Stable Matching with Externalities for Beamforming and User Assignment in Multi-cell MISO Systems

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***Abstract*—We consider the problem of distributed joint user association and beamforming in multi-cell multiple-input single-output systems. Assuming perfect local channel state information, each base station applies a distributed beamforming scheme called SLNR-MAX [1, Definition 3.5] which depends on the user association in the network. We determine the user association by a proposed stable matching with externalities algorithm which also takes the beamforming vectors at the base stations into account. The merit in the stable matching model is the distributed implementation aspects. Each user asks to be matched with a base station according to his preferences, and each base station decides independently which users to accept. Simulation results show efficient operation of the system compared to a centralized approach.**

I. Introduction

Efficient assignment of users to base stations in a multi-cell network is decisive for achieving spectral efficiency. Assuming perfect channel state information (CSI) at the base stations which are equipped with multiple antennas, the user assignment problem for maximizing the systems' sum rate is coupled with the beamforming design at the base stations [2]. In [2], this problem is addressed in multi-cell multiple-input multiple-output (MIMO) networks and an alternating optimization algorithm is proposed which reaches a local optimum of the original nonconvex problem.

In this work, we are interested in distributed algorithms for the joint user assignment and beamforming in multi-cell MISO systems. We use a beamforming scheme at the base stations according to signal-to-leakage-and-noise ratio maximizing beamforming (SLNR-MAX) [1, Definition 3.5] which is defined for a given user association to the base stations. This heuristic beamforming scheme, which has has sum rate efficiency, can be applied at the base stations requiring local CSI, i.e., each base stations needs only know the downlink channels from itself to the users. We use the beamforming scheme within a stable matching framework which we propose in this paper in order to determine the joint user assignment and beamforming in a distributed way. Specifically, the users propose (i.e., ask to be matched to) to the base stations in an order according to the channel norms. The base stations decide on which users to accept based on the achieved power gains with the SLNR-MAX beamforming scheme.

Since a base station's user choice depends on the users matched to the other base stations, the proposed framework relates to matching with externalities [3], [4]. Applications of stable matching with externalities for user association in single-antenna interference networks can be found in [5], [6]. In a multi-cell MIMO setting, a stable matching with externalities algorithm has been developed in [7] incorporating difference efficient precoding schemes.

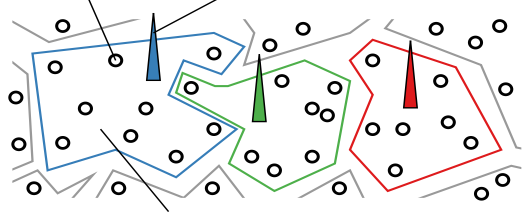

Fig. 1. System Model

Notations: Column vectors and matrices are given in lowercase and uppercase boldface letters, respectively. $\|\mathbf{a}\|$ is the Euclidean norm of $\mathbf{a} \in \mathbb{C}^N$. $|b|$ and $|\mathcal{S}|$ denote the absolute value of $b \in \mathbb{C}$, and the cardinality of a set $\mathcal{S}$, respectively. $(\cdot)^\dagger$ denotes the Hermitian transpose. The power set of $\mathcal{A}$ is $2^{\mathcal{A}}$.

II. System Model and Problem Formulation

We consider a set of base stations $\mathcal{K} = \{1, \dots, K\}$ and a set of users $\mathcal{U} = \{1, \dots, U\}$. Each base station k uses N_k antennas. The channel vector from base station k to user l is $\mathbf{h}_{k,l} \in \mathbb{C}^{N_k}$. We assume that each base station has local channel state information (CSI). That is, each base station knows the channel vectors from itself to the users.

We assume that each user can be assigned to at most one base station and each base station can serve multiple users. The user association will be determined by a *matching* defined as follows [8]:

Definition 1: A matching M is a mapping from $\mathcal{U} \cup \mathcal{K}$ to $2^{\mathcal{U} \cup \mathcal{K}}$ which satisfies

i. $\mathsf{M}(k) \in 2^{\mathcal{U}}$ and $|\mathsf{M}(k)| \leq U$ if $k \in \mathcal{K}$,
ii. $\mathsf{M}(l) \in 2^{\mathcal{K}}$ and $|\mathsf{M}(l)| \leq 1$ if $l \in \mathcal{U}$,
iii. $l \in \mathsf{M}(k)$ if and only if $k = \mathsf{M}(l)$.

