On Stable Many-to-Many Matching for Distributed Medium Access with Reuse of Spectral Resources

Bernd Holfeld* and Eduard Jorswieck†
*Fraunhofer Heinrich Hertz Institute, Einsteinufer 37, 10587 Berlin, Germany
†Technische Universität Dresden, Communications Theory, 01062 Dresden, Germany

***Abstract*—Recently, direct device-to-device (D2D) communication has gained broad attention. This paper studies the resource allocation in frequency domain within a D2D network under permission of spectral reuse. We present a novel distributed and self-organized protocol which incorporates resource-fairness among the transmitter-receiver pairs and an energy-efficient use of the resources. The low-complex algorithm is based on the game-theoretic framework of stable many-to-many matching. Simulation results evaluate the performance of the algorithm against a centralized resource allocation scheme.**

***Index Terms*—Resource allocation; Medium access; Spectral reuse; Network-assisted D2D; Distributed algorithms; Stable matching**

I. Introduction

Direct communication among wireless devices in proximity to each other provides a number of benefits over infrastructure-based communication within cellular networks. It enables low end-to-end latencies due to short-range paths in device-to-device (D2D) communication, a reduced number of communication hops and less processing. Proximity links may improve the spectral efficiency by a more efficient utilization of radio resources. Besides, energy efficiency can be greatly improved. 3GPP is currently introducing concepts for the support of proximity-based services based on D2D communication in the Long Term Evolution (LTE) standard (Rel. 12 and beyond) [1]. Here, a network-assisted D2D operation mode assumes the split of control and user planes. While control data is routed via a coordinator entity, e.g. the LTE base station, the devices may exchange user data directly. One specific application area, also discussed in the 5G context, is the use of D2D techniques for machine-type communications.

In this paper, we discuss the allocation of frequency resources in an assisted D2D setup when spectrum reuse is allowed. We assume a dynamic and flexible resource allocation rather than a static scheme with periodically reserved resources. In order to reduce the overhead in the control channels, we study a distributed protocol for the medium access. We intend few information exchange and minor coordination effort. A set of active devices may negotiate the allocation of the resource pool among each other based on individual utilities. It may not be necessary to extensively exchange channel state information (CSI) of the D2D links. The D2D coordinator is provided with as little global information as needed to coordinate the negotiation. We utilize the framework of stable matching to a give low-complex and fast terminating medium access algorithm [2], [3]. Moreover, we apply a many-to-many version of stable matching in order to incorporate resource budgets of D2D devices on the one hand and spectrum reuse, i.e. multiple D2D links per resource, on the other hand. The frequency resources shall be matched sufficiently fair, i.e. the finally allocated resource budgets shall be almost equal among the D2D links. Furthermore, the decision-making process among the devices shall be driven by energy-efficiency measures. Here, the energy-efficiency will be determined by each D2D based on the transmit power needed to reach a target SINR for a reliable transmission.

A. Related Work

In the context of wireless communications, many-to-many stable matching was rarely explored or applied so far. It was recently used in [4] for distributed CSI selection in MIMO interference channels. Besides, some variants of many-to-one stable matchings were proposed for distributed medium access schemes in cognitive radios, see [5]–[8]. In a cognitive radio, secondary communication links can be operated as an underlay to a primary network, i.e. a spectral resource is shared with one known primary user. In [7], a truncated matching algorithm is used for resource allocation in cognitive networks. Here, truncation implies that the applied stable matching algorithm terminates after a given number of negotiation rounds, hence, giving an almost stable allocation only. In [8], the secondary system is a D2D underlay to a cellular network. A multi-stage stable matching concept is applied that incorporates matching under both resource requirements (lower quotas) and resource budgets (upper quotas) of the D2D links.

B. Outline of the Paper

The outline of this work is as follows: Section II presents the system model and a formulation of our resource allocation problem based on multiple objectives. In Section III, we define the stable many-to-many matching and give an algorithm for its distributed implementation. We discuss the local utilities of the D2D links and the utility functions of the D2D coordinator in Section IV. Finally, simulation results are presented in Section V for an indoor D2D network.

