A Study on Source-Relay Cooperation for the Outage Constrained Relay Channel

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***Abstract*—We study the influence of source-relay cooperation on the outage constrained capacity bounds of the Gaussian relay channel. As was observed, coherent source-relay transmission does not lead to improvement for the *decode and forward* (DF) achievable rate in the presence of Rayleigh fading. We show that this is in sharp contrast to the case with known channel means. Then, transmission gains highly from coherent source-relay to destination transmission.**

I. Introduction

The general concept of relaying was introduced by van der Meulen in [1]. Until now, the general expression for the capacity of the relay channel is not known. An important contribution on the information-theoretic investigation of the relay channel was provided by Cover and El Gamal in [2]. They provided upper and lower bounds for the capacity. Among others, the *decode-and-forward* (DF) strategy and the *cut-set-bound* (CSB) were defined to bound the capacity from below and above, respectively.

In our study, we assume that only the receiving nodes have full *channel state information* (CSI) while the transmitting nodes have only access to the channels' statistics or an estimate of the channel. The approach for evaluating the system performance under such conditions varies upon the assumed fading model. In this work, we assume slow fading of the channel and therefore investigate the outage capacity of the relay channel [3].

Bounds on the outage probability of the relay channel have been studied by Kramer et al. in [4] for a full-duplex setup and phase fading with a given rate. Høst-Madsen and Zhang extended the results to a half-duplex setup in [5]. Other works, e.g., [6], [7], focused their study on low-SNR regions.

Similar to [8], we consider the reverse problem, namely deriving rate bounds for a restricted outage probability. This work extends the results from the aforementioned paper and provides a detailed discussion about the question when source-relay cooperation is advantageous and supports communication.

While the work in [8] concentrated on the Rayleigh channel model, in this paper, we assume either a *line of sight* (LOS) component for the channel distribution, i.e., a *Rician fading* model, or some known channel estimate.

The analysis of the Gaussian relay channel in [8] assured, that the DF strategy does not benefit from cooperation between the relay and the source if Rayleigh channels between the terminals are assumed. In particular, coherent source-relay transmission does not result in increased DF rate for a small outage requirement. The aim of our study is to show that the situation is different for known channel mean or estimates. For a wide class of such channels, cooperation leads to gains in the outage constrained DF rates.

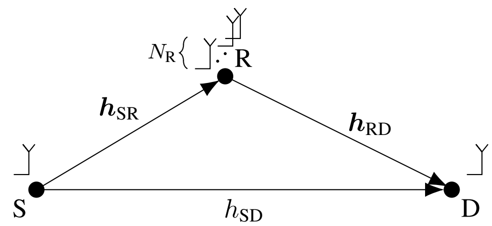

Fig. 1. Setup with multi-antenna relay

In the remainder of the extended abstract, we introduce the model of the system, motivate the research and show some preliminary results. Detailed derivations are omitted here but will be presented in the final paper (see also Section VI).

II. System Model

We assume a three-node Gaussian relay system as shown in Fig. 1. The investigations are for a setup with single-antenna source and destination and with multiple antennas at the relay.

The signals received at the relay and destination read as

$$\begin{aligned} \boldsymbol{y}_{\mathrm{R}} &= \boldsymbol{h}_{\mathrm{SR}} x_{\mathrm{S}} + \boldsymbol{n}_{\mathrm{R}}, \\ y_{\mathrm{D}} &= h_{\mathrm{SD}} x_{\mathrm{S}} + \boldsymbol{h}_{\mathrm{RD}}^{\mathrm{H}} \boldsymbol{x}_{\mathrm{R}} + n_{\mathrm{D}}. \end{aligned} \tag{1}$$

The noise components are independent of each other as well as of the transmitter signals, i.e., $\boldsymbol{n}_{\mathrm{R}} \sim \mathcal{N}_{\mathbb{C}}(\boldsymbol{0}, \boldsymbol{I}), n_{\mathrm{D}} \sim \mathcal{N}_{\mathbb{C}}(0, 1)$. Without loss of generality and optimality, we assume zero-mean channel inputs x_{S} and $\boldsymbol{x}_{\mathrm{R}}$ constrained with the available power budget $\mathrm{E}[|x_{\mathrm{S}}|^2] \leq P_{\mathrm{S}}$ and $\mathrm{E}[||\boldsymbol{x}_{\mathrm{R}}||_2^2] \leq P_{\mathrm{R}}$.

