Spatial Sigma-Delta Signal Acquisition for Wideband Beamforming Arrays

Ryan M. Corey and Andrew C. Singer
University of Illinois at Urbana-Champaign

***Abstract*—We consider a spatial sigma-delta modulation structure for wideband signal acquisition with dense sensor arrays. Using closely arranged sensors, the array oversamples the signal in space and can apply coarse quantization to each sensor output. A sigma-delta structure propagates quantization errors between sensors to shape the errors into high spatial frequencies which are then filtered by a delay-and-sum beamformer. In this work, we also introduce higher-order sigma-delta structures and hybrid space-time architectures that can achieve greater noise shaping in both space and time. The coarsely quantized sigma-delta structure can be used to build high-resolution arrays with lower power and complexity.**

I. Introduction

Sensor arrays, which can be used for directional signal processing, are essential in many communication, acoustic, and imaging applications [1]. Recent research, particularly in the area of wireless communication, has explored the benefits of "massive" arrays [2]; that is, arrays with large numbers of elements. These arrays offer improved spatial resolution, among other benefits, but require more mixed-signal hardware resources and computational power to process the large number of input signals. In this work, we propose an efficient array design with a large number of sensors that each produce a low-resolution (e.g., 1 bit per sample) data stream. These streams can be combined in a delay-and-sum beamformer to form a high-resolution output signal.

There has been some recent work on coarsely quantized arrays for communication applications. Low-resolution analog-to-digital converters (ADCs) are advantageous for large arrays because they have lower power and complexity than higher-resolution converters. In [3], a the authors considered a wideband multiple-input multiple-output array with single-bit ADCs. The coarse quantization caused minimal degradation in channel capacity in the low signal-to-noise ratio regime. There have been a number of studies [4]–[6] on channel estimation based on single-bit measurements; the spatial redundancy of large arrays and sparsity of the channels helps to compensate for the reduced resolution of the converters. The single-bit measurements can also be used directly for some spatial signal processing applications, such as direction-of-arrival estimation [7]. Low-resolution converters have been found to be especially advantageous in compact arrays with closely spaced elements [8], which are the subject of this work.

Single-bit signals have long been common in oversampled ADCs, such as the celebrated sigma-delta converter [9]. A sigma-delta modulator uses a low-resolution quantizer with a feedback loop to shape the quantization noise spectrum such that quantization effects are shifted to higher frequencies outside the signal band. These errors are removed by a discrete-time filter during decimation. Since a delay-and-sum beamformer is essentially a discrete filter sampled in space rather than time, it should be possible to apply the same error-shaping technique to sensor arrays. In a conventional linear beamforming array, the spacing between sensors is chosen to be close to half of the shortest wavelength in the signal of interest – the spatial equivalent of the Nyquist rate. In a spatially oversampled array, the spacing is much smaller than a half wavelength. Noise shaping is accomplished by propagating quantization errors from one sensor to the next. These errors accumulate at high spatial frequencies and are filtered by the delay-and-sum operation.

There have been a few previous studies on spatial sigma-delta noise shaping. In [10], the authors proposed a joint space-time sigma-delta vector quantization scheme for transmit arrays. However, the design resembles a conventional time-domain modulator as quantization errors are fed back in time, not in space. The nearest precedents to the present work are [11], [12] and related papers, in which the complex weights (phase shifts) of a phased array are coarsely quantized and the quantization errors of the weights are shaped by spatial feedback. There are two major differences between [12] and this work: first, we consider sigma-delta modulation of the received signal itself rather than of the phase shifts; second, we consider wideband delay-and-sum beamforming rather than narrowband phased-array beamforming.

Our focus will be on low-frequency applications, such as microphone arrays, in which the delay-and-sum beamforming operations are performed directly on the information signal and not on a high-frequency carrier. In these systems, beamforming and other spatial processing operations, such as direction-of-arrival estimation and source separation, can be performed in the digital domain using the sampled data [13]. The proposed architecture applies sigma-delta modulation to the received signal, propagating quantization errors between adjacent sensors and producing a set of coarsely quantized output signals. After analyzing the noise-shaping characteristics of the first-order spatial sigma-delta structure, we will consider higher-order modulators and hybrid time-space noise shaping, which can

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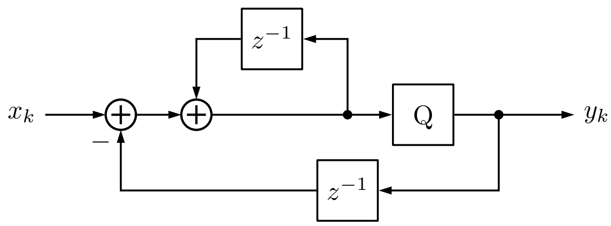

Figure 1. A generic first-order sigma-delta modulator. The "Q" block is a low-resolution quantizer.

further improve resolution for wideband systems.