II. Problem Statement

We consider a set of D2D links $d \in \mathcal{D}$ from transmitter d_T to receiver d_R. The D2D pairs are to be matched to a set of frequency resources $r \in \mathcal{R}$ based on utility functions $u_d(r)$

and $u_r(d)$ which are use case specific, see Section IV. From the utility functions, strict preference relations over favorable assignments are to be defined for each set of agents. We denote by $M(d)$ the set of resources matched to D2D link d and by $M(r)$ the set of D2D links matched to resource r. Further, we allow a many-to-many matching, i.e. frequency reuse with a factor $q_r^{\text{reuse}} \leq |\mathcal{D}|$, $q_r^{\text{reuse}} \in \mathbb{N}$, per resource and multiple resources per D2D link. Further, our aim is to achieve resource-fairness through an *almost uniform* distribution of the resources allocated to D2D links over many realizations, aiming for the average score

$$p = \frac{\sum_{r=1}^{|\mathcal{R}|} q_r^{\text{reuse}}}{|\mathcal{D}|} \,, \tag{1}$$

which is bounded by the integers

$$\underline{p} = \lfloor p \rfloor \in \mathbb{N} \;, \;\; \overline{p} = \lceil p \rceil \in \mathbb{N} \;. \tag{2}$$

Hence, we seek $\mathbb{E}[\,|M(d)|\,] \approx p$, $\forall d$.

A realization of our two-sided matching problem can be stated as a multi-objective binary programming (MOBP) problem [9]:

$$\underset{\mathbf{x}}{\text{maximize}} \quad \mathbf{f}(\mathbf{x}) = \left[f_1(\mathbf{x}), f_2(\mathbf{x}), ..., f_N(\mathbf{x})\right]^T \tag{3a}$$

$$\text{subject to} \quad \mathbf{x} \in \mathcal{X} \,, \tag{3b}$$

where $\mathcal{X} \subseteq \{0,1\}^{|\mathcal{R}|\,|\mathcal{D}|}$ is the set of feasible points in the binary domain, see the refinement in (4), and N is the number of objectives. In the following, we formulate (3) as a weighted sum utility optimization given by

$$\underset{\mathbf{x}}{\text{maximize}} \quad \sum_{r\in\mathcal{R}} \sum_{d\in\mathcal{D}} \Big(\omega_d\, u_d(r) + \omega_r\, u_r(d)\Big)\, x_{rd} \tag{4a}$$

$$\text{subject to} \quad \underline{p} \leq \sum_{r\in\mathcal{R}} x_{rd} \leq \overline{p},\; \forall d \in \mathcal{D}, \tag{4b}$$

$$\sum_{d\in\mathcal{D}} x_{rd} \leq q_r^{\text{reuse}},\; \forall r \in \mathcal{R}, \tag{4c}$$

$$x_{rd} \in \{0,1\},\; \forall (r,d) \in \mathcal{R}\times\mathcal{D}\,, \tag{4d}$$

where ω_d, $\omega_r \in [0,1]$ are weights to adjust the operating point and $\sum_{d\in\mathcal{D}} \omega_d + \sum_{r\in\mathcal{R}} \omega_r = 1$. Note, that (4) is a linear assignment problem (AP) which can be efficiently solved by centralized algorithms provided that all necessary information is globally available, see Section V-B. The weighted utility maximization ends up in an operating point on the convex hull of the achievable utility region and, hence, serves as an upper bound for the proposed distributed matching implementation in Section III.

III. Distributed Medium Access Scheme

A. Stable Many-to-Many Matching

We distributively solve the AP on the basis of locally available information, using two-sided stable matching [2], [3]. Stable matching is a game-theory inspired framework to solve decision-making problems with multiple agents based on a stable outcome of the system. The goal is to terminate in a state in which each agent is assigned a partner (or a set of partners)

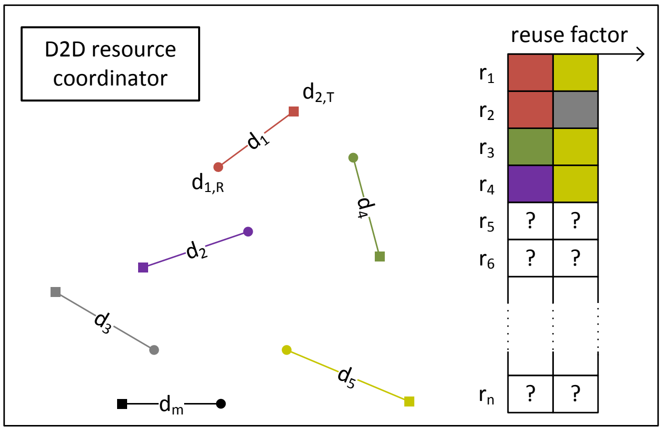

Fig. 1: Considered D2D network with frequency reuse. In order to get frequency resources assigned for data transmission over direct peer-to-peer links, the D2D users communicate with each other over control channels supported by a coordinator.

which is considered its best mutually beneficial choice. Hence, it is impossible to strictly improve the performance of all agents jointly. Below, we give some definitions in order to define stability in the context of many-to-many matchings for our setup, with upper matching quotas q_r $\forall r$ and q_d $\forall d$. We assume non-negative real-valued utilities, i.e., $u_d(\cdot) \in \mathbb{R}_+$ and $u_r(\cdot) \in \mathbb{R}_+$.