In our work, we provide results for a channel model with known mean or estimate. For example, we assume one direct path and multiple scattered paths. Omitting the subscripts referring to the links, the formal description reads as

$$\boldsymbol{h} = \bar{\boldsymbol{h}} + \hat{\boldsymbol{h}}, \quad \hat{\boldsymbol{h}} \sim \mathcal{N}_{\mathbb{C}}(\boldsymbol{0}, \sigma_{\hat{\boldsymbol{h}}}^2 \boldsymbol{I}). \tag{2}$$

The Rician K factor is defined as the ratio of the power of the direct path and the power of the scatterers $K = \frac{||\bar{\boldsymbol{h}}||_2^2}{\sigma^2_{\hat{h}}}$.

We see the extension of the analysis in [8] to the channels in (2) as an important input to the discussion on practical applications of relaying systems. For example, in mmWave systems, which are considered to be included in the 5G standards, it is agreed that a strong LOS path is required for maintaining connectivity between the terminals.

III. Problem statement

The capacity bounds for the relay channel with perfect CSI were given by Cover and El Gamal in [2]. Gaussian full-power signaling maximizes the DF achievable rate as well as the CSB expression for the Gaussian relay channel. Thus, the DF rate and CSB expression can be written as (c.f. [8])

$$C_{\text{CSB}}(\boldsymbol{h}) = \max_{\beta} \min \left\{ C^{(1)}_{\text{CSB}}(\beta, \boldsymbol{h}), C^{(2)}_{\text{CSB}}(\beta, \boldsymbol{h}) \right\}, \quad (3)$$

$$R_{\text{DF}}(\boldsymbol{h}) = \max_{\beta} \min \left\{ R^{(1)}_{\text{DF}}(\beta, \boldsymbol{h}), R^{(2)}_{\text{DF}}(\beta, \boldsymbol{h}) \right\} \quad (4)$$

where

$$C^{(1)}_{\text{CSB}}(\beta, \boldsymbol{h}) = \log_2 \left(1 + \left(1 - \beta^2\right) \left(\|\boldsymbol{h}_{\text{SR}}\|_2^2 + |h_{\text{SD}}|^2\right) P_{\text{S}}\right), \quad (5)$$

$$R^{(1)}_{\text{DF}}(\beta, \boldsymbol{h}) = \log_2 \left(1 + \left(1 - \beta^2\right) \|\boldsymbol{h}_{\text{SR}}\|_2^2 P_{\text{S}}\right), \quad (6)$$

$$C^{(2)}_{\text{CSB}}(\beta, \boldsymbol{h}) = R^{(2)}_{\text{DF}}(\beta, \boldsymbol{h}) = = \log_2 \left(1 + [h^*_{\text{SD}}, \boldsymbol{h}^{\text{H}}_{\text{RD}}] \boldsymbol{C} [h_{\text{SD}}, \boldsymbol{h}^{\text{T}}_{\text{RD}}]^{\text{T}}\right), \quad (7)$$

with $\beta = \frac{||\boldsymbol{r}_{\text{SR}}||_2}{\sqrt{P_{\text{S}} P_{\text{R}}}}$, the matrix $\boldsymbol{C} = \begin{bmatrix} P_{\text{S}} & \boldsymbol{r}^{\text{H}}_{\text{SR}} \\ \boldsymbol{r}_{\text{SR}} & \boldsymbol{R}_{\text{R}} \end{bmatrix}$ is the joint covariance matrix of the source and the relay, $\boldsymbol{R}_{\text{R}} = \text{E}[\boldsymbol{x}_{\text{R}} \boldsymbol{x}^{\text{H}}_{\text{R}}]$ is the covariance matrix of the relay and $\boldsymbol{r}_{\text{SR}} = \text{E}[\boldsymbol{x}_{\text{R}} x^*_{\text{S}}]$.