For $k \in \mathcal{K}$, $\mathsf{M}(k)$ is the set of users assigned to base station k. Similarly, $\mathsf{M}(l)$ is the base station assigned to user l (see Fig. 1). In Definition 1, (i) restricts that each BS can serve a set of users from the set $\mathcal{U}$, (ii) restricts that each user is served by at most one BS, and (iii) ensures symmetry in the matching. Note that if $\mathsf{M}(k) = \emptyset$, then base station k is unmatched and

$$r_l(\mathsf{M},\{\mathbf{w}\}) = \log_2\left(1+\left\|\mathbf{h}_{k,l}^{\dagger}\mathbf{w}_{k,l}\right\|^2\Big(\underbrace{\sum\nolimits_{j\in\mathsf{M}(k)\setminus\{l\}}\left\|\mathbf{h}_{k,l}^{\dagger}\mathbf{w}_{k,j}\right\|^2}_{\text{intra-cell inteference}}+\underbrace{\sum\nolimits_{n\in\mathcal{K}\setminus\{k\}}\sum\nolimits_{m\in\mathsf{M}(l)}\left\|\mathbf{h}_{n,l}^{\dagger}\mathbf{w}_{n,m}\right\|^2}_{\text{inter-cell interference}}+\sigma^2\Big)^{-1}\right) \quad \text{for } k=\mathsf{M}(l) \quad (1)$$

thus switches its transmission off (the same applies for an unmatched user).

Given a matching M, the signal received at a user l is

$$y_l = \sum\nolimits_{k\in\mathcal{K}}\sum\nolimits_{j\in\mathsf{M}(k)}\mathbf{h}_{k,l}^{\dagger}\mathbf{w}_{k,j}x_j + z_l \quad (2)$$

where $\mathbf{w}_{k,j}\in\mathbb{C}^{N_k}$ is the transmit beamforming vector associated with user j at base station k, $x_j\sim\mathcal{CN}(0,1)$ is the signal intended for user j, and $z_l\sim(0,\sigma^2)$ is additive white Gaussian noise at receiver l. The achievable rate of user l, matched to base station $k=\mathsf{M}(l)$, is given in (1).

We are interested in the problem of maximizing the sum rate in the network through joint beamforming design and user association:

$$\begin{aligned} \underset{\mathsf{M}\in\mathcal{M},\{\mathbf{w}\}}{\text{maximize}} \quad & \sum\nolimits_{l\in\mathcal{U}} r_l(\mathsf{M},\{\mathbf{w}\}) \\ s.t. \quad & \sum\nolimits_{j\in\mathsf{M}(k)}\|\mathbf{w}_{k,j}\|^2\le 1, \text{ for all } k\in\mathcal{K}, \end{aligned} \quad (3)$$

In Problem (3), $\mathcal{M}$ is the set of all feasible matchings, and we assume a total power constraint at each base station of one. Problem (3) is NP-hard [2] also for a fixed matching M [9].

Our approach in this paper emphasizes on a distributed implementation for joint beamforming and user association. The matching M will be determined by a stable matching algorithm (proposed in the next section) and will depend on the beamforming scheme at the base stations. For a given matching M, we fix the beamforming scheme to SLNR-MAX beamforming [1, Definition 3.5] at a base station k serving user l as:

$$\mathbf{v}_{k,l}^{\text{SLNR}}(\mathsf{M}) = \frac{\left(\sum_{j\in\bigcup_{m\in\mathcal{K}}\mathsf{M}(m)}\mathbf{h}_{k,j}\mathbf{h}_{k,j}^{\dagger}+\sigma^2\mathbf{I}\right)^{-1}\mathbf{h}_{k,l}}{\left\|\left(\sum_{j\in\bigcup_{m\in\mathcal{K}}\mathsf{M}(m)}\mathbf{h}_{k,j}\mathbf{h}_{k,j}^{\dagger}+\sigma^2\mathbf{I}\right)^{-1}\mathbf{h}_{k,l}\right\|} \quad (4)$$

with $l\in\mathsf{M}(k)$. Including power control, the beamforming vector at base station k for user l is written as

$$\mathbf{w}_{k,l}^{\text{SLNR}}(\mathsf{M}) = \sqrt{p_{k,l}(\mathsf{M}(k))}\,\mathbf{v}_{k,l}^{\text{SLNR}}(\mathsf{M}), \quad (5)$$

where the power allocation $p_{k,j}(\mathsf{M}(k))$ at base station k depends on the users in its cell $\mathsf{M}(k)$ and is determined using [1, Theorem 3.16].

