/2}\right\|_{\mathrm{F}}^2 \leq P_l, \forall l. \tag{6}$$

Substituting (4) into (3), we obtain

$$\overline{\mathrm{MSE}}_k^{\mathrm{DL}} = M_k - \mathrm{E}\left[\mathrm{tr}\left(\boldsymbol{B}_k^{\mathrm{H}} \boldsymbol{H}_k \boldsymbol{W}_k^{-1} \boldsymbol{H}_k^{\mathrm{H}} \boldsymbol{B}_k\right)\right]. \tag{7}$$

This average MSE is bounded by $0 < \overline{\mathrm{MSE}}_k^{\mathrm{DL}} \leq M_k$, where the upper bound occurs when $\boldsymbol{B}_k = \boldsymbol{0}_{N \times M_k}$. Due to this bound, some of the MSE to target ratios $\overline{\mathrm{MSE}}_k^{\mathrm{DL}}/\varepsilon_k \leq M_k/\varepsilon_k$ may fulfill the restriction with equality at the optimum. In particular, only the users with sufficiently small targets ε_k are balanced while transmissions to users with too large ε_k are switched off due to $\overline{\mathrm{MSE}}_k \leq M_k$. Assume $\varepsilon_1 \leq \varepsilon_2 \leq \ldots \leq \varepsilon_K$ and an optimum balancing value of ε for (6). Moreover, let $\ell \leq K$ be the lowest index with $\varepsilon\varepsilon_\ell \geq M_\ell$. Then, only the MSEs of users $1, \ldots, \ell - 1$ are balanced. This is in contrast to the related SINR and rate balancing, where all the users are active and balanced in the optimum, e.g., $\frac{R_1}{\varrho_1} = \ldots \frac{R_K}{\varrho_K} > 0$ (e.g., see [15, Section III.] and [16, Theorem 1]). The same reasoning also applies for MSE balancing with equal targets.

Despite this knowledge, the balancing optimization itself is difficult to handle. Even though a closed form representation may be found for the expectation in (7), e.g., see [17] for zero-mean Gaussian channels, it is still non-convex in the precoders. To overcome this difficulty, an AO process can be used to find a local solution for (6). Let us now split up the precoders into $\boldsymbol{B}_k = \sqrt{p_k} \tilde{\boldsymbol{B}}_k$, with p_k being the transmit power for user k and $\|\tilde{\boldsymbol{B}}\|_{\mathrm{F}}^2 = 1$, and define the expected values $\bar{\boldsymbol{H}}_k = \mathrm{E}[\boldsymbol{H}_k \tilde{\boldsymbol{F}}_k]$, $\boldsymbol{R}_k = \mathrm{E}[\boldsymbol{H}_k \tilde{\boldsymbol{F}}_k \tilde{\boldsymbol{F}}_k^{\mathrm{H}} \boldsymbol{H}_k^{\mathrm{H}}]$ and $\sigma_k^2 = \mathrm{tr}(\mathrm{E}[\tilde{\boldsymbol{F}}_k^{\mathrm{H}} \boldsymbol{C}_{\boldsymbol{\eta}_k} \tilde{\boldsymbol{F}}_k])$. Accordingly, (3) reads as

$$\begin{aligned} \overline{\mathrm{MSE}}_k^{\mathrm{DL}} = & M_k - 2\Re\left\{v_k^* \,\mathrm{tr}\left(\bar{\boldsymbol{H}}_k^{\mathrm{H}} \boldsymbol{B}_k\right)\right\} \\ & + |v_k|^2 \sum_{i=1}^{K} \mathrm{tr}\left(\boldsymbol{B}_i^{\mathrm{H}} \boldsymbol{R}_k \boldsymbol{B}_i\right) + |v_k|^2 \sigma_k^2. \end{aligned} \tag{8}$$

The AO exploits that (8) is biconvex [18] in the precoders and the equalizer functions within the expectations. The following steps are repeated until convergence (cf. [11]):

1) The equalizer functions $\boldsymbol{F}_k^{\mathrm{MMSE}}$ and powers p_k are first found based on (7) for fixed $\tilde{\boldsymbol{B}}_k$, $k = 1, \ldots, K$.
2) The expected values $\bar{\boldsymbol{H}}_k$, $\boldsymbol{R}_k$, and σ_k^2 are computed.
3) Then, the downlink precoders $\boldsymbol{B}_k$ are optimized as equalizers in the dual uplink, based on (8).

