Pilot Contamination Precoding Assisted Sum Rate Maximization for Multi-cell Massive MIMO Systems

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***Abstract*- This paper focuses on the pilot contamination precoding (PCP) assisted sum rate maximization for multi-cell massive MIMO system with finite number of antennas at base station (BS). Considering the case where BSs with finite number of antennas, the impact of the noise, the channel estimation error, channel uncertainty caused by usage of statistical channel state information (CSI) and channel non-orthogonality on the system performance can not be neglected any more compared with the pilot contamination, we in this work jointly consider all these factors and formulate an optimization problem to maximize the sum rate of all users. We derive the expression of the PCP matrix maximizing sum rate (MSR) of all users. Based on the obtained expression, an iterative algorithm is proposed to get a suboptimal solution to maximize the sum rate of all users. Simulations have been done to verify its superiority and results show that the proposed MSR-PCP outperforms the existing zero forcing PCP (ZF-PCP) and max-min PCP with acceptable computational complexity, especially for the case where users are located at cell edges and suffer from strong interference.**

Keywords*- *Massive MIMO; pilot contamination precoding; sum rate maximization

I. INTRODUCTION

Massive MIMO is seen as one of the key technologies in 5G communications systems, because of its advantage in spectral and energy efficiency improvement, vast spatial diversity and so on [1, 2]. In multi-cell massive MIMO systems, when the number of antennas at base station (BS) is very large, the noise and intra-cell interference have inappreciable effect on the system performance and pilot contamination becomes the only factor that restricts the system performance [3]. A technique called pilot contamination precoding (PCP) is proposed in [4] to combat the effect of pilot contamination in multi-cell massive MIMO systems. The main idea of PCP is that BSs recombine data from all users according to PCP matrix before downlink precoding. The results and conclusions in [4]-[8] have proved that the PCP technique is effective. The authors in [4] proposed a zero forcing PCP (ZF-PCP) scheme based on the estimated channel state information (CSI). ZF-PCP can eliminate the effect of pilot contamination when the number of antennas at BS is infinite. It is well known that the accuracy of CSI is critical for achieving high system performance. So, [5] proposed a beamforming training (BT)-PCP scheme to improve the system performance by taking the downlink channel training into account to get more accurate CSI. Both the PCP schemes in [4] and [5] are based on the assumption that the number of antennas at base station is infinite or approaches infinity. In practice, the number of antennas at base station cannot be arbitrarily large because of the limitation of space and the complexity of antenna devices. For practical consideration, a PCP scheme (max-min PCP) considering finite number of antennas at BS is proposed in [8] to maximize the minimum rate among all users in the system, which offers better fairness among users. However, taking the fairness into consideration is bound by sacrificing the performance of the whole system. To get a better performance of the whole system, we try to maximize the sum rate (MSR) of all users in the system through PCP with finite number of antennas at BS.

Analysis has shown that when the number of antennas at BS is finite, the channels among users are not orthogonal any more. In this case, the noise, the channel estimation error as well as the channel uncertainty are not equal to zero anymore [4]. And the impact of these factors on the system performance can not be ignored either. So, in this work, we jointly consider the noise, channel estimation error, the channel uncertainty caused by the usage of statistical CSI, the channel nonorthogonality of users due to the finite number of antennas at BS as well as the pilot contamination among cells to maximize the sum rate of the whole system. In detail, we formulate an optimization problem to maximize the sum rate of the whole system, and based on which the MSR-PCP is designed. Since it is difficult to get optimal solution of the formulated problem, an iterative algorithm is proposed to get a suboptimal solution instead. Simulations are done to investigate the performance of the designed algorithm and results show that the proposed scheme can greatly improve the sum rate of whole system with acceptable computational complexity especially for the case where users are located at cell edges suffering from strong interference.

Throughout the paper, the following notations are employed: $(\cdot)^T$, $(\cdot)^*$, $E(\cdot)$ and $Tr(\cdot)$ denote the transposition of matrix, the Hermitian transposition of matrix, the expectation of matrix and the trace of matrix, respectively. $(A)_{i,j}$ is used to represent the element of i th row and j th column of a matrix A.

