On EIRP Control in Downlink Precoding for Massive MIMO Arrays

Niklas Doose and Peter A. Hoeher
Information Theory and Coding
Faculty of Engineering, University of Kiel, 24143 Kiel, Germany
Email: {nd,ph}@tf.uni-kiel.de

***Abstract*—The usual performance comparisons of wireless systems are based on a constrained sum-power at the transmitter. However, many wireless systems are actually constrained in their equivalent isotropic radiated power (EIRP). So far, research effort in the area of beamforming under constrained EIRP seems to be done solely for single-user systems. As an original contribution, this contribution develops a theoretical description of an EIRP-limited multi-user system with joint consideration of the EIRP and the capacity. Accordingly, the EIRP is interpreted as a function of the precoding matrix instead of a static measure of the antenna. The theoretical formulations is applicable to arbitrary antenna designs, including multi-mode antennas. Down-scaling of any coventional linear precoding solution is proposed as a simple strategy to comply with the EIRP limit. Numerical results are provided in the context of massive MIMO with simple linear precoding techniques.**

I. Introduction

The received signal power to noise power ratio (SNR) is a crucial parameter with respect to the capacity of wireless communication systems. Therefore, it is necessary and suitable to compare different systems in terms of some sort of power constraint to prevent a capacity increasement by simple up-scaling of the transmit power. The most common approach is the assumption of a constrained sum-power at the transmitter side of the system. However, in the beginning of the successful multiple-input multiple-output (MIMO) era an equivalent isotropic radiated power (EIRP) constraint has been investigated. The radiation characteristic has been taken into account in terms of the array factor. In [1] and [2] the EIRP constraint is mentioned to be a regulatory condition for WLAN-type systems. Again, in [3] the EIRP is studied as one possible constraint in the context of WLAN. Surprisingly, since then joint precoding and power control has rarely been taken into account in research papers on WLAN systems, although these radio systems are restricted with respect to their EIRP.

With the upcoming interest in ultra-wideband (UWB) systems, EIRP constraints were reconsidered for example in [4]–[8]. UWB systems have been standardized by the FCC, ETSI and several other institutions for license-free communication purposes [9], [10]. In order to limit interference, a regulation of the EIRP is particularly important for UWB systems, because they operate in the same frequency bands as primary, licensed systems.

The publications mentioned so far considering EIRP-limited beamforming assume only one user and a single data stream to be transmitted. However, with the target of an ultra-high speed link to a single user it is crucial to split the signal into multiple streams to relax the requirements for the decoding in the receiver, which becomes a bottleneck in such systems [11]. Most other research in the area of beamforming and/or precoding does not even mention the EIRP or deals with the EIRP as a static property of the system. To our best knowledge multi-layer beamforming with explicit consideration of the variable EIRP has not been investigated in the research literature so far. This contribution introduces a joint consideration of the capacity of a multi-layer beamforming system together with an EIRP limitation. Therefore, a theoretic problem formulation is derived to give insight to the radiation characteristics of precoded systems. In this context the resulting EIRP is a function of the precoding and not a static measure of the antenna configuration and the power. The performance gain of the consideration of the variable EIRP compared to a static EIRP is investigated in terms of numerical simulations.

The employment of so-called multi-mode antennas as anticipated in [12] will be included in the investigations of this contribution. The multi-mode antenna approach aims at very compact multiple-element antennas. The design process makes use of the theory of characteristic modes that enables multiple—theoretically orthogonal—radiation patterns on single physical elements as shown in [13]. Just the same as for spatially separated, discrete single antenna elements, multiple multi-mode antennas can be used to form an array as introduced in [14], where a 484 port array has been realized with 11×11 physical elements. Note that the utilization of multi-mode antennas results in a generalization of the considerations rather than being a special case.

Section II introduces the system model under consideration and gives insight to the configuration and flexibility of the system design. In Section III the problem formulation is derived and solved by a simple down-scaling scheme. In Section IV precoding methods for massive MIMO systems are investigated in the context of a constrained EIRP.

