Achievable Rates of Hybrid Architectures with Few-Bit ADC Receivers

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Abstract—**Hybrid analog/digital precoding and combining with low-resolution analog-to-digital converters (ADCs) are two promising solutions that were proposed to reduce the cost and power consumption in millimeter wave (mmWave) systems. In this paper, we consider a new transceiver architecture that combines some elements of hybrid precoding and low-resolution ADCs. For this architecture, we derive lower and upper bounds on the achievable rates. Further, we investigate the interplay between numbers of RF chains and ADC quantization bits, which provide some insights into the energy-rate trade-offs for the adopted architecture. Results illustrate that hybrid architectures can achieve comparable rates to the fully-digital solutions in the low-to-medium SNR ranges, even with few-bit ADC receivers.**

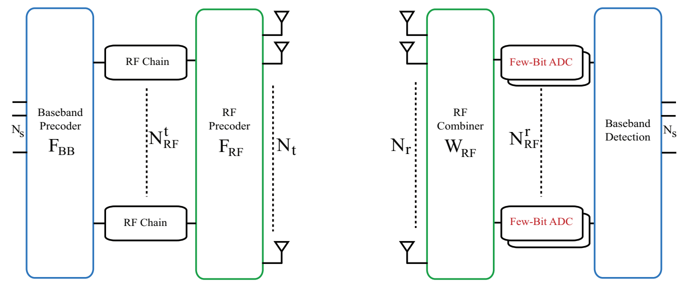

Fig. 1. A MIMO system with hybrid precoding and few-bit ADCs. Note that we assume the DACs at the transmitter have full precision.

I. Introduction

Directional beamforming with large antenna arrays needs to be employed at both the transmitter and receiver in mmWave systems, to provide sufficient received signal power [1], [2]. Unfortunately, the high hardware cost and power consumption of mixed-signal components makes the fully-digital precoding solution, that allocates an RF chain per antenna, infeasible [3], [4]. To overcome this challenge, new architectures that relax the requirement of associating an RF chain per antenna need to be developed [5].

To reduce the number of RF chains, and to support multi-stream multiplexing, hybrid precoding, that divides precoding processing between analog and digital domains, was recently proposed for mmWave large-MIMO systems [6]. In [6], hybrid precoding/combining was shown to achieve close performance to fully-digital architectures with much less number of RF chains compared to the number of antennas. The hybrid combining in [6], though, still assumes that receive RF chains include high-resolution analog-to-digital converters (ADCs), which consumes high power [3]. An alternative to high resolution ADCs is to live with ultra low resolution ADCs (1-4 bits), which reduces power consumption and cost. In [7]–[9], receiver architecture where the received signal at each antenna is directly quantized by low resolution ADCs without any analog combining was considered. In that architectur, however, the number of RF chains were assumed to be equal to the number of antennas, which keeps the hardware cost relatively high for large antenna systems.

In this paper, we consider a new architecture that combines hybrid precoding and combining with few-bit ADCs, to achieve a better compromise between the high achievable rates and the low power consumption. For this architecture, we derive upper and lower bounds on the mutual information. The derived lower bounds correspond to two proposed transmission schemes bases on channel inversion and matched filter precoding. For high SNR regimens, we show that a relatively small gap exists between the lower and upper bounds. Further, results illustrate that the considered architecture can achieve a performance comparable to that obtained with fully-digital precoding in the low-to-medium SNR range, which is of a special importance for mmWave communications.

II. System Model

In this paper, we consider the system model in Fig. 1, where the transmitter employs a hybrid analog/digital precoding and the receiver deploys a hybrid combiner, but with few-bit ADC-s. The transmitter and the receiver are assumed to have N_{t} and N_{r} antennas, respectively. Further, the transmitter is assumed to have $N_{\mathrm{RF}}^{\mathrm{t}}$ RF chains, while the receiver employs $N_{\mathrm{RF}}^{\mathrm{r}}$ RF chains with few-bit (e.g., 1-4 bits) ADCs. Similar to the hybrid architectures in [6], [10], the number of antennas and RF chains are assumed to satisfy $(N_{\mathrm{RF}}^{\mathrm{t}} \le N_{\mathrm{t}}, N_{\mathrm{RF}}^{\mathrm{r}} \le N_{\mathrm{r}})$. The transmitter and receiver communicate via N_{s} data streams, with $N_{\mathrm{s}} \le \min(N_{\mathrm{RF}}^{\mathrm{t}}, N_{\mathrm{RF}}^{\mathrm{r}})$. Compared to the fully-digital architecture where the receiver has N_{r} pairs of high resolution ADCs, the adopted receiver architecture contains only $N_{\mathrm{RF}}^{\mathrm{r}}$ pairs of few-bit ADCs, which has a potential reduction of the cost and power consumption.