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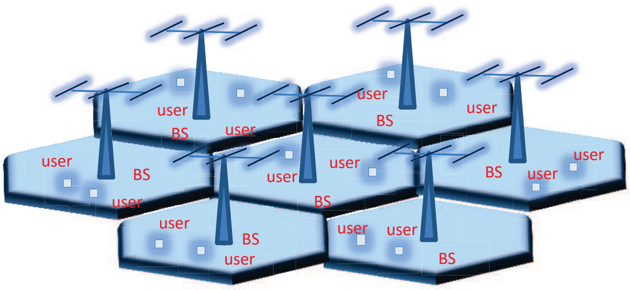

Fig. 1. System Model.

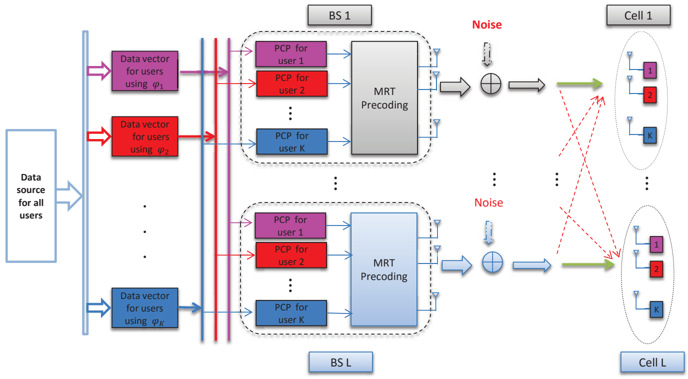

Fig. 2. System diagram with Pilot contamination precoding (PCP).

II. System model

We consider a multi-cell massive MIMO system with L cells as illustrated in Fig. 1, where each cell is equipped with one BS serving K single antenna users and the BSs are all equipped with M antennas. We assume that the pilot sequences of users are orthogonal in the same cell and reused among cells. Meanwhile, the system works in TDD mode and reciprocity holds between uplink and downlink channels. Let superscript $[kl]$ represent the user k in cell l and the downlink channel vector from the BS j to the user k in cell l is denoted by

$$\mathbf{g}_j^{[kl]} = \sqrt{\beta_j^{[kl]}}\mathbf{h}_j^{[kl]}, \tag{1}$$

where $\beta_j^{[kl]}$ represents the large scale fading coefficient which is determined by the distance and environment between BS j and user k in the cell l. $\mathbf{h}_j^{[kl]} = [h_{j1}^{[kl]}, h_{j2}^{[kl]}, \cdots, h_{jM}^{[kl]}]$ represents the small scale fading vector between BS j and user k in cell l. The entry $h_{jM}^{[kl]}$ in vector $\mathbf{h}_j^{[kl]}$ means the small scale fading factor between the Mth antenna of BS j and user k in cell l, which is a complex Gaussian variable with zero mean and unit variance, i.e., $\mathcal{CN}(0,1)$.

We define the pilot matrix as $\Psi = [\psi_1, \psi_2, \cdots, \psi_K]^T$, in which columns $\psi_1, \psi_2, \cdots, \psi_K$ are pairwisely orthogonal with each other. In uplink training, all users transmit uplink pilot sequences of length τ to BS synchronously. At receiving terminal, the received signals at BS j are

$$\mathbf{Y}_j = \sqrt{\rho_u \tau}\sum_{k=1}^{K}\sum_{l=1}^{L}\sqrt{\beta_j^{[kl]}}(\mathbf{h}_j^{[kl]})^T\psi_k + \mathbf{N}_j, \tag{2}$$

where ρ_u is the average uplink training power of every pilot symbol and $\mathbf{N}_j$ denotes the Gaussian noise matrix in cell j with i.i.d. $\mathcal{CN}(0,1)$ entries. According to the received signals and pilot sequences, BS j can decouple the signal from the user k in its own cell and the decoupled signal has the form

$$\mathbf{y}_j^{[k]} = \sqrt{\rho_u \tau}\sum_{l=1}^{L}\sqrt{\beta_j^{[kl]}}(\mathbf{h}_j^{[kl]})^T + \mathbf{n}_j^{[k]}, \tag{3}$$

where $\mathbf{n}_j^{[k]} = \mathbf{N}_j\psi^{[k]*}$ is the noise vector with i.i.d. $\mathcal{CN}(0,1)$ entries. Assume that the minimum mean square error (MMSE) channel estimator [8] is employed at the receiver sides to estimate the uplink channel vector, the estimated uplink CSI can be expressed as

$$(\hat{\mathbf{g}}_j^{[kl]})^T = \frac{\sqrt{\rho_u\tau}\beta_j^{[kl]}}{1+\rho_u\tau\sum_{s=1}^{L}\beta_j^{[ks]}}(\sqrt{\rho_u\tau}\sum_{i=1}^{L}\sqrt{\beta_j^{[ki]}}(\mathbf{h}_j^{[ki]})^T + \mathbf{n}_j^{[k]}).$$

