Performance of Linear Receivers for Wideband Massive MIMO with One-Bit Analog-to-Digital Converters

– Extended Summary of Proposed Paper for WSA-2016 –

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***Abstract*—Among analog-to-digital converters, the ones with one-bit resolution consume the least power. Their use has been suggested for massive MIMO base stations, where analog-to-digital conversion is power consuming because of the large number of antenna branches. It is believed that the nonlinear distortion of one-bit quantization makes channel estimation and symbol equalization in such systems computationally complex and resource demanding. It is shown that, for massive MIMO when the number of channel taps is large, low-complexity linear channel estimation and symbol equalization become feasible. The performance of the proposed linear detector, based on the obtained channel estimate, is close to that of the optimal detection method in the regime of low transmit power. Further it is shown that a massive MIMO system with one-bit analog-to-digital converters would have to use 2–4 times more antennas than an unquantized system, which, considering the power savings in the analog-to-digital conversion, still might be less power consuming.**

I. Background and Motivation

Equipping a base station that serves multiple single-antenna users with a massive antenna array, in which each antenna element can be controlled individually, has been termed massive MIMO (Multiple-Input Multiple-Output) [1]. Massive MIMO has many benefits over single-antenna systems, such as the possibility to serve many users concurrently over the same time-frequency resource, which gives increased spectral efficiency, and to lower the total amount of power that the users and base station radiate. Whereas the radiated power can be reduced in massive MIMO by a factor equal to the square root of the number of antennas compared to systems with single-antenna base stations, this does not necessarily mean a reduction of the total consumed power. The power consumption of massive MIMO depends on the power consumption and power efficiency of each antenna branch. For example if the power consumption of each antenna branch decreases linearly in the number of antennas, then the massive MIMO base station consumes as much power as the single-antenna system despite radiating less power. It is therefore important to lower the power consumption of each component of the antenna branches to keep the power consumption low.

One component that each antenna branch has to be equipped with is the analog-to-digital converter (ADC), which converts the analog received signal to a digital one. The power consumption of this component mostly depends on the number of quantization levels that it uses—fewer quantization levels lowers the power consumption of the ADC. As a rough rule of thumb, the power consumption is linear in the number of quantization levels [2], which would make an ADC in a single-antenna system consume the same amount of power as each ADC of the massive MIMO system, if they have the same number of quantization levels. Therefore, to keep the power consumed by the ADCs the same, or to lower it, in the massive MIMO system as in the single-antenna system, the number of quantization levels has to decrease proportionally to the number of antennas. Going from a single-antenna system with $2^8 = 256$ quantization levels to a massive MIMO system with 128 antennas, the number of quantization levels thus has to decrease to 2 in order to keep the power consumption of the analog-to-digital conversion constant. An ADC with 2 quantization levels is called a one-bit ADC, because the output of the ADC can be described by one bit.

The practicality of a one-bit ADC, however, is debated, since its coarse quantization results in heavy signal distortion. Despite this, it has been shown that, in a single-input single-output system with a one-bit ADC, the capacity only decreases by a factor $2/\pi$ compared to a system, where the analog-to-digital conversion has infinite resolution, in the low SNR regime [3]; this is also true in a MIMO system [4], [5]. It has also been shown that, in a MIMO system, the capacity approaches a limit that grows linearly in the number of receiver antennas as the SNR grows large [5]. In a massive MIMO base station, it is thus theoretically possible to still reach high data rates also with one-bit ADCs. In this previous work, prefect knowledge of the channel was assumed. In reality, the base station has to estimate the channel from the signals it receives. So it never has perfect channel state knowledge, especially not if all received signals are quantized by one-bit ADCs.

Prior work has presented solutions both for equalization of the quantized received signals and for channel estimation, see [6]–[10] for example. The equalization methods presented in prior work usually employ algorithms, whose complexity becomes increasingly difficult to handle when the number of antennas and users grows and when the channel is frequency-

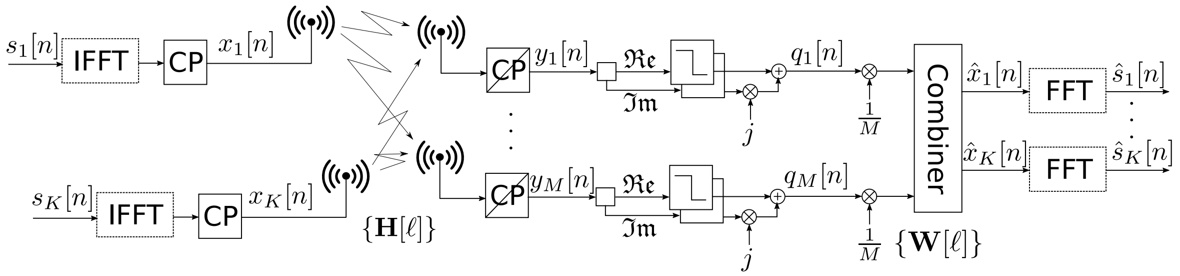

Fig. 1. The system model for both single-carrier (without IFFT and FFT) and OFDM transmission.