III. Stable Matching with Externalities

In a stable matching problem, there exists two sets of agents. Each agent in one set wants to be matched with one or more agents in the other set. In our case, the two sets correspond to the set of users $\mathcal{U}$ and the set of base stations $\mathcal{K}$. A matching between the two sets is defined in Definition 1.

Let a set of users $\mathcal{L}$ want to be matched with a base station k. The *choice function* $Ch_k(\mathsf{M},\mathcal{L})\subseteq\mathcal{L}$ of base station k selects the users out of $\mathcal{L}$ which it prefers most. We model this choice to depend on the beamforming scheme in (5) as follows: A user $l\in\mathcal{L}$ is in $Ch_k(\mathsf{M},\mathcal{L})$ if and only if

$$\left|\mathbf{h}_{k,l}^{\dagger}\mathbf{w}_{k,l}^{\text{SLNR}}(\mathsf{M}_{(k,\mathcal{L})})\right|^2 \ge \alpha\max_{j\in\mathcal{L}}\left\{\left|\mathbf{h}_{k,j}^{\dagger}\mathbf{w}_{k,j}^{\text{SLNR}}(\mathsf{M}_{(k,\mathcal{L})})\right|^2\right\}, \quad (6)$$

where $\alpha\in[0,1]$ is a design parameter[1] and $\mathsf{M}_{(k,\mathcal{L})}$ is the matching induced from M according to the following definition [3]:

Definition 2: Given a matching M and a pair $(k,\mathcal{L})$ with $k\in\mathcal{K}$ and $\mathcal{L}\subseteq\mathcal{U}$, define the matching $\mathsf{M}_{(k,\mathcal{L})}$ as

i. if $l\in\mathcal{L}$, then $\mathsf{M}_{(k,\mathcal{L})}(l)=\{k\}$
ii. if $l\in\mathsf{M}(k)\setminus\mathcal{L}$, then $\mathsf{M}_{(k,\mathcal{L})}(l)=\emptyset$
iii. if $l\notin\mathsf{M}(k)\cup\mathcal{L}$, then $\mathsf{M}_{(k,\mathcal{L})}(l)=\mathsf{M}(l)$

In Definition 2, the matching $\mathsf{M}_{(k,\mathcal{L})}$ is induced from M by matching $\mathcal{L}$ with base station k and "unmatching" the users $\mathsf{M}(k)\setminus\mathcal{L}$ from base station k.

In order to calculate the choice function, base station k needs to calculate the beamforming vectors according to SLNR-MAX beamforming in (5), which requires local CSI only. It is additionally required that each base station knows which users are matched to the other base stations in order to determine the interference directions. This information should then be exchanded between the base stations.

Due to the existance of externalities in the choice functions of the base stations, i.e., the decision at a base station depends on which users are matched to the other base stations, then we need to design a user proposal method which takes the externalities into accout. For this purpose, we define for each user $l\in\mathcal{U}$ a *proposal budget* $b_l\in\mathbb{N}$ which limits the total number of times this user asks to be matched with a base station. Using the proposal budget, we model the utility of a user l with base station k to depend on the channel norm, which is available information at the users:

$$u_{l,k}(\mathbf{p}_l) = \begin{cases} \|\mathbf{h}_{k,l}\| & \text{if } \sum_{j\in\mathcal{K}}p_{l,j}\le b_l \text{ and } p_{l,k}=\min\limits_{j\in\mathcal{C}}\{p_{l,j}\} \\ & \text{where } \mathcal{C}:=\{j\in\mathcal{K}\mid\|\mathbf{h}_{j,l}\|>\left\|\mathbf{h}_{\mathsf{M}(l),l}\right\|\} \\ 0 & \text{otherwise} \end{cases} \quad (7)$$

In (7), $p_{l,k}$ is the number of times a user $l\in\mathcal{U}$ proposes to base station $k\in\mathcal{K}$. Note that the user's utility in (7) can

[1]For large α, the choice function is more restrictive than for smaller α which will affect the size of $Ch_k(\mathsf{M},\mathcal{L})$.