By employing the channel reciprocity, the downlink channel vector can be obtained as

$$\hat{\mathbf{g}}_j^{[kl]} = \frac{\sqrt{\rho_u\tau}\beta_j^{[kl]}}{1+\rho_u\tau\sum_{s=1}^{L}\beta_j^{[ks]}}(\sqrt{\rho_u\tau}\sum_{i=1}^{L}\sqrt{\beta_j^{[ki]}}\mathbf{h}_j^{[ki]} + \mathbf{n'}_j^{[k]}), \tag{4}$$

where $\mathbf{n'}_j^{[k]} = (\mathbf{n}_j^{[k]})^T$. Based on the obtained downlink CSI, the downlink precoding can be designed accordingly to combat the interference among users within a cell. In this work, it is assumed that conjugate beamforming is employed. Let $\mathbf{w}^{[nj]}$ be the conjugate beamforming vector for user n in the cell j, it is expressed as

$$\mathbf{w}^{[kl]} = (\hat{\mathbf{g}}_l^{[kl]})^*. \tag{5}$$

It is assumed that the CSIs and signals of all users in the system are shared among BSs and all BSs work together. Before downlink precoding, the signals of all users in the cellular network are jointly processed. Pilot contamination precoder is employed to play the role of the processor and the signal processing diagram are shown in Fig. 2. Let $q^{[ni]}$ be the signal intended for user n in cell i and the combined coefficient for user n in the ith cell used by BS j is $\alpha_j^{[ni]}$. Then, the transmitted signal for user n in cell j is $\sum_{i=1}^{L}\alpha_j^{[ni]}q^{[ni]}$, which is a combination of the signals intended for the user n that use the same pilot sequence in different cells. After pilot contamination precoding, the downlink precoding is done and the transmitted signals by BS j are $\sum_{n=1}^{K}\mathbf{w}^{[nj]}\sum_{i=1}^{L}\alpha_j^{[ni]}q^{[ni]}$. At user terminals, the received signal of user k in cell l is

$$x^{[kl]} = \sqrt{\frac{\rho_d}{\gamma}}\sum_{j=1}^{L}\sum_{n=1}^{K}\sum_{i=1}^{L}\mathbf{g}_j^{[kl]}\mathbf{w}^{[nj]}\alpha_j^{[ni]}q^{[ni]} + n^{[kl]}, \tag{6}$$

where ρ_d is the downlink transmission power. Considering the power normalization for each BS, we set the scaling factor γ as [4]

$$\gamma = \frac{M}{L}\sum_{k=1}^{K}\sum_{j=1}^{L}\sum_{l=1}^{L}(1+\rho_u\tau\sum_{s=1}^{L}\beta_j^{[ks]})|\alpha_j^{[kl]}|^2. \tag{7}$$

and $n^{[kl]}$ denotes the complex Gaussian noise obeying $\mathcal{CN}$ $(0, 1)$.

Based on the expression of the received signal, we know that it is composed of not only the desired signal but also noise and interference. Besides pilot contamination, the interference also includes the channel estimation error, the channel uncertainty and the channel nonorthogonality. These interference mentioned above can not be neglected compared with pilot contamination. Taking all these into account, we in the following formulate an optimization problem to maximize the sum rate of the whole system.

III. Problem formulation and MSR-PCP design

By decoupling the interference caused by noise, the channel estimation error, the channel uncertainty, the channel nonorthogonality as well as the pilot contamination from the desired signal, the received signal in (6) can be rewritten as

$$x^{[kl]} = \bar{g}^{[kl]} q^{[kl]} + n_1^{[kl]} + n_2^{[kl]} + n_3^{[kl]} + n_4^{[kl]} + n^{[kl]} \tag{8}$$

where $\bar{g}^{[kl]}$ represents the effective statistical channel information estimated by user k in cell l, which is expressed as

$$\bar{g}^{[kl]} = \sqrt{\frac{\rho_d}{\gamma}} \sum_{j=1}^{L} E\{\hat{\mathbf{g}}_j^{[kl]} \mathbf{w}^{[kj]}\} \alpha_j^{[kl]}. \tag{9}$$