Definition 1: Two agents are *mutually acceptable* for a matching if $u_d(r) > 0 \wedge u_r(d) > 0$ for $(r,d) \in \mathcal{R}\times\mathcal{D}$.

Definition 2: [3] The matching M is *individually rational* if $u_d(r^\star) > u_d(d)$, $r^\star \in M(d)$, for some $d \in \mathcal{D}$ or $u_r(d^\star) > u_r(r)$, $d^\star \in M(r)$, for some $r \in \mathcal{R}$.

Accordingly, individually rational matching ensures that no agent would prefer being matched to himself than with its current matching.

Definition 3: [3], [10] The matching M is *blocked* if there exists a mutually acceptable pair $(r,d) \in \mathcal{R}\times\mathcal{D}$ which is unmatched, $r \notin M(d)$, $d \notin M(r)$, and

- **(i)** the agents prefer each other over their matched partners, i.e., $u_d(r) > u_d(r^\star)$ for some $r^\star \in M(d)$ and $u_r(d) > u_r(d^\star)$ for some $d^\star \in M(r)$,
- **(ii)** the agents prefer each other to an unfilled position, i.e., $|M(d)| < q_d$ or $|M(r)| < q_r$.

A matching is *pairwise stable* if it is individually rational and not blocked by any pair of agents.[1]

[1]Note, that pairwise stability is different from group stability which requires that a matching is not blocked by any coalition of agents. A coalition might consist of multiple D2D links and/or resources. Group stability was first defined in the context of many-to-one matchings [3] and later on extended for many-to-many matchings, see the definition of setwise stability in [10], [11] and credible group stability in [12]. In general, the relation between the stability concepts is: group-stable sets $\subseteq$ setwise-stable sets $\subseteq$ credibly group-stable sets $\subseteq$ pairwise-stable sets. In this paper, we do not consider preference strategies (utilities) over coalitions and, hence, stick to pairwise stability.

B. Distributed Implementation

We apply the matching algorithm presented in [13] and adopt it to meet our system model. For that reason, we seek to achieve a pairwise stable matching that involves uniformity as far as possible, meaning that constraint set (4b) of allocating between $\underline{p}$ and $\overline{p}$ resources per D2D link is envisaged, however, it may be underfulfilled. Therefore, we set $q_d = \overline{p}\ \forall d$ to target $|M(d)| \leq \overline{p}$. The proposed matching algorithm fulfills constraint set (4c) with equality, giving $|M(r)| = q_r^{\text{reuse}}\ \forall r$. We assume that the device pairs d rank their preferred resources in descending order in lists l_d based on $u_d(r)$. Similar, the coordinator managing the resources holds preference lists l_r over the D2D links based on $u_r(d)$. Basic assumptions are: (i) only agents that both find each other acceptable appear on the preference lists, (ii) agents are not indifferent in their preferred matches and (iii) $|l_d| \geq \overline{p}$ is satisfied for each $d \in \mathcal{D}$. We give the uniform many-to-many matching approach in Algorithm 1, where each D2D pair proposes at most once to be matched to a resource from its list. Thus, the complexity of the algorithm is $\mathcal{O}(|\mathcal{D}||\mathcal{R}|)$ in terms of the number of proposals.

Algorithm 1 D2D-proposing stable matching with almost uniform outcome.

Initial Phase:

1: *Proposals:* Every D2D $d \in \mathcal{D}$ sends to coordinator the index of $\overline{p}$ most preferred resources (via control channels). These indices are cleared from preference list l_d.

2: *Decision:* Coordinator accepts at most q_r^{reuse} proposals per resource $r \in \mathcal{R}$ subject to preference list l_r and rejects proposals when the reuse factor is overfulfilled.