Algorithm 1 Stable matching with proposal budget.

Initialize: matching M such that $\mathsf{M}(l) = \emptyset$ for all $l \in \mathcal{U}$

repeat

  for all $l \in \mathcal{U}$ **do**

    user l proposes to it best base station

$$l_k^* = \underset{k \in \mathcal{K} \setminus \mathsf{M}(l)}{\arg\max} \quad u_{l,k}(\mathbf{p}_l) \tag{8}$$

    update proposal count $p_{l,l_k^*} = p_{l,l_k^*} + 1$

  for all $k \in \mathcal{K}$ **do**

    set of users proposing to base station k

$$\mathcal{P}_k := \{\mathcal{A} \subseteq \mathcal{U} \mid l \in \mathcal{A} \text{ if } l_k^* = k\} \cup \mathsf{M}(k). \tag{9}$$

    accept $Ch_k(\mathsf{M}, \mathcal{P}_k)$

    reject $\mathcal{P}_k \setminus Ch_k(\mathsf{M}, \mathcal{P}_k)$

    update $\mathsf{M} = \mathsf{M}_{(k, Ch_k(\mathsf{M}, \mathcal{P}_k))}$

until no proposal from any user is made

be computed locally. The condition $p_{l,k} = \min_{j \in \mathcal{C}} \{p_{l,j}\}$ where $\mathcal{C} := \{j \in \mathcal{K} \mid \|\mathbf{h}_{j,l}\| > \|\mathbf{h}_{\mathsf{M}(l),l}\|\}$ indicates that user l is interested only in the base stations which have higher channel norms than that of the current matching as well as to the base stations to which it has proposed the least number of times.

Next, we describe the stability rquirements in a stable matching.

Definition 3: Matching M is individually rational if for all users $l \in \mathcal{U}$ we have $u_{l,\mathsf{M}(l)}(\mathbf{p}_l) > 0$ and all base stations $k \in \mathcal{K}$, $Ch_k(\mathsf{M}(k), \mathsf{M}) = \mathsf{M}(k)$.
Individually rationality ensures that each user prefers being in its current matching rather than being unmatched, and that each base station should be matched to the users determined by its choice function.

Definition 4: Matching M is pairwise stable if there does not exist a pair $(l, k) \in \mathcal{U} \times \mathcal{K}$ such that $u_{l,k}(\mathbf{p}_l) > u_{l,\mathsf{M}(l)}(\mathbf{p}_l)$ and $l \in Ch_k(\mathsf{M}(k) \cup \{l\})$.
Pairwise stability requires that there exist no base station k and no user l which are not matched to each other but prefer a matching between themselves.

Definition 5: A matching M is *stable* if it is individually rational and pairwise stable.

Algorithm 1 has similarities with the deferred acceptance algorithm [8] which reaches a stable matching is settings without externalities. First, each user proposes to its best base station according to the utility model in (7). Given the proposals from the users, each base station selects its best users according to its choice function. The algorithm terminates when no user proposes to any base station.

The reached matching satisfies the individual rationality condition and pairwise stability since the algorithm iterates over all possible opportunities for pairing any user and base station which prefer each other. Note that algorithm 1 converges to a stable matching with a worst case total number of proposals $B_m = \sum_{l \in \mathcal{U}} b_l$ having the proposal budget in the utility model in (7).

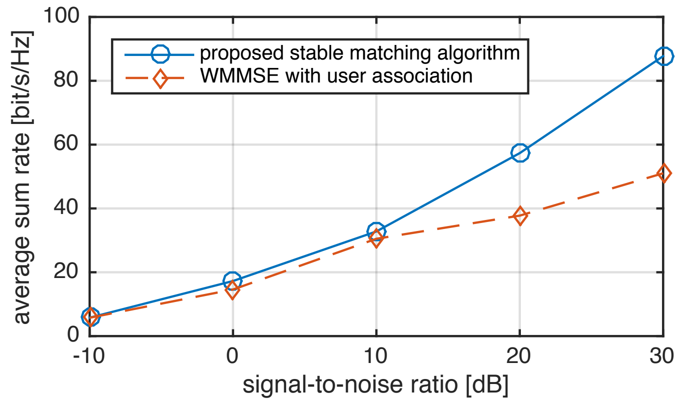

Fig. 2. Sum-rate for a setting with 10 existing users and 5 base stations equipped with 10 antennas each.