$n_1^{[kl]}$ represents the interference caused by the pilot contamination caused by pilot reuse, and writes

$$n_1^{[kl]} = \sqrt{\frac{\rho_d}{\gamma}} \sum_{j=1}^{L} \sum_{i \neq l}^{L} E\{\hat{\mathbf{g}}_j^{[kl]} \mathbf{w}^{[kj]}\} \alpha_j^{[ki]} q^{[ki]}. \tag{10}$$

$n_2^{[kl]}$ represents the interference caused by the channel estimation error and is written as

$$n_2^{[kl]} = \sqrt{\frac{\rho_d}{\gamma}} \sum_{j=1}^{L} \sum_{i=1}^{L} \sum_{n=1}^{K} \tilde{\mathbf{g}}_j^{[kl]} \mathbf{w}^{[nj]} \alpha_j^{[ni]} q^{[ni]}. \tag{11}$$

where $\tilde{\mathbf{g}}_j^{[kl]}$ denotes the channel estimation error vector, and we have $\mathbf{g}_j^{[kl]} = \hat{\mathbf{g}}_j^{[kl]} + \tilde{\mathbf{g}}_j^{[kl]}$.
Item $n_3^{[kl]}$ is caused by the channel nonorthogonality of users in the case that the number of antennas at the BS is not large enough, which is written as

$$n_3^{[kl]} = \sqrt{\frac{\rho_d}{\gamma}} \sum_{j=1}^{L} \sum_{i=1}^{L} \sum_{n \neq k}^{K} \hat{\mathbf{g}}_j^{[kl]} \mathbf{w}^{[nj]} \alpha_j^{[ni]} q^{[ni]}. \tag{12}$$

Interference $n_4^{[kl]}$ is caused by the channel uncertainty, i.e., the mis-match between the real CSI and the statistical CSI when the number of antennas at BSs is finite. So, it is expressed as

$$n_4^{[kl]} = \sqrt{\frac{\rho_d}{\gamma}} \sum_{j=1}^{L} \sum_{i=1}^{L} (\hat{\mathbf{g}}_j^{[kl]} \mathbf{w}^{[kj]} - E\{\hat{\mathbf{g}}_j^{[kl]} \mathbf{w}^{[kj]}\}) \alpha_j^{[ki]} q^{[ki]}. \tag{13}$$

Based on the signal model analyzed above, the signal-to-noise-ratio (SINR) of user k in cell l, $\mathit{SINR}^{[kl]}$, can be expressed as

$$\mathit{SINR}^{[kl]} = \frac{E|\bar{g}^{[kl]}|^2}{1 + E\{\sum_{t=1}^{4} |n_t^{[kl]}|^2\}}. \tag{14}$$

Substituting (10)-(14) into (15), $\mathit{SINR}^{[kl]}$ [8] is simplified as

$$\mathit{SINR}^{[kl]} = \frac{\left|\sum_{j=1}^{L} \beta_j^{[kl]} \alpha_j^{[kl]}\right|^2}{D_1 + \frac{1}{M} D_2} \triangleq \frac{\eta^{[kl]}}{\sigma^{[kl]}}, \tag{15}$$

where

$$D_1 = \sum_{i=1, i \neq l}^{L} \left| \sum_{j=1}^{L} \beta_j^{[kl]} \alpha_j^{[ki]} \right|^2,$$

$$D_2 = \sum_{j=1}^{L} \sum_{n=1}^{K} \left(\frac{1}{L\rho_d} + \beta_j^{[kl]} \right) \left(\frac{1}{\rho_u \tau} + \sum_{s=1}^{L} \beta_j^{[ns]} \right) \sum_{i=1}^{L} |\alpha_j^{[ni]}|^2.$$

D_1 is the interference caused by the pilot contamination and D_2 is the mixed interferences caused by the rest interference, i.e., $n_2^{[kl]}$,$n_3^{[kl]}$,$n_4^{[kl]}$ and $n^{[kl]}$. Then, we can get the low bound of achievable sum rate of the system

$$R = \sum_{l=1}^{L} \sum_{k=1}^{K} \log \left(1 + \frac{\eta^{[kl]}}{\sigma^{[kl]}} \right). \tag{16}$$