Both the CSB expression in (3) and the DF rate in (4) can be seen as the minimum rate of two links. In the first link, the source is transmitting and either the relay and destination are jointly receiving (for the CSB) or only the relay is receiving (for the DF). In the second link (for both CSB and DF), the source and relay are jointly transmitting and the destination terminal serves as the receiver. In our work, we place emphasis on the analysis of the joint source-relay transmission. The degree of cooperation is modeled by β and the specifics of the cooperation is included in $\boldsymbol{r}_{\text{SR}}$.

As pointed out in the introduction, we focus on setups with imperfect channel knowledge and the outage capacity as performance measure. Therefore, we define the DF rate bound and CSB on the ϵ-outage capacity as

$$R^{(\text{out})}_{\text{DF}} = \max_{\rho, \beta} \left\{\rho \in \mathbb{R} : \, p_{\text{DF}}(\rho, \beta) \geq 1 - \epsilon \right\}, \quad (8)$$

$$C^{(\text{out})}_{\text{CSB}} = \max_{\rho, \beta} \left\{\rho \in \mathbb{R} : \, p_{\text{CSB}}(\rho, \beta) \geq 1 - \epsilon \right\} \quad (9)$$

where the probabilities inside (8) and (9) are defined as

$$p_{\text{DF}}(\rho, \beta) = \Pr \left[\min_{i=1,2} \left\{ R^{(i)}_{\text{DF}}(\beta, \boldsymbol{h}) \right\} \geq \rho \right], \quad (10)$$

$$p_{\text{CSB}}(\rho, \beta) = \Pr \left[\min_{i=1,2} \left\{ C^{(i)}_{\text{CSB}}(\beta, \boldsymbol{h}) \right\} \geq \rho \right]. \quad (11)$$

Both, (8) and (9), are chance-constrained optimization problems with unknown convexity properties. In [8], these problems are studied for a single antenna at the relay terminal and Rayleigh fading channels. As mentioned, we consider the Rician fading channel model and extend the study to multiantenna setups. In the full paper, we will further provide a study that includes channel esitimation.

IV. Single Antenna Relay

We investigate the impact of source-relay cooperation on the outage constrained DF rate (4) first for the single relay antenna setup. To this end, we rewrite (7) as

$$R^{(2)}_{\text{DF}}(\beta, \boldsymbol{h}) = \log_2 \left(1 + |h_{\text{SD}}|^2 P_{\text{S}} + h_{\text{RD}} P_{\text{R}} + 2\text{Re}(h^*_{\text{SD}} h_{\text{RD}} r_{\text{SR}})\right). \quad (12)$$

For perfect CSI, the r_{SR} that maximizes (4) is available in closed form and reads as (cf. [2])

$$r_{\text{SR}} = \sqrt{P_{\text{S}} P_{\text{R}}} \frac{h^*_{\text{SD}} h_{\text{RD}}}{|h_{\text{SD}}||h_{\text{RD}}|} \beta \quad (13)$$

where optimal β leads to equal $R^{(1)}_{\text{DF}}(\beta, \boldsymbol{h})$ and $R^{(2)}_{\text{DF}}(\beta, \boldsymbol{h})$, if possible. Thus, the non-coherent transmission maximizes the first rate expression since $R^{(1)}_{\text{DF}}(\beta, \boldsymbol{h})$ only depends on β^2 and (genie-aided) coherent transmission maximizes the second rate expression. The situation becomes less obvious, when only the channel statistic are available. Then, if we model the cross-covariance as