Iterative Phase:

1: **while** $\exists\, d \in \mathcal{D} : l_d \neq \emptyset$ (not yet proposed to all resources) and $|M(d)| < \overline{p}$ (undersubscribed) **do**

2: *Proposals:* D2D d sends to coordinator the index of next $\overline{p} - |M(d)|$ resources in preference list. These indices are cleared from preference list l_d.

3: *Decision:* Coordinator accepts at most q_r^{reuse} proposals per resource subject to preference list l_r and rejects proposals when the reuse factor is overfulfilled.

4: **end while**

In the following, we show some theorems and their proofs for Algorithm 1 as discussed in [13].

Theorem 1: Algorithm 1 gives a pairwise stable matching which is D2D-optimal, i.e., the best feasible stable matching for the D2D links.

Proof: The proof will be provided in Appendix A. ■

Theorem 2: If Algorithm 1 gives a stable matching where at least one D2D link obtains an unfulfilled resource score $\underline{p}$, then no stable matching exists in which *every* D2D link is assigned $\underline{p}$ or $\overline{p}$ resources.

Proof: The proof will be provided in Appendix B. ■

IV. Utility Functions

A. D2D Utilities based on Local Information

We consider SISO transmission throughout the network. Furthermore, each D2D pair d shall adjust its power budget for transmission at a *target SINR*, if this is achievable under peak power constraints. The SINR calculation for the link between receiver d_R and transmitter d_T depends on the assignment $\mathbf{x}$ of other D2D users on the same resource and is given for time instance t on resource r as

$$\gamma_d^{[r]}\big(\mathbf{x}(t)\big) = \frac{P_{d_T}^{[r]}(t)\, |h_{d_R d_T}^{[r]}(t)|^2}{\sigma_n^2 + I_{d_R}^{[r]}\big(\mathbf{x}(t)\big)}\,, \tag{5}$$

where the interference term is

$$I_{d_R}^{[r]}\big(\mathbf{x}(t)\big) = \sum_{d' \in \mathcal{D}\setminus\{d\}} P_{d'_T}^{[r]}(t)\, \big|h_{d_R d'_T}^{[r]}(t)\big|^2\, x_{rd'}\,. \tag{6}$$

Above, $|h_{d_R d_T}^{[r]}(t)|^2$ is the instantaneous channel gain between Tx and Rx of D2D link d, including pathloss and the respective fading model, while $P_{d_T}^{[r]}(t)$ is the transmit power, including antenna gains. For the moment, we assume perfect channel knowledge for the link $h_{d_R,d_T}^{[r]}$. Furthermore, $|h_{d_R d'_T}^{[r]}(t)|^2$ denotes the instantaneous gain of the interference channel between Tx $d' \neq d$ and Rx d, $P_{d'_T}^{[r]}(t)$ is the transmit power of interferer d' and σ_n^2 is the noise floor. Consequently, (6) gives the interference term which depends on the final resource assignment vector $\mathbf{x}$ at time instance t.

Due to the interdependence of the SINR and the assignment decision, a resource allocation problem based on (5) is extremely difficult and not solvable by efficient assignment mechanisms. To simplify the problem, we propose defining an expected *mean interference portion* from the active interference links $d' \neq d$, forming set $\mathcal{I}$. This set is determined by the reuse factor $q_{r,\Delta t}^{\text{reuse}}$ being applied over Δt time slots in the past on resource r. Depending on whether a D2D device $d \in \mathcal{D}$ that observes the interference was transmitting or not, there are $q_{r,\Delta t}^{\text{reuse}} - 1$ or $q_{r,\Delta t}^{\text{reuse}}$ parallel interferers to consider. The sum interference can be easily measured by each device over longer time periods in which $q_{r,\Delta t}^{\text{reuse}}$ stayed constant. Therefore, we set

$$\bar{I}_{d_R}^{[r]} = \frac{1}{|\mathcal{I}|}\, \mathbb{E}\Bigg[\sum_{d' \in \mathcal{I}} P_{d'_T}^{[r]}\, \big|h_{d_R d'_T}^{[r]}\big|^2\Bigg]. \tag{7}$$

Then, in time instance t the simplified SINR term for a chosen reuse factor $q_{r,t}^{\text{reuse}}$ is

$$\tilde{\gamma}_d^{[r]}(t) = \frac{P_{d_T}^{[r]}(t)\, |h_{d_R d_T}^{[r]}(t)|^2}{\sigma_n^2 + (q_{r,t}^{\text{reuse}} - 1)\, \bar{I}_{d_R}^{[r]}}\,. \tag{8}$$