Assuming a narrowband channel model, the received signal at the antenna ports can be written as

$$\mathbf{y} = \mathbf{H}\mathbf{F}_{\mathrm{RF}}\mathbf{F}_{\mathrm{BB}}\mathbf{s} + \mathbf{n}, \tag{1}$$

where $\mathbf{s}$ is the digital baseband signal with the covariance $\mathbb{E}[\mathbf{s}\mathbf{s}^*] = \frac{P_{\mathrm{t}}}{N_{\mathrm{s}}}\mathbf{I}_{N_{\mathrm{s}}}$ where P_{t} is the average total transmit power, $\mathbf{n} \sim \mathcal{CN}(0, \mathbf{I}_{N_{\mathrm{r}}})$ is the white Gaussian noise, $\mathbf{F}_{\mathrm{RF}} \in$

$\mathbb{C}_{N_{\mathrm{t}} \times N_{\mathrm{RF}}^{\mathrm{t}}}$ and $\mathbf{F}_{\mathrm{BB}} \in \mathbb{C}_{N_{\mathrm{RF}}^{\mathrm{t}} \times N_{\mathrm{s}}}$ is the analog and digital precoding matrix, respectively.

After the analog combining and low-resolution quantization, the signal is

$$\mathbf{r} \quad = \quad \mathcal{Q}\left(\mathbf{W}_{\mathrm{RF}}^{*} \mathbf{H} \mathbf{F}_{\mathrm{RF}} \mathbf{F}_{\mathrm{BB}} \mathbf{s} + \mathbf{W}_{\mathrm{RF}}^{*} \mathbf{n}\right), \tag{2}$$

where $\mathbf{W}_{\mathrm{RF}} \in \mathbb{C}_{N_{\mathrm{r}} \times N_{\mathrm{RF}}^{\mathrm{r}}}$ is the analog combining matrix and $\mathcal{Q}()$ is the quantization function applied component-wise and separately to the real and imaginary parts. In this paper, we assume that uniform mid-rising quantization is used. For a real-valued scalar y, $r = \mathcal{Q}(y)$ if

$$r = \operatorname{sign}(y)\, \Delta \left(\min\left(\left\lceil \frac{|y|}{\Delta} \right\rceil, 2^{b-1} \right) - \frac{1}{2} \right). \tag{3}$$

where Δ is the quantization stepsize and chosen to follow the values given in [11].

The effective noise $\widetilde{\mathbf{n}} \triangleq \mathbf{W}_{\mathrm{RF}}^{*} \mathbf{n}$ has covariance $\mathbf{W}_{\mathrm{RF}}^{*} \mathbf{W}_{\mathrm{RF}}$. Throughout the paper, it is assume that $\mathbf{W}_{\mathrm{RF}}^{*} \mathbf{W}_{\mathrm{RF}} = N_{\mathrm{r}} \mathbf{I}$ and therefore the effective noise is still white Gaussian noise. The same idea appeared in [12].

III. Problem Formulation

Since analog precoding and combining are implemented by analog phase shifters, each element of $\mathbf{F}_{\mathrm{RF}}$ and $\mathbf{W}_{\mathrm{RF}}$ is limited to have unit-norm. The optimization problem is to maximize the mutual information between $\mathbf{s}$ and $\mathbf{r}$ as follows.

$$\begin{aligned}
\max_{\mathbf{F}_{\mathrm{BB}}, \mathbf{F}_{\mathrm{RF}}, \mathbf{W}_{\mathrm{RF}}} \quad & I(\mathbf{s}; \mathbf{r}) && (4)\\
s.t. \quad & |\mathbf{F}_{\mathrm{RF}}(i,j)| = 1, \quad \forall i, j, && (5)\\
& |\mathbf{W}_{\mathrm{RF}}(i,j)| = 1, \quad \forall i, j, && (6)\\
& \|\mathbf{F}_{\mathrm{RF}} \mathbf{F}_{\mathrm{BB}}\|_F^2 \le N_{\mathrm{s}}, && (7)
\end{aligned}$$

where (7) is due to the transmission power constraint, i.e., $\|\mathbf{F}_{\mathrm{RF}} \mathbf{F}_{\mathrm{BB}} \mathbf{s}\|^2 \le P_{\mathrm{t}}$.

The problem is difficult to solve. First, the constraints in (5) and (6) is non-convex. Second, the nonlinear quantization function $\mathcal{Q}$ makes it difficult to evaluate the mutual information $I(\mathbf{s}; \mathbf{r})$ [8].

IV. Preliminary Results

In this paper, we derive the upper and lower bounds of the rate. The lower bounds correspond to two proposed transmission schemes based on channel inversion (CI) and matched filter (MF) precoding, respectively.

We simulate a narrowband MIMO channel with 16 transmit antennas and 8 receive antennas. The wireless channel is assumed to be a clustered channel with 4 clusters, each of which consists of 10 rays.