Define $\mathbf{A}$ as the PCP matrix which is a block diagonal matrix with $\mathbf{A}_{(k-1)L+j,(k-1)L+l} = \alpha_j^{[kl]}$, the PCP matrix $\mathbf{A}$ maximizing the sum rate can be designed as

$$\mathbf{A}_{\text{MSR}} = \arg\max_{\mathbf{A}} R \tag{17}$$

Theorem 1: The optimal $\mathbf{A}_{\text{MSR}}$ is of the expression $\mathbf{A}_{\text{MSR}} = (\mathbf{B}^* \mathbf{D} \mathbf{B} + \mathbf{C} \mathbf{N})^{-1} \mathbf{B}^* \Delta$, where the block diagonal matrix $\mathbf{B}$ and the diagonal matrices $\mathbf{D}$, $\mathbf{C}$, $\mathbf{N}$, Δ are defined separately as the following.

Matrix $\mathbf{B}$:

$$\mathbf{B} \triangleq \text{diag}\left[\mathbf{B}^{[1]}, \mathbf{B}^{[2]}, \cdots, \mathbf{B}^{[K]}\right],$$

where $\mathbf{B}^{[k]}$ is composed of large scale fading factor as follows:

$$\mathbf{B}^{[k]} = \begin{bmatrix} \beta_1^{[k1]} & \beta_2^{[k1]} & \cdots & \beta_L^{[k1]} \\ \beta_1^{[k2]} & \beta_2^{[k2]} & \cdots & \beta_L^{[k2]} \\ \vdots & \vdots & \vdots & \vdots \\ \beta_1^{[kL]} & \beta_2^{[kL]} & \cdots & \beta_L^{[kL]} \end{bmatrix}.$$

Matrix $\mathbf{D}$:

$$\mathbf{D} \triangleq diag\left[\mathbf{D}^{[1]}, \mathbf{D}^{[2]}, \cdots, \mathbf{D}^{[K]}\right],$$

where

$$\mathbf{D}^{[k]} = diag\left[d^{[k1]}, d^{[k2]}, \cdots, d^{[kL]}\right],$$

$$d^{[kj]} = \frac{\eta^{[kj]}}{\sigma^{[kj]}(\eta^{[kj]} + \sigma^{[kj]})}.$$

Matrix $\mathbf{C}$:

$$\mathbf{C} \triangleq \text{diag}\left[\mathbf{O}, \mathbf{O}, \cdots, \mathbf{O}\right],$$

where

$$(\mathbf{O})_{j,j} = \mathrm{Tr}(\mathbf{P}_j\mathbf{D}),$$

$$(\mathbf{P}_j)_{(k-1)L+l,(k-1)L+l} = \frac{1}{L\rho_d} + \beta_j^{[kl]}.$$

Matrix $\mathbf{N}$:

$$\mathbf{N} \triangleq diag\left[\mathbf{N}^{[1]}, \mathbf{N}^{[2]}, \cdots, \mathbf{N}^{[K]}\right],$$

where

$$\mathbf{N}^{[k]} = diag\left[n^{[k1]}, n^{[k2]}, \cdots, n^{[kL]}\right],$$

$$n^{[kj]} = \frac{1}{\rho_u \tau} + \sum_{s=1}^{L} \beta_j^{[ks]}.$$

Matrix Δ :

$$\Delta \triangleq \mathrm{diag}\left[\Delta^{[1]}, \Delta^{[2]}, \cdots, \Delta^{[K]}\right],$$

where $\Delta^{[k]}$ is diagonal matrix,

$$\Delta^{[k]} = \mathrm{diag}\left[\delta^{[k1]}, \delta^{[k2]}, \cdots, \delta^{[kL]}\right],$$

$$\delta^{[kl]} = \frac{\sum_{j=1}^{L} \beta_j^{[kl]} \alpha_j^{[kl]}}{\sigma^{[kl]}}.$$