II. Array Model

Sensor array processing is analogous in many ways to discrete-time signal processing: in an array, signals are sampled at regular intervals in space and filtered to isolate signals from a particular direction. This analogy is valuable in understanding spatial sigma-delta modulation. Consider a linear array of N elements spaced distance d apart. For simplicity, we assume plane wave propagation in a dispersionless linear medium. Let $x_n(t)$ denote the continuous-time signal received at sensor n for $n = 0, \ldots, N-1$, and let $X_n(\Omega)$ be its continuous-time Fourier transform (CTFT). Let $r = \frac{d}{c}\cos\theta$ be the relative time delay between sensors for signals arriving at angle θ relative to the array axis. Thus, $r = \frac{d}{c}$ at endfire and 0 at broadside. The output of a delay-and-sum beamformer steered to an angle corresponding to delay r is

$$\tilde{x}_r(t) = \frac{1}{N}\sum_{n=0}^{N-1} x_n(t - nr). \tag{1}$$

In the CTFT domain, the beamformer output is

$$\tilde{X}_r(\Omega) = \frac{1}{N}\sum_{n=0}^{N-1} X_n(\Omega)\, e^{-j\Omega nr}, \tag{2}$$

which is analagous to a discrete-time Fourier transform with $r\Omega$ as the spatial frequency.

III. Sigma-Delta Architecture

A. Sigma-delta modulation

Figure 1 shows a first-order sigma-delta modulation structure. A continuous-valued input sequence x_k is quantized to produce a discrete-valued output sequence y_k. The quantization error, q_k, is fed back and subtracted from the next input. It is straightforward to show that the output can be written

$$y_k = x_k + q_k - q_{k-1}. \tag{3}$$

Thus, the modulator applies unity gain to the signal but a highpass filter, $H(z) = 1 - z^{-1}$, to the quantization error. This highpass filter is called the noise transfer function (NTF). In a sigma-delta ADC, the oversampled binary sequence y_k would then be lowpass filtered and decimated to produce a multibit Nyquist-rate signal.

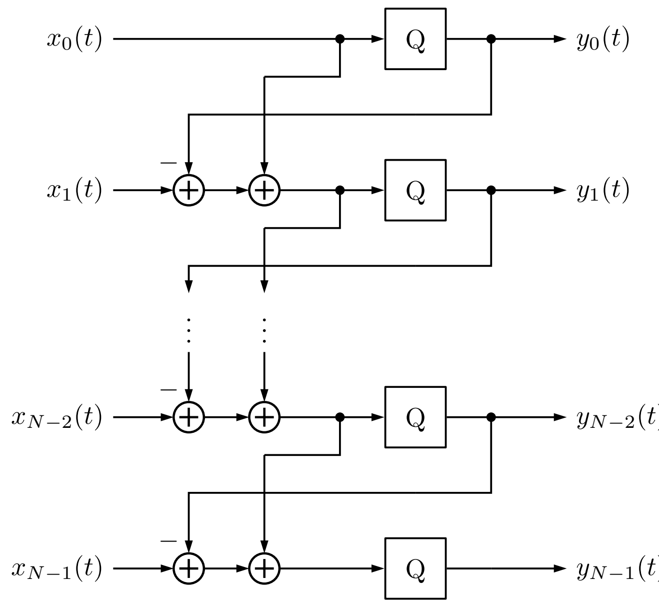

Figure 2. A first-order spatial sigma-delta array with N elements.

B. First-order spatial sigma-delta structure

Figure 2 shows a first-order spatial sigma-delta array. It is similar to the time-domain structure of Figure 1 except that time delays are replaced by propagation to the next element. Furthermore, due to the finite extent of the array, the input is defined only for $0 \le n \le N-1$. This system is characterized by the input-output relation

$$y_n(t) = x_n(t) + q_n(t) - q_{n-1}(t), \tag{4}$$

where $q_n(t)$ is the error introduced by the n^{th} quantizer for $0 \le n \le N-1$ and $q_n(t) = 0$ for $n < 0$. The delay-and-sum beamformer output is then

$$\tilde{y}_r(t) = \frac{1}{N}\sum_{n=0}^{N-1} y_n(t - nr) \tag{5}$$

$$= \frac{1}{N}\sum_{n=0}^{N-1}\left[x_n(t-nr) + q_n(t-nr) - q_{n-1}(t-nr)\right] \tag{6}$$

$$= \tilde{x}_r(t) + \tilde{q}_r(t) - \frac{1}{N}\sum_{n=0}^{N-2} q_n(t - (n+1)r) \tag{7}$$

$$= \tilde{x}_r(t) + \tilde{q}_r(t) - \tilde{q}_r(t-r) + \frac{1}{N} q_{N-1}(t - Nr). \tag{8}$$