$$r_{\text{SR}} = \sqrt{P_{\text{S}} P_{\text{R}}} \frac{\bar{h}^*_{\text{SD}} \bar{h}_{\text{RD}}}{|\bar{h}_{\text{SD}}||\bar{h}_{\text{RD}}|} \beta, \quad (14)$$

the expression $\text{Re}(h^*_{\text{SD}} h_{\text{RD}} r_{\text{SR}})$ can be less than zero for certain channel realizations. We note that the probability of this event increases with decreasing Rician K-factor. The limit case, i.e., with K equal to zero results in Rayleigh fading channels. For this channel we know from [8] that noncoherent transmission is optimal. On the other hand, for K equal to infinity, we know that the channel is perfectly known and thus the transmission profits from cooperation. We expect that we will benefit in various degrees from source-relay cooperation for Rician K-factors in between.

Our Monte-Carlo simulations agree with this suggestion. For the simulations, we use the line network model [8] and set $\epsilon = 0.25$. In Fig. 2, we show a 3D plot with the source-relay distance on the x-axis, the Rician K-factor on the y-axis and the benefit from cooperation on the z-axis. We see that for each relay position, decreasing K results in a decrease of the cooperation gain. We also see, that for large distances between the source and the relay, i.e., $d_{\text{SR}} > 0.5$, we get no benefit from cooperation. This is because the first rate expression in (10) becomes the main limiting factor in this region.

Figures 3, 4, and 5 give an insight into the results for three values of the Rician K-factor, i.e., $K \in \{0.25, 1, 4\}$. We compare the outage constrained DF rates when only the channel statistics are available at the transmitters [and r_{SR} is as in (14)] with two extreme cases. The first one assumes perfect CSI at the transmitters and thus r_{SR} as in (13). In the

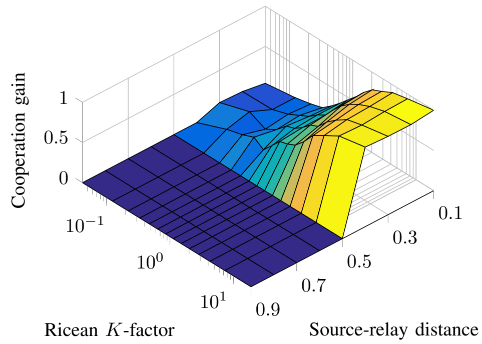

Fig. 2. Gain of source-relay cooperation with respect to the source-relay distance and the Rician K-factor for $\epsilon = 0.25$

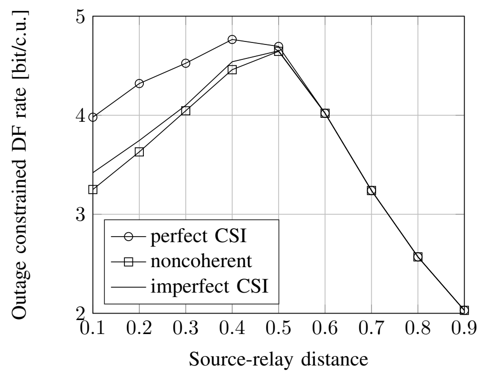

Fig. 3. Outage constrained DF rates for source and single antenna relay cooperating as well as for the noncoherent transmission. $K = 0.25, \epsilon = 0.25$

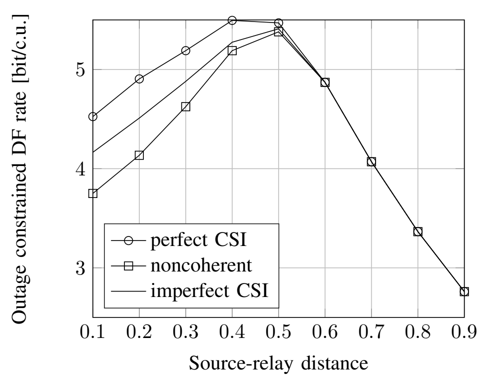

Fig. 4. Outage constrained DF rates for source and single antenna relay cooperating as well as for the noncoherent transmission. $K = 1, \epsilon = 0.25$

second one, noncoherent source-relay transmission is applied. Similarly as in Fig. 2, we see that gains from cooperation are possible only when the relay is close to the source, i.e., $d_{\mathrm{SR}} \leq 0.5$. Moreover, the transmission profits from the knowledge of the channel statistics only if the channel mean is sufficiently strong, i.e., for higher values of K.