Proof: If $\mathbf{A}_{\text{MSR}}$ is the optimal solution of (18), it must satisfy

$$\begin{aligned}
\frac{\partial R}{\partial \alpha_a^{[bc]}} =& \sum_{l=1}^{L}\sum_{k=1}^{K} \frac{(\eta^{[kl]})'\sigma^{[kl]} - \eta^{kl}(\sigma^{[kl]})'}{\left(\eta^{[kl]} + \sigma^{[kl]}\right)\sigma^{[kl]}} \\
=& \frac{\beta_a^{[bc]}\left|\sum_{j=1}^{L} \beta_j^{[bc]}\alpha_j^{[bc]}\right|}{\sigma^{[bc]}} \\
&- \sum_{l=1}^{L} \frac{\eta^{[bl]}\beta_a^{[bl]}\left|\sum_{j=1}^{L} \beta_j^{[bc]}\alpha_j^{[bc]}\right|}{\sigma^{[bl]}(\sigma^{[bl]} + \eta^{[bl]})} \\
&- \sum_{k=1}^{K}\sum_{l=1}^{L} \frac{1}{M} d^{[kl]} n^{[ba]} |\alpha_a^{[bc]}| \left(\frac{1}{L\rho_f} + \beta_a^{[kl]}\right) \\
=& 0
\end{aligned}$$

Rewrite the equation above in matrix form as $\mathbf{B}^*\Delta - \mathbf{B}^*\mathbf{DBA} - \mathbf{CNA} = \mathbf{0}$ and we can get $\mathbf{A} = \left(\mathbf{B}^*\mathbf{DB} + \mathbf{CN}\right)^{-1}\mathbf{B}^*\Delta$. ■

Since the PCP matrix $\mathbf{A}$ is embedded in $\mathbf{C}$, $\mathbf{D}$, Δ and $\left(\mathbf{B}^*\mathbf{DB} + \mathbf{CN}\right)^{-1}\mathbf{B}^*\Delta$ is a function of $\mathbf{A}$, it is difficult to get the closed form solution of $\mathbf{A}$. By adopting the fixed-point iterative algorithm, a sub-optimum solution can be obtained. The algorithm is summarized in Algorithm 1. For this algorithm, the initialization dominates the sub-optimum results, since for different initialization the solution will convergent to different local optimal points. In this work, the initialization is set to be $\mathbf{D}_0 = \mathbf{I}, \Delta_0 = \mathbf{I}$ just like the initialization of Method 2.1 in [9]. The simulation results presented in the following section show that such initialization guarantees a local convergence. The computational complexity of Algorithm 1 is about $\mathcal{O}(N_{iter}KL^3)$, N_{iter} denotes the number of iterations. It is easily shown that the complexity mainly depends on the iterative calculations of $KL \times KL$ block diagonal matrix inversion and multiplication.

Algorithm 1 Iterative MSR-PCP design algorithm

Given $\mathbf{B}$, initiate $\mathbf{D}_0 = \mathbf{I}, \Delta_0 = \mathbf{I}, i = 0, R_{-2} = 10, R_{-1} = 100$.

Repeat while $|R_{i-1} - R_{i-2}| \geq 10^{-3}$.

(1) $\mathbf{A}_i = \left(\mathbf{B}^*\mathbf{D}_i\mathbf{B} + \mathbf{C}_i\mathbf{N}\right)^{-1}\mathbf{B}^*\Delta_i$.

(2) $\eta^{[kl]} = \left|[\mathbf{BA}_i]_{(k-1)L+l,(k-1)L+l}\right|^2$,
$(\mathbf{Z}^{[kl]})_{j,j} = \frac{1}{\rho_d L} + \beta_j^{[kl]}$,
$\mathbf{Q}^{[kl]} = \mathrm{diag}\left[\mathbf{Z}^{[kl]}, \mathbf{Z}^{[kl]}, \cdots, \mathbf{Z}^{[kl]}\right]$.
$\sigma^{[kl]} = \sum_{v\neq l}\left|(\mathbf{BA}_i)_{(k-1)L+l,(k-1)L+v}\right|^2 + \frac{1}{M}\mathrm{Tr}((\mathbf{A}_i)^*\mathbf{Q}^{[kl]}\mathbf{NA}_i)$.

(3) $R_i = \sum_{l=1}^{L}\sum_{k=1}^{K}\log\left(1 + \frac{\eta^{[kl]}}{\sigma^{[kl]}}\right)$,

(4) $d_{i+1}^{[kl]} = \frac{\eta^{[kl]}}{\sigma^{[kl]}(\eta^{[kl]}+\sigma^{[kl]})}$, $\delta_{i+1}^{[kl]} = \frac{\sum_{j=1}^{L}\beta_j^{[kl]}\alpha_j^{[kl]}}{\sigma^{[kl]}}$.

(5) Calculate $\mathbf{D}_{i+1}$,$\mathbf{C}_{i+1}$,Δ_{i+1}.