selective. The channel estimation techniques proposed for non-sparse channels usually requires long training sequences to obtain good enough channel state knowledge. Apart from the number of antennas and users being large in massive MIMO, in most practical wideband scenarios, the channel is also heavily frequency selective, which implies that the complexity of these schemes will be difficult to implement in reality. Furthermore, a typical mobile channel has a coherence interval of only a few thousand symbols. For example, if the delay spread of the channel is 5 µs and the Doppler shift is 250 Hz, the coherence interval is approximately 800 complex symbols. Since massive MIMO is usually implemented with time-division duplex, both the downlink and uplink has to fit in this interval besides the training sequences [11]. The training sequences thus have to be short.

In this paper, we investigate the possibility to use low-complexity linear combiners for equalization of the quantized signals and a low-complexity pilot-based LMMSE (Linear Minimum-Mean-Square Error) technique for channel estimation. This implementation is the kind of architecture commonly considered for massive MIMO with ADCs with perfect resolution [12]. The implementation is thus feasible in terms of complexity. We use a technique similar to [13] to derive a closed form expressions for the rate achievable with our implementation in a wideband system and show that the power loss incurred by using one-bit ADCs and this low-complexity implementation is close to the $2/\pi$ limit at low transmit power when the channel is frequency selective.

The main contributions of this work are:

- It is proposed to also use linear combiners and pilot-based low-complexity LMMSE channel estimation, which are commonly considered for unquantized massive MIMO, also in massive MIMO systems with one-bit ADCs.
- The performance of low-complexity detectors and channel estimation methods for the massive MIMO uplink with one-bit ADCs is quantified with analytical expressions for the rate achievable.
- It is shown that frequency-selective channels can be helpful in massive MIMO systems with one-bit quantizers, in that such channels improve the performance of linear detectors.

In a related paper, we proved that the estimates of the linear receivers in one-bit ADC massive MIMO are consistent. That paper has been submitted to IEEEs conference International Conference on Acoustics, Speech and Signal Processing, 2016 [14].

II. System Model

We consider the uplink of the massive MIMO system in Figure 1, where the base station is equipped with M antennas and there are K single-antenna users. All signals are modeled in complex baseband and in symbol-sampled discrete-time, i.e., where the uniform sampling period is equal to the symbol duration.

The users transmit the signals $\mathbf{x}[n] \triangleq (x_1[n], \dots, x_K[n])^\mathsf{T}$ at symbol duration n over the frequency-selective channel, whose small-scale fading is described by the L-tap impulse response $\{\mathbf{H}[\ell]\}_{\ell=0}^{L-1}$, where $\mathbf{H}[\ell]$ is an $M \times K$-dimensional matrix. The elements $\{h_{mk}[\ell]\}_{\ell=0}^{L-1}$ at position (m, k) of these matrices form the impulse response between the k-th user and the m-th antenna of the base station. We assume the impulse response follows a uniform power delay profile, i.e., $\mathsf{E}[|h_{mk}[\ell]|^2] = \frac{1}{L}$ for all ℓ.

In a wideband system, the number L is in the order of tens. For example, a system that uses 15 MHz of bandwidth over a channel with 1 µs of maximum excess delay, which corresponds to a moderately frequency-selective channel, has $L = 15$ channel taps, c.f. [15] where the "Extended Typical Urban Model" has a maximum excess delay of 5 µs.

The received signal at antenna m at symbol duration n is

$$y_m[n] = \sum_{k=1}^{K} \sum_{\ell=0}^{L-1} \sqrt{\beta_k P_k} h_{mk}[\ell] x_k[n-\ell] + z_m[n], \qquad (1)$$

where P_k is the transmit power of user k, β_k is the large-scale fading (including path loss and shadowing) of user k, and $z_m[n]$ is a complex zero-mean Gaussian random vector with variance N_0 that models the thermal noise of the base station hardware. It is assumed that $z_m[n]$ is IID over n and m. The path losses $\{\beta_k\}$ are varying only slowly. It is therefore assumed that the base station is able to estimate them perfectly.