V. Multiantenna Relay

Next, we consider the multiantenna relay setup. The source-relay cooperation is then modeled with the vector $\boldsymbol{r}_{\mathrm{SR}}$ and

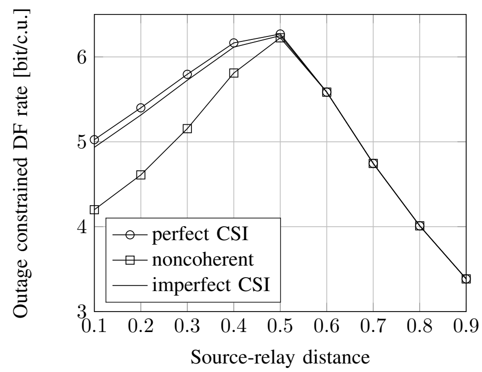

Fig. 5. Outage constrained DF rates for source and single antenna relay cooperating as well as for the noncoherent transmission. $K = 4, \epsilon = 0.25$

the relay transmit strategy is defined by the relay covariance matrix $\boldsymbol{R}_{\mathrm{R}}$. For perfect CSI, the $\boldsymbol{r}_{\mathrm{SR}}$ and $\boldsymbol{R}_{\mathrm{R}}$ that maximize (7) are given in closed form by

$$\boldsymbol{r}_{\mathrm{SR}} = \sqrt{\frac{P_{\mathrm{S}}P_{\mathrm{R}}}{N_{\mathrm{R}}}} \frac{h_{\mathrm{SD}}^{*}\boldsymbol{h}_{\mathrm{RD}}}{|h_{\mathrm{SD}}| \|\boldsymbol{h}_{\mathrm{RD}}\|_2} \beta, \tag{15}$$

$$\boldsymbol{R}_{\mathrm{R}} = \frac{\boldsymbol{h}_{\mathrm{RD}}\boldsymbol{h}_{\mathrm{RD}}^{\mathrm{H}}}{\|\boldsymbol{h}_{\mathrm{RD}}\|_2^2} P_{\mathrm{R}} \tag{16}$$

where optimal β leads to equal $R_{\mathrm{DF}}^{(1)}(\beta, \boldsymbol{h})$ and $R_{\mathrm{DF}}^{(2)}(\beta, \boldsymbol{h})$, if possible.

If only the channel statistics are available, we follow the strategy from Section IV and model $\boldsymbol{r}_{\mathrm{SR}}$ as

$$\boldsymbol{r}_{\mathrm{SR}} = \sqrt{\frac{P_{\mathrm{S}}P_{\mathrm{R}}}{N_{\mathrm{R}}}} \frac{\bar{h}_{\mathrm{SD}}^{*}\bar{\boldsymbol{h}}_{\mathrm{RD}}}{|\bar{h}_{\mathrm{SD}}| \|\bar{\boldsymbol{h}}_{\mathrm{RD}}\|_2} \beta. \tag{17}$$

We investigate the system performance for two relay transmit strategies. In the first one, we match $\boldsymbol{R}_{\mathrm{R}}$ to the channel mean $\bar{\boldsymbol{h}}_{\mathrm{RD}}$, i.e.,

$$\boldsymbol{R}_{\mathrm{R}} = \frac{\bar{\boldsymbol{h}}_{\mathrm{RD}}\bar{\boldsymbol{h}}_{\mathrm{RD}}^{\mathrm{H}}}{\|\bar{\boldsymbol{h}}_{\mathrm{RD}}\|_2^2} P_{\mathrm{R}}. \tag{18}$$