We target $\tilde{\gamma}_d^{[r]}(t) = \tilde{\gamma} = \text{const.}$, $\forall d \in \mathcal{D}$, $\forall r \in \mathcal{R}$. This is to allow a unified interface for resource block association on higher layers and a simplified physical layer modulation and coding scheme selection. The extension to different SINR targets is straightforward. Following, the transmit power of each D2D transmitter to achieve the target SINR computes as

$$P_{d_T}^{[r]}(t) = \min \left\{ \tilde{\gamma} \, \frac{\sigma_n^2 + (q_{r,t}^{\text{reuse}} - 1)\, \bar{I}_{d_R}^{[r]}}{|h_{d_R d_T}^{[r]}(t)|^2} \, , \, P_{d_T}^{\max} \right\}, \quad (9)$$

where $P_{d_T}^{\max}$ is the maximum peak power per resource of the devices which limits the finally achievable SINR. Realizing power budget $P_{d_T}^{[r]}(t)$ translates into the achievable communication rate for D2D link d on resource r

$$\Gamma_d^{[r]}\big(P_{d_T}^{[r]}(t)\big) = \eta \, BW \log_2 \Big(1 + \tilde{\gamma}_d^{[r]}\big(P_{d_T}^{[r]}(t)\big)\Big) \quad (10)$$

in [bit/s], where BW is the bandwidth of the resource and η is a scaling factor to consider signaling overhead.

Using (9) and (10), we define the utility of the D2D links $d \in \mathcal{D}$ by their resource-wise *energy efficiency (EE)* [14] in [bit/Joule]:

$$u_d(r) = EE_d^{[r]} = \frac{\Gamma_d^{[r]}\big(P_{d_T}^{[r]}(t)\big)}{\alpha_d P_{d_T}^{[r]}(t) + \frac{P_{d,\text{HW}}}{|\mathcal{R}|}} \, . \quad (11)$$

Above, $\alpha_d \geq 1$ accounts for amplifier non-idealities and $P_{d,\text{HW}}$ is the power consumed by hardware components such as DA/AD converters, modulation filters etc. Using EE as a measure, the D2D users aim for an efficient use of their battery power and extended battery life under rate guarantees.

B. Coordinator Utilities based on Globally Collected Information

We assume that the coordinator receives from each transmitter of D2D pair d a subset of the locally available information, e.g. only the applicable power budgets per resource or the resource-wise energy efficiency measure. In the following, we imply the global collection of the latter parameter.

Then, we define the utility of the resources $r \in \mathcal{R}$ managed by the coordinator by a prioritization of the communicated EE values. With user priority ρ_d, e.g. according to buffer queues or tariffs, we have

$$u_r(d) = \rho_d \, EE_d^{[r]} \, . \quad (12)$$

V. Simulation Assumptions and Initial Results

A. D2D Network Layout and Channel Model

We simulate a network with $|\mathcal{D}| = 8$ D2D links that compete for $|\mathcal{R}| = 25$ orthogonal frequency resources. The considered simulation scenario corresponds to a large industrial hall with dimensions shown in the area plot in Fig. 2. We use the indoor path loss model presented in [15] for 5.2 GHz. It is given by $PL_{[dB]}(x) = 70.28 + 25.9 \log_{10}(x_{[m]}/15)$, where x is the distance. Also, we apply additional geo-correlated shadow fading with $\sigma_{\text{SF}} = 6$ dB. The transmitter-receiver pairs have a communication distance between 6 m and 12 m. Other simulation parameters are listed in Table I.

Parameter	**Value**
Scenario	Industrial indoor environment
Carrier frequency	5.2 GHz
D2D Tx-Rx Pairs	8
Frequency resources	25 PRBs
BW	180 kHz
Rx noise figure	6 dB
Thermal noise spectral density	-174 dBm/Hz
η	0.7
$P_{d,\text{HW}}$	10 dBm
α_d	1.2

TABLE I: Basic configuration of the performed simulations.

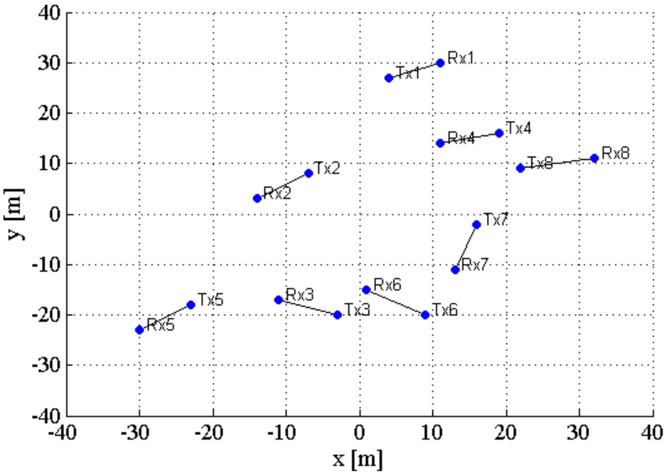

Fig. 2: Indoor D2D network with 8 transmitter-receiver pairs.