Upon reception, the signal is quantized. We assume that the inphase and the quadrature parts of the signal are separately sampled by one out of two one-bit ADCs:

$$\big[y_m[n]\big] \triangleq \frac{1}{\sqrt{2}} \operatorname{sign}\big(\mathfrak{Re}\, y_m[n]\big) + j \frac{1}{\sqrt{2}} \operatorname{sign}\big(\mathfrak{Im}\, y_m[n]\big), \qquad (2)$$

where $[\cdot]$ denotes quantization. Here we have assumed that the threshold of the one-bit ADC is zero. Other thresholds are also possible, c.f. [16]. The scaling of $\big[y_m[n]\big]$ is arbitrary but chosen such that $\big[y_m[n]\big]$ has unit power. For convenience, we denote the quantized signal $q_m[n] \triangleq \big[y_m[n]\big]$.

We study two transmission modes: single-carrier and OFDM transmission. We observe the transmission for a block of N symbols. During the symbol periods $n = 0, \dots, N-1$, we assume that the users transmit

$$\mathbf{x}[n] = \begin{cases} \mathbf{s}[n], & \text{if single-carrier} \\ \frac{1}{\sqrt{N}} \sum_{\nu=0}^{N-1} \mathbf{s}[\nu] e^{-j2\pi n\nu/N}, & \text{if OFDM} \end{cases}, \quad (3)$$

where $\mathbf{s}[n] = (s_1[n], \dots, s_K[n])^\mathsf{T}$ is the vector of the n-th data symbols from the K users. We assume that the symbols have zero-mean and unit-power, i.e., $\mathsf{E}[\, s_k[n] \,] = 0$ and $\mathsf{E}[\, |s_k[n]|^2 \,] = 1$ for all k, n. The users also transmit a cyclic prefix that is $L-1$ symbols long:

$$\mathbf{x}[n] = \mathbf{x}[N+n], \quad -L < n < 0. \quad (4)$$

III. Outline of Proposed Channel Estimation

The channel estimation is done by letting the users transmit pairwise orthogonal pilot sequences of length $N_\text{p} \triangleq \eta K L$, where K is the number of users and L the number of taps in the frequency-selective channel. The parameter η is the fraction of extra pilot symbols as compared to the shortest length that still allows for pairwise orthogonal pilots, which is KL. The base station then performs an LMMSE estimation based on the $\eta K L$ observed quantized signals and each antenna estimates the channels from all users.

We study a scenario, where the channel is IID Rayleigh fading, i.e., where the channel tap $h_{mk}[\ell] \sim \mathcal{CN}(0, \frac{1}{L})$. It is found out that if the pilot sequences are designed such that the received energy is spread uniformly over the training period, then the error variance of the estimate of each channel tap is

$$E = \frac{1}{L}(1 - \delta_k), \quad (5)$$

where the power loss

$$\delta_k \triangleq \frac{\beta_k P_k N_\text{p}}{\beta_k P_k N_\text{p} + L(N_0 + Q)} \quad (6)$$

and, when the number of taps is large, the quantization noise Q of the one-bit ADC is closely approximated by the limit

$$Q \to P_\text{rx}(\frac{\pi}{2} - 1), \quad L \to \infty, \quad (7)$$

where $P_\text{rx} \triangleq \mathsf{E}\big[|y_m[n]|^2\big]$ is the power of the signal received at one of the antennas. Compared to an unquantized system, where $Q = 0$, the degradation of the channel estimates due to the one-bit ADCs is

$$\frac{\delta_k\big|_{Q=0}}{\delta_k\big|_{Q=P_\text{rx}(\frac{\pi}{2}-1)}} = \frac{\pi}{2} \approx 2\,\text{dB}, \quad (8)$$

when $\eta = 1$. Furthermore, it is found out that the error of the channel estimation approaches zero as the fraction of extra pilots η grows large and that the convergences is fast, which can be seen in Figure 2.

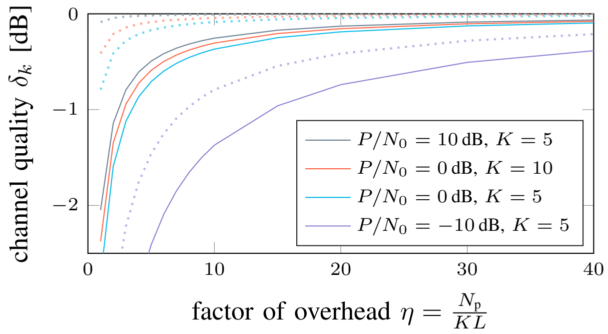

Fig. 2. The power loss δ_k of the estimation of a channel, where the number of taps is large, for a one-bit ADC system in solid lines and for an unquantized system in dotted lines.