In the second one, we set the covariance matrix to a scaled identity matrix

$$\boldsymbol{R}_{\mathrm{R}} = \frac{P_{\mathrm{R}}}{N_{\mathrm{R}}} \boldsymbol{I}. \tag{19}$$

For high values of the Ricean K-factor, we expect that the system achieves better performance with rank–one $\boldsymbol{R}_{\mathrm{R}}$ as in (18) since the channel mean is “close” to the channel itself. In contrast, for low values of K, we expect better results with the scaled identity $\boldsymbol{R}_{\mathrm{R}}$ in (19). Figures 6, 7, and 8 agree with this suggestion. We also observe that the transmission profits from cooperation depending on the value of K similarly to the single antenna setup. Moreover, compared to the single antenna setup, the cooperation helps in the transmission for a larger range of d_{SR}, i.e., even for $d_{\mathrm{SR}} = 0.5$.

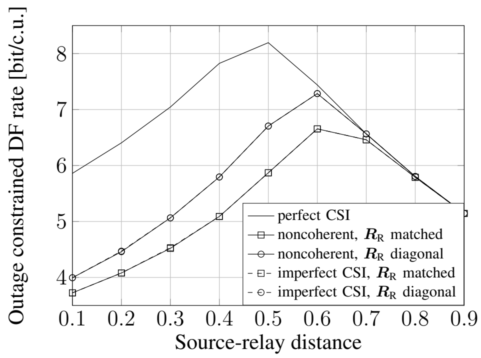

Fig. 6. Outage constrained DF rates for source and (multiantenna) relay cooperating as well as for the noncoherent transmission. $K = 0.1, \epsilon = 0.25, N_\mathrm{R} = 4$

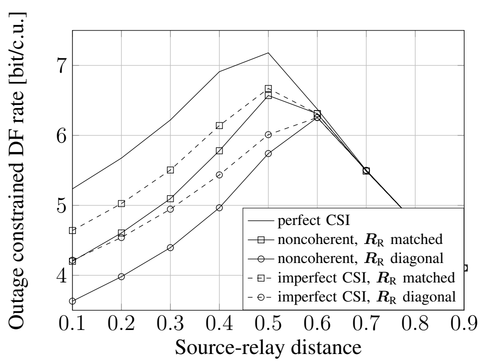

Fig. 7. Outage constrained DF rates for source and (multiantenna) relay cooperating as well as for the noncoherent transmission. $K = 4, \epsilon = 0.25, N_\mathrm{R} = 4$

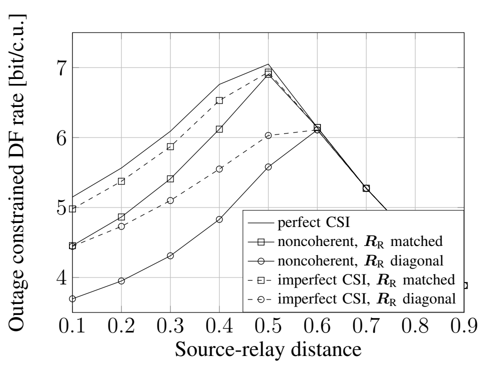

Fig. 8. Outage constrained DF rates for source and (multiantenna) relay cooperating as well as for the noncoherent transmission. $K = 15, \epsilon = 0.25, N_\mathrm{R} = 4$

VI. Outlook

In the final paper, we will further investigate the gains of source-relay cooperation. We will extend the work on the single-antenna and multiantenna setup at the relay terminal. We will provide a detailed description of choices for $\boldsymbol{r}_\mathrm{SR}$ for the multiantenna setup and analyze the optimal choice of β. We will also investigate setups with very high number of antennas at the relay. Moreover, we will present results if the channel model in (2) stems from estimation. For the CSB expressions, we will extend the loosened CSB expressions from [8] to the case with single and multiple antennas at the relay terminal and channel mean/estimation at the transmitting nodes. For the DF rates, we will give closed form outage probability expressions. We will also provide detailed derivations of the computations required for the simulations in this extended abstract.

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