The power allocation in step 1 is responsible for switching users on if ε becomes sufficiently small. The corresponding optimization can equivalently be formulated as

$$\min_{\varepsilon, \boldsymbol{p} \geq \boldsymbol{0}_K} \varepsilon \text{ s.t. } \boldsymbol{p} \geq \boldsymbol{I}\left(\varepsilon, \boldsymbol{p}\right), \ \tilde{\boldsymbol{A}} \boldsymbol{p} \leq \boldsymbol{1}, \tag{9}$$

where $\boldsymbol{p} = [p_1, \ldots, p_K]^{\mathrm{T}}$ is the power allocation, $\mathbf{1}$ an all-ones vector, and $\boldsymbol{I}(\varepsilon, \boldsymbol{p}) = \mathrm{diag}(\gamma_1(\varepsilon), \ldots, \gamma_K(\varepsilon))\boldsymbol{Z}(\boldsymbol{p})$, with

$$\gamma_k(\varepsilon) = \max\{0, M_k - \varepsilon\varepsilon_k\},$$

the function $\boldsymbol{Z}(\boldsymbol{p}) = [Z_1(\boldsymbol{p}), \ldots, Z_K(\boldsymbol{p})]^{\mathrm{T}}$ is given by

$$Z_k(\boldsymbol{p}) = \left(\mathrm{E}\left[\mathrm{tr}\left(\tilde{\boldsymbol{B}}_k^{\mathrm{H}} \boldsymbol{H}_k \boldsymbol{W}_k^{-1}(\boldsymbol{p}) \boldsymbol{H}_k^{\mathrm{H}} \tilde{\boldsymbol{B}}_k\right)\right]\right)^{-1}$$

where $\boldsymbol{W}_k(\boldsymbol{p}) = \boldsymbol{H}_k^{\mathrm{H}} \sum_{i=1}^K \tilde{\boldsymbol{B}}_i \tilde{\boldsymbol{B}}_i^{\mathrm{H}} p_i \boldsymbol{H}_k + \boldsymbol{C}_{\boldsymbol{\eta}_k}$, and

$$[\tilde{\boldsymbol{A}}]_{\ell,k} = P_\ell^{-1} \|\tilde{\boldsymbol{B}}_k^{\mathrm{H}} \boldsymbol{A}_{k,l}^{1/2}\|_{\mathrm{F}}^2.$$

We remark that $\boldsymbol{Z}(\boldsymbol{p})$ satisfies the properties of standard interference functions. Therefore, the solution $(\varepsilon^\star, \boldsymbol{p}^\star)$ of (9) is uniquely characterized by the two properties

$$\begin{aligned} \boldsymbol{p}^\star &= \boldsymbol{I}(\varepsilon^\star, \boldsymbol{p}^\star) \\ \varepsilon^\star &= \min\left\{\varepsilon \in \mathbb{R}_+ : \tilde{\boldsymbol{A}}\boldsymbol{I}(\varepsilon, \boldsymbol{p}^\star) \leq \mathbf{1}\right\}. \end{aligned} \tag{10}$$

This fixed-point is found, for example, via various adaptations of Yates procedure in [19], Schubert and Boches approach in [1], or using a Newton like method. The details of the applied method will be provided in the full paper.

Assume the solution to (9) is $\boldsymbol{p} \geq \mathbf{0}_K$, with $p_k > 0$ for $k = 1, \ldots, \ell - 1$ and $p_k = 0$ for $k = \ell, \ldots, K$. Based on this solution, we compute the MMSE receive filters and the required expectations for (8). To keep the flexibility for switching precoders $\boldsymbol{B}_k$ on (or off) within step 3 of the AO iteration, we also compute the receive filters $\boldsymbol{F}_k^{\mathrm{MMSE}}$, $\bar{\boldsymbol{H}}_k$, $\boldsymbol{R}_k$, and σ_k^2, for users $k \geq \ell$, but under the assumption that $p_k = 1$. The objective of the precoder optimization in step 3 of the AO is to minimize the maximum ratios of the MSEs and targets, which inherently contains the decision whether either of the MSEs for users $k = \ell, \ldots, K$ are balanced as well. In particular, the following optimization problem is solved with given expectations based on the MMSE filters, $\boldsymbol{F}_k^{\mathrm{MMSE}} = v_k \tilde{\boldsymbol{F}}_k^{\mathrm{MMSE}}$:

$$\min_{\{v_k, \boldsymbol{B}_k\}_{k=1}^K} \max_j \frac{\overline{\mathrm{MSE}}_j^{\mathrm{DL}}}{\varepsilon_j} \text{ s.t. } \sum_{k=1}^K \left\|\boldsymbol{B}_k^{\mathrm{H}} \boldsymbol{A}_{k,l}^{1/2}\right\|_{\mathrm{F}}^2 \leq P_l, \forall l. \tag{11}$$

The solution may be found via a sequence of convex power minimization problems, each of which defines a second order cone program, or alternatively, using uplink-downlink duality.