The simulations are averaged over 800 channel realizations. For each realization, we model SISO-OFDM channel coefficients with multipath fading according to the ITU indoor office model, see ITU-R M.1225.[2]

B. Centralized Numerical Solution of the Linear AP

We use the solver tool GLPK[3] [17] to find an efficient (near-optimal) solution for the binary program in (4). For a fair comparison with the proposed distributed algorithm, we fulfill (4c) with equality and distinguish between two cases of the binary constraint set (4b), namely:

(Almost) Uniform Resource Distribution	Relaxation
$\underline{p} \leq \sum_{r \in \mathcal{R}} x_{rd} \leq \overline{p}, \ \forall d \in \mathcal{D}$	$\sum_{r \in \mathcal{R}} x_{rd} \leq \overline{p}, \ \forall d \in \mathcal{D}$

The centralized relaxation delivers an upper bound to our distributed solution that may violate the lower quotas $\underline{p}$ as well. Hence, both schemes are well comparable. However, the centralized uniform scheme that incorporates $\underline{p}$ finds a solution

[2] In the final version of this paper, the tap delay parameters may be adapted to a multipath model which fits better to an industrial indoor use case.

[3] Please refer to [16] for a general performance comparison between GLPK and other noncommercial solvers for mixed-integer linear programming.

on the actual feasible domain of (4). Due to its circumscribed solution space, the performance of the centralized uniform scheme may be or may not be better than the performance of the distributed solution. This can also be seen in Fig. 4 provided in the next paragraph.

For reasons of simplification, we assume $\rho_d = 1 \; \forall d$ herein. Then, the utilities of the D2D links and the coordinator are based on the same $EE_d^{[r]}$ measure. For this specific case, the weights in (4a) can be dropped.

C. Simulation Results with Distributed Stable Solution

We evaluate the performance of the distributed medium access scheme using the sum utility achieved by the resource allocation for a reuse factor $q_r^{\text{reuse}} = 2 \; \forall r$. Fig. 3 shows the average sum EE in the D2D network. It can be seen that the distributed solution based on the simplified interference metric (7) is worse than the true performance involving perfect interference knowledge of the negotiated matching. Also, we see that the centralized uniform solution based on the simplified expected interference term gives the upper bound. How much overall performance degradation in the network occurs by using the distributed algorithm is shown in Fig. 4 for different SINR targets.

In general, the sum EE performance (bit per energy spent) is limited for high SINR targets by the max. peak power constraint per resource that restricts the achievable rate and by the increasing interference from high-power transmissions of other transmitters. Then, the target SINR cannot be reached anymore, see Fig. 5. For low SINR targets, the sum EE is limited by the power consumption in the D2D hardware components, see (11), even though low transmit power might be needed to achieve the SINR. From Fig. 3, an optimal operation region for the energy-efficient resource allocation in the considered D2D network is for network-wide SINR targets in the range of 5 to 15 dB. Here, more than 50 % of the allocated resources fulfill the SINR and the scheme is not much limited by the peak power constraint.

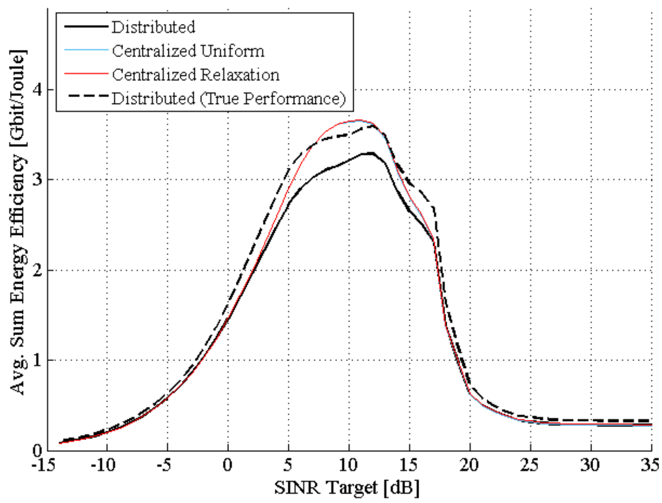

Fig. 3: Sum EE in the D2D network over different SINR targets $\tilde{\gamma}$ for reuse factor 2. Solid lines correspond to the optimization based on expected interference while the dotted line shows the performance by actual interference.