The noise shaping effect is clear when the output is written in the frequency domain:

$$\tilde{Y}_r(\Omega) = \tilde{X}_r(\Omega) + \left(1 - e^{-j\Omega r}\right)\tilde{Q}_r(\Omega) + \frac{1}{N} Q_{N-1}(\Omega)\, e^{-j\Omega Nr}. \tag{9}$$

Note that due to the finite length of the array, the quantization error introduced by the last sensor is not shaped. For each

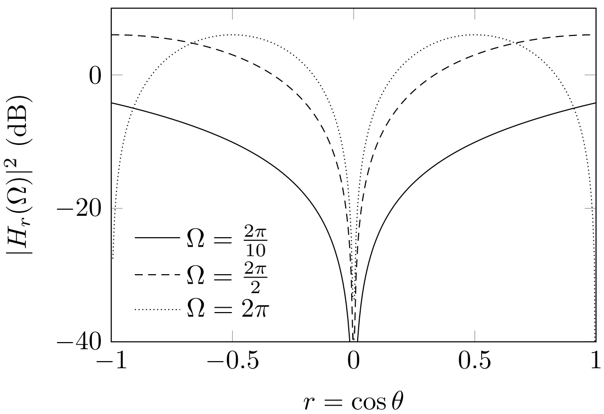

Figure 3. The spatial noise transfer function (11) of a first-order spatial sigma-delta modulator for an array with $c/d = 1$. The noise is most strongly attenuated near broadside.

other error signal, the NTF is $H_r(\Omega) = 1 - e^{-j\Omega r}$, which has gain

$$|H_r(\Omega)|^2 = \left|1 - e^{-j\Omega r}\right|^2 \tag{10}$$

$$= 4\sin^2\left(\frac{\Omega r}{2}\right) \tag{11}$$

$$= 4\sin^2\left(\frac{d}{2c}\Omega\cos\theta\right). \tag{12}$$

For a fixed temporal frequency Ω, $H_r(\Omega)$ shapes the noise to higher values of r, i.e., away from broadside, as shown in Figure 3. The spatial noise shaping effect is strongest for frequencies much smaller than c/d. Indeed, for fixed nonzero r, (11) shifts quantization errors to higher temporal frequencies, much like a time-domain sigma-delta modulator.

C. Higher-order modulation

The first-order sigma-delta structure of Figure 1 is rarely used in high-resolution applications such as audio recording. Its noise-shaping power can be improved using additional integration stages. For example, a second-order sigma-delta modulator has the input-output relation

$$y_k = x_k + q_k - 2q_{k-1} + q_{k-2}, \tag{13}$$

which corresponds to $H(z) = \left(1 - z^{-1}\right)^2$. The second-order spatial sigma-delta structure is analagous, except that the last sensor uses a first-order structure. Higher-order noise shaping is also possible using modified feedback architectures to prevent instability in the nonlinear quantization error loop.

One drawback of spatial sigma-delta modulation is that it shapes the quantization noise only across spatial frequency. Thus, in practice, the beamformer output may have substantial quantization error at low temporal frequencies. It is therefore beneficial to apply oversampling and sigma-delta modulation in time as well as in space. The resulting NTF will depend on both r and the sample period τ_s. For example, a first-order temporal modulator combined with a second-order spatial modulator would have the noise transfer function

$$H_\theta(\Omega) = \left(1 - e^{-j\Omega\tau_s}\right)\left(1 - e^{-j\Omega r}\right)^2 \tag{14}$$

for all but the final two sensors. These hybrid architectures can be realized using cascaded sigma-delta modulators, also known as multi-stage noise shaping (MASH) modulators. In this work, we will explore these higher-order and hybrid architectures in detail.

IV. Contributions

We will propose and analyze spatio-temporal sigma-delta modulation architectures for wideband beamforming. By oversampling in space, the sigma-delta array can use lower-resolution quantization at each element, saving both power and signal bandwidth. To the best of our knowledge, this work represents the first spatial sigma-delta architecture to operate directly on a received signal from a wideband beamforming array. We will present analysis and simulations of noise-shaping characteristics in first- and second-order spatial sigma-delta structures as well as hybrid time-space structures. We will assess the performance of the sigma-delta array for applications such as blind source separation, direction-of-arrival estimation, and adaptive beamforming.

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