The rates are shown in Fig. 2. First, compared to the fully digital architecture where $N_{\mathrm{RF}}^{\mathrm{t}} = N_{\mathrm{t}} = 16$, $N_{\mathrm{RF}}^{\mathrm{r}} = N_{\mathrm{r}} = 8$ and infinite-bit ADCs are assumed, the hybrid architecture with few-bit ADC introduces small loss in low and medium SNR regimes. In addition, the MF precoding is better than the CI precoding at low SNR but worse than that at medium and high SNR. The figure also shows that the rate of the CI precoding converges to $2N_{\mathrm{s}}$ bps/Hz at high SNR.

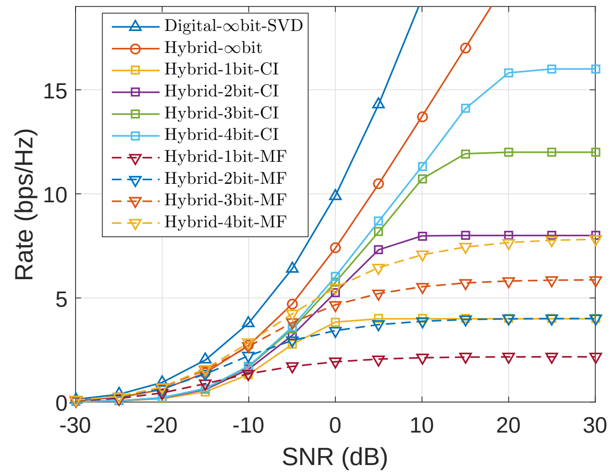

Fig. 2. Rates comparison of different receiver architectures when $N_{\mathrm{t}} = 16$, $N_{\mathrm{r}} = 8$, $N_{\mathrm{RF}}^{\mathrm{t}} = 4$, $N_{\mathrm{RF}}^{\mathrm{r}} = N_{\mathrm{s}} = 2$.

We also compare our algorithm with a close prior art which is hybrid precoding but with perfect ADCs considered in [6]. The gap between the curve of 'Digital-∞bit-SVD' and 'Hybrid-∞bit' represents the loss of hybrid precoding while the gap between the curve 'Hybrid-∞bit' and the proposed algorithm is the loss due to low resolution ADCs. We can see the loss due to low resolution ADCs is small at low and medium SNRs. For example, the gap between the curve 'Hybrid-∞bit' and 'Hybrid-4bit-CI' is less than 3 dB when the SNR is less than 15 dB.

References

[1] Z. Pi and F. Khan, "An introduction to millimeter-wave mobile broadband systems," *IEEE Commun. Mag.*, vol. 49, no. 6, pp. 101–107, 2011.

[2] T. S. Rappaport, R. W. Heath Jr, R. C. Daniels, and J. N. Murdock, *Millimeter wave wireless communications*. Englewood Cliffs, NJ, USA: Prentice Hall, 2015.

[3] B. Murmann, "ADC performance survey 1997-2015," 2015. [Online]. Available: http://www.stanford.edu/ murmann/adcsurvey.html

[4] W. Hong, K.-H. Baek, Y. Lee, Y. Kim, and S.-T. Ko, "Study and prototyping of practically large-scale mmWave antenna systems for 5G cellular devices," *IEEE Commun. Mag.*, vol. 52, no. 9, pp. 63–69, September 2014.

[5] A. Alkhateeb, J. Mo, N. Gonzalez-Prelcic, and R. W. Heath Jr, "MIMO precoding and combining solutions for millimeter-wave systems," *IEEE Commun. Mag.*, vol. 52, no. 12, pp. 122–131, December 2014.

[6] O. El Ayach, S. Rajagopal, S. Abu-Surra, Z. Pi, and R. W. Heath Jr, "Spatially sparse precoding in millimeter wave MIMO systems," *IEEE Trans. Wireless Commun.*, vol. 13, no. 3, pp. 1499–1513, March 2014.

[7] A. Mezghani and J. Nossek, "Capacity lower bound of MIMO channels with output quantization and correlated noise," in *IEEE International Symposium on Information Theory Proceedings (ISIT)*, 2012.

[8] J. Mo and R. W. Heath Jr., "Capacity analysis of one-bit quantized MIMO systems with transmitter channel state information," *IEEE Trans. Signal Processing*, vol. 63, no. 20, pp. 5498–5512, Oct 2015.

[9] S. Wang, Y. Li, and J. Wang, "Multiuser detection in massive spatial modulation MIMO with low-resolution ADCs," *IEEE Trans. Wireless Commun.*, vol. 14, no. 4, pp. 2156–2168, April 2015.

[10] S. Han, C.-L. I, Z. Xu, and C. Rowell, "Large-scale antenna systems with hybrid analog and digital beamforming for millimeter wave 5G," *IEEE Communications Magazine*, vol. 53, no. 1, pp. 186–194, Jan. 2015.

[11] J. Max, "Quantizing for minimum distortion," *IRE Transactions on Information Theory*, vol. 6, no. 1, pp. 7–12, 1960.

[12] A. Molisch and X. Zhang, "FFT-based hybrid antenna selection schemes for spatially correlated MIMO channels," *IEEE Commun. Lett.*, vol. 8, no. 1, pp. 36–38, Jan 2004.