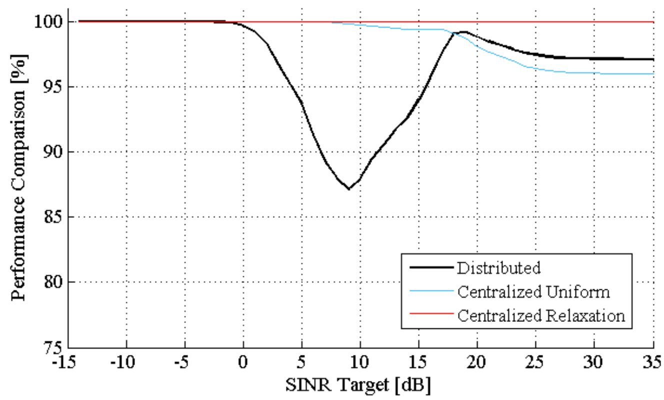

Fig. 4: Performance of the distributed medium access scheme in comparison to centralized solutions. With the distributed scheme, at least 87% sum EE performance of the centralized optimum is reached.

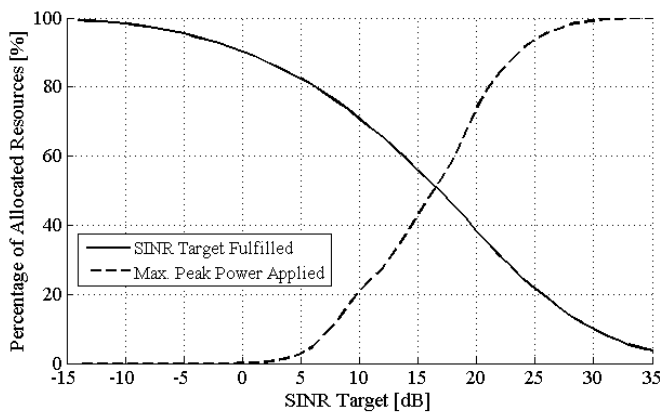

Fig. 5: Distributed medium access: Average number of fulfilled SINR targets (solid) and number of applied max. peak powers (dotted) in the allocated resources over increasing SINR requirements in the D2D network.

VI. Conclusion and Outlook

In this paper, we presented a framework for distributed medium access in network-assisted D2D networks where spectrum reuse is allowed and resource-fairness is a goal. The framework is based on many-to-many stable matching. Analytical proofs on stability for the proposed matching algorithm are given as well as the utility functions needed to derive the matching preferences. The utilities are based on an energy-efficiency measure under the condition of a target SINR. Hence, we aim for the energy-efficient use and re-use of resources. System-level simulations for an exemplified D2D indoor network are presented and the results of the distributed implementation are compared to a centralized solution of the resource allocation problem. In the final version of the paper, additional results on the scalability and the algorithmic complexity of the distributed scheme will be presented. Also, we like to improve the interference measure.

Appendix A
Proof of Theorem 1

The following proof is from [13].

Pairwise stability: Assume a blocking pair (r, d), $r \notin M(d)$, $d \notin M(r)$, and (i) $\exists\, r^{\star} \in M(d) : u_d(r) > u_d(r^{\star})$ and (ii) $\exists\, d^{\star} \in M(r) : u_r(d) > u_r(d^{\star})$. Then, two cases are to be discussed:

- d never proposed to r. It means that $u_d(r)$ is worse than the utility over any matched resource in $M(d)$ which contradicts (i). Thus, (r, d) cannot be a blocking pair.
- d proposed to r but was finally rejected. It means that r preferred other q_r^{reuse} D2D links (and traded d for a more preferable one) which contradicts (ii). Thus, (r, d) cannot be a blocking pair.

D2D optimality: Assume an alternative matching M' which is better for some D2D link d and assume $\exists\, r' \in M'(d)$, $r' \notin M(d)$ and $\exists\, r \in M(d)$ such that $u_d(r') > u_d(r)$. Then, again, either d never proposed to r' which contradicts the mechanism of Algorithm 1 or r' rejected the proposal to achieve a more preferable stable trade. Hence, M' cannot be stable and M is the best stable matching for the D2D links.