IV. MMSE and Rate Regions

We next discuss the effect of MSE balancing via a graphical example. To this end, we consider a *multiple-input single-output* (MISO) BC, where the BS is equipped with $N = 2$ antennas and sends data to $K = 2$ single-antenna receivers, i.e., $M_1 = M_2 = 1$. Due to imperfect CSI at the BS, the sum-MSE is lower bounded by $\mathrm{MSE}_1 + \mathrm{MSE}_2 \geq 1$. We sketch this MSE attainable region for unbounded transmit power, $\mathcal{M}$, in Fig. 1. The bound of $\mathcal{M}$ is asymptotically reached when the transmit power approaches infinity.

For the MSE balancing formulation in (6), we move along a straight line with 45 degree slope by varying the balance factor ε within $(0, 1]$ if $\varepsilon_1 = \varepsilon_2$. In the case of asymmetric targets, e.g., $\varepsilon_2 = 2\varepsilon_1 = 0.5$, transmission to user 2 is inactive

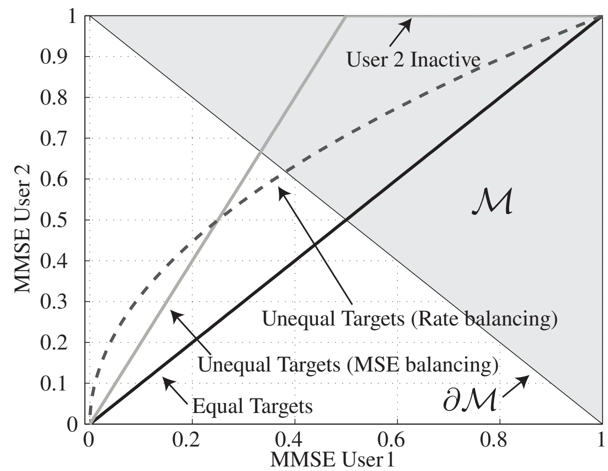

Fig. 1. MSE Region

for $\varepsilon \geq 2$ since $\varepsilon\varepsilon_2 \geq M_2 = 1$. In this case, the achived MSE for transmission to user 1 moves along the horizontal line on the top of the figure as still $\mathrm{MSE}_1 = \varepsilon_1\varepsilon \leq M_1$. This behavior is only possible when balancing is performed using the MSE metric. The obtained curve for balancing the ratios of the rates R_i over the targets $\varrho_i = -\log_2(\varepsilon_i)$, $i = 1, 2$ is also depicted in Fig. 1. As previously pointed out, transmission to the users is active for all of them in the optimal point.

The corresponding rate region and balancing curves to the setup considered is depicted in Fig. 2. The attainable rate region $\mathcal{R}$ is shown, whose upper right bound is reached asymptotically for unconstrained transmit power. The MSE balancing formulation yields a horizontal line at the R_1 axis as long as transmission to user 2 is inactive, i.e., for $-\log_2(\varepsilon_2\varepsilon) \leq -\log_2(M_2) = 0$. When $-\log_2(\varepsilon_2\varepsilon) \geq -\log_2(M_2) = 0$, the MSE balancing solution forms a straight line with 45 degree slope starting at $(-\log_2(\frac{\varepsilon_1/M_1}{\varepsilon_2/M_2}), 0)$ for $\varepsilon_1/M_1 \leq \varepsilon_2/M_2$. In contrast, the rate balancing line for the different targets $\varrho_i = -\log_2(\varepsilon_i)$, $i = 1, 2$ starts at the origin and has the slope ϱ_2/ϱ_1. This means that all users are active when optimizing with respect to the rates.

V. Outlook

The final paper further discusses the following issues:

- The detailed steps to solve the power minimization within step 1 and the precoder update within step 3 of the AO.
- An analysis of the region of MSE targets where transmission to all users is active using simulations of the proposed algorithm.
- The figures for the MSE and rate regions will be improved to include the achievable region for limited sum and per-antenna transmit powers.
- Moreover, we will provide a detailed analysis of the differences between rate and MSE balancing. Furthermore, we will present the achievable MSE and rate curves when performing the rate balancing via the MSE lower bounds.

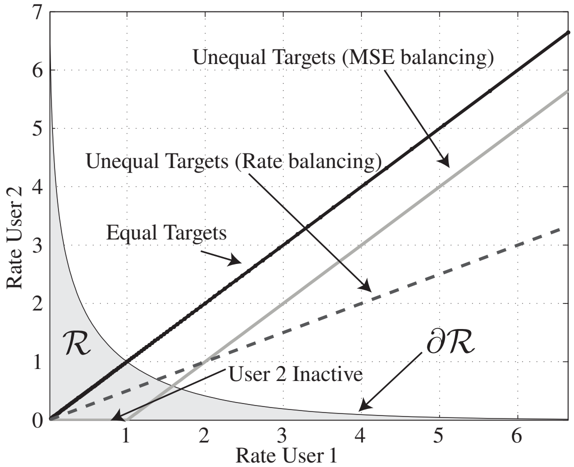

Fig. 2. Rate Region

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