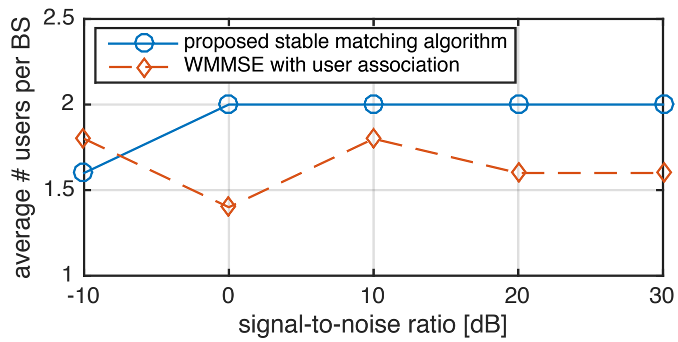

Fig. 3. Average number of users assigned to each base station in a setting with 10 existing users and 5 base stations equipped with 10 antennas each.

IV. Simulation Results

In the simulations, we consider a multi-cell system with five base stations. Each base station is equipped with ten antennas. The channel vectors are chosen as $\mathbf{h}_{k,l} \sim \mathcal{CN}(0, \mathbf{I})$ and the signal-to-noise ratio (SNR) is defined as $1/\sigma^2$. We generate 100 random channel realizations to calculate the average performance.

The users' proposal budget b_l are chosen to be the same for all users and equal to the number of base stations, $b_l = K$. The parameter α in the base stations' choice function is set to 0.5.

In Fig. 2, the average sum rate achieved with our proposed algorithm is compared to the weighted minimum mean square error (WMMSE) with user association algorithm in [2]. Remarkably, for a number of ten users and ten antennas at each base station, our algorithm outperforms that in [2], and the gains are larger at higher SNR values. Note that the algorithm in [2] is not distributed. The corresponding average number of users assigned to each base station are shown in Fig. 3. Above 0 dB SNR, our algorithm assigns all users to all base stations contrary to the algorithm in [2].

In Fig. 4 we show the effect of increasing the number of users in the network on the average sum rate. It is shown that our proposed algorithm outperforms the algorithm in [2] when the number of users is small and comparable to the number of

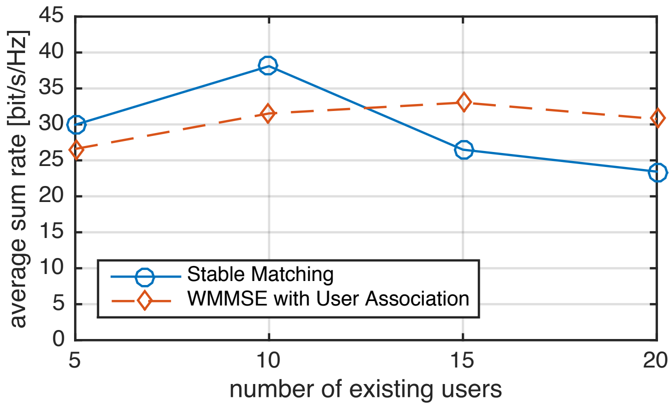

Fig. 4. Sum-rate for a setting with 5 base stations equipped with 10 antennas each. SNR is 10 dB.

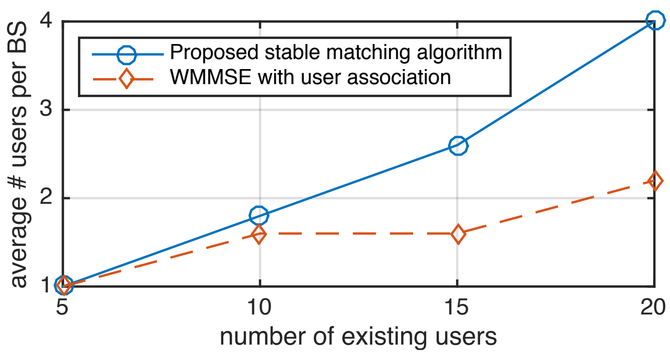

Fig. 5. Average number of users assigned to each base station in a setting with 5 base stations equipped with 10 antennas each. SNR is 10 dB.

antennas used at the base stations. The corresponding average number of users served by each base station is plotted in Fig. 5. The degradation in the average sum rate with our algorithm in Fig. 4 might be due to scheduling too many users per base station when the number of users is large.

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