(6) $i = i + 1$

end

TABLE I. SIMULATION SETUPS

Parameter	Value
L	3
K	5
r	1 km
ρ_d	10 W
ρ_u	1 W
$\beta_j^{[kl}$	$\left(d_j^{[kl]}\right)^{-3.5}$

$d_j^{[kl]}$ is the distance between BS j and user k in cell l.

IV. SIMULATION RESULTS AND ANALYSIS

In this section, to show the performance of MSR-PCP and the convergence of Algorithm 1, simulations are done based on the system model described in the system model part. In detail, in doing the simulations, a multi-cell system with 3 hexangular cells is considered where each cell serves 5 single antenna users. It is assumed that the length of the pilot sequence is equal to the number of users in each cell and the same pilot sequences are reused among all the cells. Detailed simulation setups are listed in Table I.

A. Performance Analysis

The sum rate performance of the multi-cell where all users are distributed randomly in each cell and the minimum distance between users and their corresponding BS is assumed to be no less than $0.2km$ is investigated, results are in Fig. 3. It is observed that MSR-PCP brings considerable improvement on the sum rate for all the depicted antenna number regime compared with max-min PCP, ZF-PCP and no PCP schemes. This is because the proposed MSR-PCP scheme can combat not only the pilot contamination but also the noise and interference, while the ZF-PCP only combats the interference caused by the pilot contamination. In addition, the max-min PCP proposed in [8] aims at the fairness of all users, which sacrifices the performance of sum rate to satisfy the fairness. This is the reason that MSR-PCP performs better than ZF-PCP and max-min PCP in terms of the sum rate. The sum rate performance when the system works in another scenario

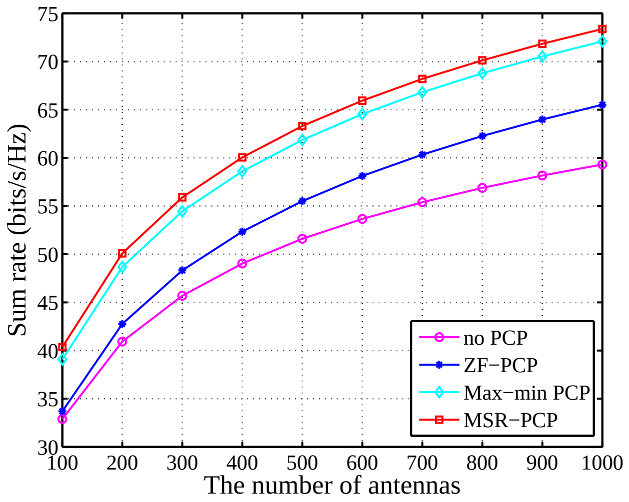

Fig. 3. Sum rate comparison of systems with various PCP techniques when users are randomly distributed in cells.

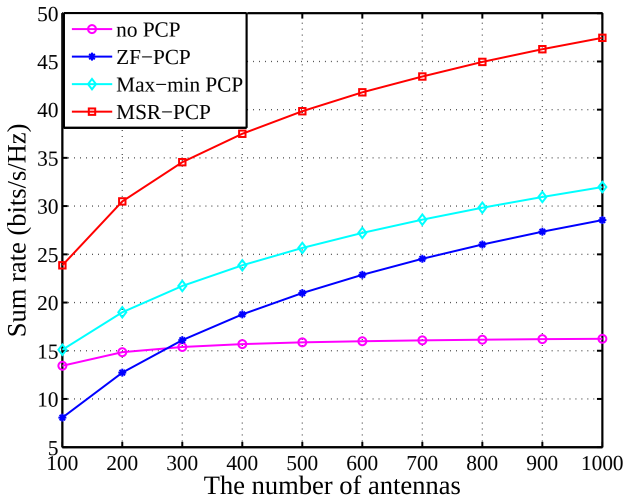

Fig. 4. Sum rate comparison of systems with various PCP techniques when users are located at the cell edges.

where the users are allocated at the area of cell edges and bear serious interference is also simulated, results are shown in Fig. 4. In this scenario, it is assumed that the minimum distance between users and its BS is not less than $0.8km$. Simulation results show that in this case the proposed MSR-PCP offers more improvement on the sum rate compared with other two PCP schemes. This is because in this case the interference is much severer, and the proposed MRS-PCP is combating all the interference while the other two are not. Comparing Fig. 3 with Fig. 4, we know that the proposed scheme is much more effective when users suffer from strong interference.