Appendix B
Proof of Theorem 2

The following proof is from [13].

Assume a D2D link d in stable matching M for which $|M(d)| < \underline{p}$. Also, assume a stable matching M' where d fulfills $\underline{p}$. Then, all $r \in M'(d)$ are strictly better off than (at least) one resource in $M(d)$ which contradicts the argument that M is D2D-optimal, see Theorem 1.

References

[1] X. Lin, J. Andrews, A. Ghosh, and R. Ratasuk, "An Overview of 3GPP Device-to-Device Proximity Services," *IEEE Commun. Mag.*, vol. 52, no. 4, pp. 40–48, April 2014.

[2] D. Gale and L. Shapley, "College Admissions and the Stability of Marriage," *The American Mathematical Monthly*, vol. 69, no. 1, pp. 9–15, Jan. 1962.

[3] A. Roth and M. Sotomayor, *Two-Sided Matching. A Study in Game-Theoretic Modeling and Analysis*. Cambridge, UK: Cambridge University Press, 1990.

[4] R. Mochaourab, R. Brandt, H. Ghauch, and M. Bengtsson, "Overhead-aware distributed CSI selection in the MIMO interference channel," in *Proc. of EUSIPCO*, Aug. 2015, pp. 1043–1047.

[5] Y. Zhang, Y. Gu, M. Pan, and Z. Han, "Distributed Matching Based Spectrum Allocation in Cognitive Radio Networks," in *Proc. of IEEE GLOBECOM*, Dec. 2014, pp. 864–869.

[6] R. Mochaourab, B. Holfeld, and T. Wirth, "Distributed Channel Assignment in Cognitive Radio Networks: Stable Matching and Walrasian Equilibrium," *IEEE Trans. Wireless Commun.*, vol. 14, no. 7, pp. 3924–3936, July 2015.

[7] L. Lu, D. He, G. Ye Li, and X. Yu, "Graph-Based Robust Resource Allocation for Cognitive Radio Networks," *IEEE Trans. Signal Process.*, vol. 63, no. 14, pp. 3825–3836, July 2015.

[8] B. Holfeld, T. Jaeuthe, and T. Wirth, "Resource Sharing with Minimum QoS Requirements for D2D Links Underlaying Cellular Networks," in *Proc. of IEEE VTC-Spring*, May 2015, pp. 1–7.

[9] E. Bjornson, E. Jorswieck, M. Debbah, and B. Ottersten, "Multiobjective Signal Processing Optimization: The way to balance conflicting metrics in 5G systems," *IEEE Signal Process. Mag.*, vol. 31, no. 6, pp. 14–23, Nov. 2014.

[10] M. Sotomayor, "Three remarks on the many-to-many stable matching problem," *Mathematical Social Sciences*, vol. 38, no. 1, pp. 55–70, July 1999.

[11] F. Echenique and J. Oviedo, "A theory of stability in many-to-many matching markets," *Theoretical Economics*, vol. 1, no. 2, pp. 233–273, June 2006.

[12] H. Konishi and M. Utku Unver, "Credible Group Stability in Many-to-Many Matching Problems," *Journal of Economic Theory*, vol. 129, no. 1, pp. 57–80, July 2006.

[13] M. Freer and M. Titova, "Many-to-Many Matching Problem with Quotas," Interdisciplinary Center for Economic Science, George Mason University, Discussion Paper, Feb. 2015.

[14] A. Zappone and E. Jorswieck, "Energy Efficiency in Wireless Networks via Fractional Programming Theory," *Foundations and Trends in Communications and Information Theory, Now Publishers Inc.*, vol. 11, no. 3-4, pp. 185–396, 2015.

[15] E. Tanghe, W. Joseph, L. Verloock, L. Martens, H. Capoen, K. Van Herwegen, and W. Vantomme, "The Industrial Indoor Channel: Large-Scale and Temporal Fading at 900, 2400, and 5200 MHz," *IEEE Trans. Wireless Commun.*, vol. 7, no. 7, pp. 2740–2751, July 2008.

[16] J. Linderoth and T. Ralphs, "Noncommercial Software for Mixed-Integer Linear Programming," Technical Report 04T-023, Dept. of Industrial and Systems Engineering, Lehigh University, Tech. Rep., Dec. 2004.

[17] A. Makhorin, "GLPK: GNU Linear Programming Kit, version 4.57," Nov. 2015.