IV. Outline of Proposed Symbol Detection

Multiuser symbol detection is done by convolution of the received quantized signals $q_m[n]$ and the impulse response of the receiver combiner $w_{km}[\ell]$:

$$\hat{x}_k[n] = \frac{1}{M} \sum_{m=1}^{M} \sum_{\ell=\ell_\text{min}}^{\ell_\text{max}} w_{km}[\ell] q_m[n - \ell]. \quad (9)$$

The impulse response of the receiver combiner is computed based on the estimated channel. The maximum-ratio and zero-forcing combiners are special cases of this linear detector. An achievable rate

$$R_k = \log\left(1 + \frac{\big|\,\mathsf{E}\left[x_k^*[n]\hat{x}_k[n]\right]\big|^2}{\mathsf{E}\left[|\hat{x}_k[n]|^2\right] - \big|\,\mathsf{E}\left[x_k^*[n]\hat{x}_k[n]\right]\big|^2}\right), \quad (10)$$

for user k in this system setup is given and the limit R'_k of this achievable rate as the number of channel taps grows large is derived in closed form:

$$R_k \to R'_k, \quad L \to \infty, \quad (11)$$

where, for maximum-ratio combining

$$R'_k = \log\left(1 + \frac{2}{\pi} \frac{\delta_k \beta_k P_k M}{N_0 + \sum_{k'=1}^{K} \beta_{k'} P_{k'}}\right), \quad (12)$$

and zero-forcing combining

$$R'_k = \log\left(1 + \frac{2}{\pi} \frac{\delta_k \beta_k P_k (M - K)}{N_0 + \sum_{k'=1}^{K} \beta_{k'} P_{k'} (1 - \delta_{k'} \frac{2}{\pi})}\right). \quad (13)$$

V. Preliminary Results

It is shown, in a numerical analysis, that this limit R'_k closely approximates the achievable rate R_k already for moderately frequency-selective channels, see Figure 3. By comparing the achievable rate of the quantized system with the one for the unquantized system for maximum-ratio and zero-forcing combining, which is given in [17], we find that, with perfect channel state knowledge, the power loss due to the one-bit ADCs approaches $2/\pi$ for both the maximum-ratio and zero-forcing combiners as the transmit powers grows small. This coincides with the power loss of the capacity that was derived in previous works. With estimated channels, the difference in performance between the quantized and unquantized systems depends on many parameters, the length of the pilot sequences,

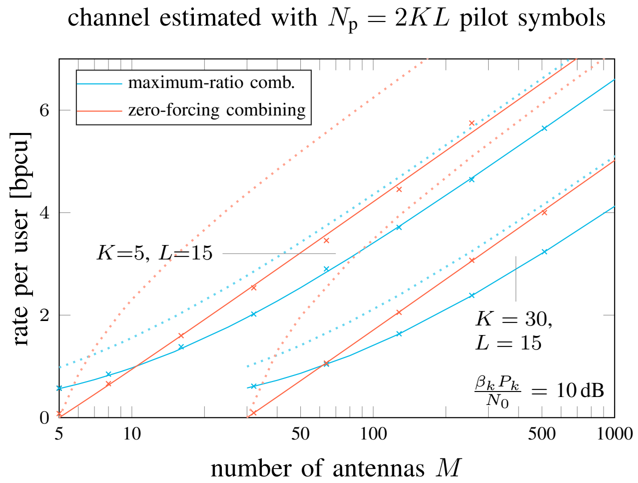

Fig. 3. The achievable rate R_k is marked ×, the limit R'_k is drawn with a solid line and the rate R_k for an unquantized system is drawn with a dotted line for different systems and IID Rayleigh fading. The length of the pilot sequences is $N_p = 2KL$. The curves for single-carrier and OFDM transmission coincide both for maximum-ratio and zero-forcing combining. The received power $\frac{\beta_k P_k}{N_0} = 10\,\text{dB}$ is the same for all users.

the transmit power and the number of users and antennas. For example, in numerical studies of a system with five users and high transmit power, the quantized system with the maximum-ratio combiner needs 2.0 times more antennas to reach the same data rate as the unquantized system and 4.0 time more antennas with the zero-forcing combiner; at low transmit power it needs 2.4 times more antennas than the unquantized system. Having in mind that the power saving in each one-bit ADC is in the order $2^{b-1} \approx 12\,\text{dB}$, where $b \approx 5$ is the lowest number of bits that still results in a performance close to the unquantized system, a 4-fold increase in the number of antennas still results in a lower total power consumption for the ADCs.

We show that, with many antennas, it is possible to use low-complexity linear receivers and channel estimators even with one-bit ADCs when the channel is frequency selective. The loss in performance that this incurs as compared to the optimal detector is small at low transmit powers. To achieve the same performance as the unquantized system, the one-bit ADC system has to employ 2–4 times more antennas at the base station in representative cases. Such an increase in the number of antennas would still result in a decrease in total power consumption of all the ADCs of the system.

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