B. Convergence and Complexity Investigation

Since it is difficult to prove that the algorithm is convergent, we investigate the convergence property of Algorithm 1 through simulations. The simulation is done under the condition that users are distributed randomly within its cell and the number of antennas at BS is 200. The convergence of the algorithm is simulated for five different random channel realizations, simulation results are included in Fig. 5. It is shown that the sum rate tends to be flat after 30 to 40 iterations. To examine whether the algorithm is convergent for all channel realizations or not, we also investigated the cumulative distribution function (CDF) of the number of iterations under 10000 channel realizations, which is shown in Fig. 6. When the number of iterations is 30, Algorithm 1 is convergent for 97.88% of the channel realizations. As it is known, the complexity of ZF-PCP is about $\mathcal{O}(KL^3)$ determined by the inversion of the block diagonal matrix in the matrix field $\mathbb{R}^{KL\times KL}$. Compared with ZF-PCP, the complexity of the proposed MSR-PCP is somewhat high. But considering the improved performance, the increased complexity is still acceptable. For the complexity of max-min PCP, we find the complexity is at least $\mathcal{O}(n_{iter}KL^4)$ where n_{iter} is the number of iterations to get the uplink power allocation, and is about 20 or more. Besides, max-min PCP also involves a complexity of the eigenvalue decomposition in the matrix field $\mathbb{R}^{KL\times KL}$. Although the MSR-PCP sacrifices the fairness, we can still use MSR-PCP if higher sum rate performance is of demand sacrificing a bit complexity especially for the case where the interference is strong.

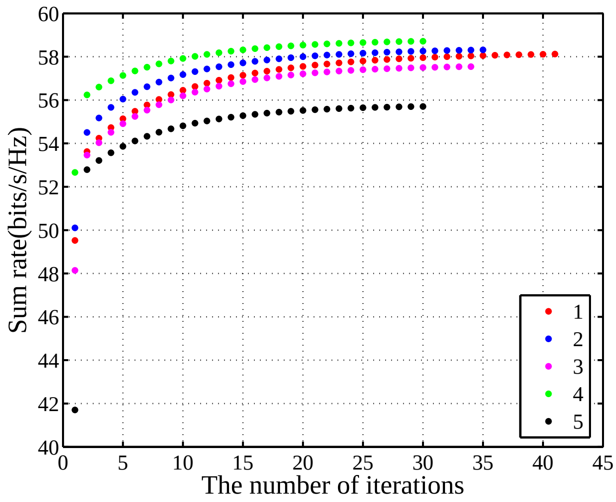

Fig. 5. Sum rate values during the procedure of Algorithm 1

V. CONCLUSION

In this paper, we consider a multi-cell massive MIMO system working in TDD mode with L cells and K users per cell. For practical consideration, it is assumed that there are finite but large number of antennas at BS. By jointly considering the noise and the interference caused by the channel estimation error, the channel uncertainty, the channel nonorthogonality and the pilot contamination, an optimization problem to maximize the sum rate of all users is formulated. The expression of the PCP matrix maximizing the sum rate has been derived and an iterative method has been proposed to search for the sub-optimum solution. Simulation results show the superior of the proposed MSR-PCP in terms of the sum rate performance through comparison with other existing PCP

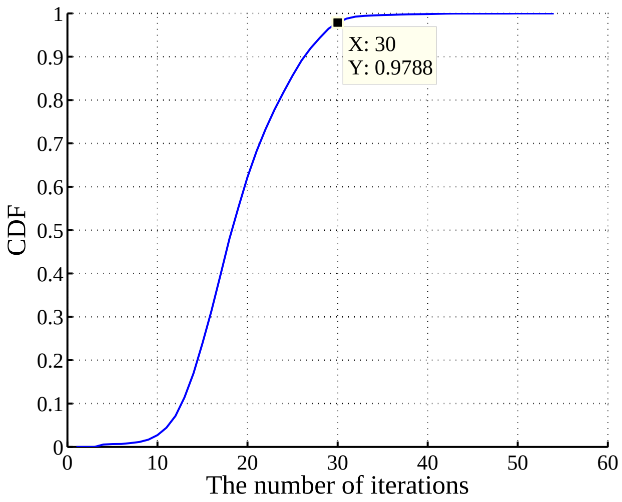

Fig. 6. CDF of the number of iteration using Algorithm 1.

schemes.The complexity of the proposed scheme has also been investigated.

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