II. System Model

Most physical wireless channels exhibit multipath propagation, which leads to dispersion and frequency-selectivity. By means of an orthogonalization in frequency like OFDM, the system equation of each orthogonal sub-channel can be written in multiplicative vector-matrix form as

$$\boldsymbol{y} = \boldsymbol{H}\boldsymbol{x} + \boldsymbol{n}. \tag{1}$$

The received vector $\boldsymbol{y}$ for one time instant and sub-channel (time index and frequency index are omitted for simplicity) is given by the multiplication of the transmit vector $\boldsymbol{x}$ with the channel matrix $\boldsymbol{H}$ and the additive noise vector $\boldsymbol{n}$. The transmit vector $\boldsymbol{x}$ is constructed by the precoding matrix $\boldsymbol{W}$ and the symbol vector $\boldsymbol{s}$ as

$$\boldsymbol{x} = \boldsymbol{W}\boldsymbol{s}. \tag{2}$$

The vector $\boldsymbol{s}$ has the dimension $N_\mathrm{S} \times 1$, with N_S being the number of streams (or layers) in the precoder. The channel matrix is assumed to be known at the transmitter side from uplink pilots and successive channel estimation. This method is commonly assumed in time-division duplex systems like massive MIMO with reciprocal uplink and downlink channel matrices [15]–[17]. The elements of $\boldsymbol{n}$ are chosen from a circular symmetric complex zero-mean Gaussian distribution.

The system setup is sketched in Fig. 1. Please note that the sketched multi-mode antennas (MMA) offer three inputs that can be used by the digital baseband processing. Therefore, it is meaningful to distinguish between the number of physical antenna elements and the number of effective antenna ports. The system offers flexibility in terms of trade-offs between multiplexing and diversity. The extreme cases are (i) full multiplexing, where the number of streams N_S is equal to the number of effective receive ports $N_\mathrm{R,eff}$ and (ii) full diversity, where just one stream is used. In the case of full multiplexing the matrix $\boldsymbol{W}$ acts as a precoder, for the case with one input stream it breaks down to a beamforming vector. The receiver has to be configured by control bits in the MAC layer to adapt the coding rates and modulation formats. If any diversity is used, the parallel to serial conversion in the receiver unit needs a scheme to combine the diversely transceived streams. Similarly, a control unit in the transmitter takes care of channel estimation from pilots, calculation of EIRP-limited precoding, adaptive modulation formats and coding rates and the loading of bits onto the adaptive number of streams. The control unit has to consider quality of service (QoS) requirements and data rate demand to adapt the parameters of the system blocks. In the context of ultra-high data rates in the region of 100 Gbps [11], the case of full multiplexing is the suitable scenario. Therefore, in this contribution we will neglect the case of diversity and narrow the scope to full multiplexing.

III. Problem Formulation

In multi-user systems the optimization criterion is usually the maximization of the signal-to-interference-plus-noise ratio (SINR). In the scenario proposed in [11], [12], the multiplexed data streams are actually used by one mobile terminal. Therefore all SINRs have to be maximized simultaneously, which is novel compared to previous EIRP-limited beamforming considerations. In a different view, the overall capacity including a specific precoding solution,

$$C(\boldsymbol{W}) = \log_2 \left[\det\left(\boldsymbol{I}_{N_\mathrm{R}} + \mathrm{SNR}_0\, \boldsymbol{H}\boldsymbol{W}\boldsymbol{W}^\mathrm{H}\boldsymbol{H}^\mathrm{H}\right)\right], \tag{3}$$

has to be maximized. The capacity is given as a function of the precoding matrix and the channel matrix, which is assumed to be perfectly known at the transmitter side. The nominal signal-to-noise ratio SNR_0 is defined as the maximal SNR that can be transmitted and is therefore given by the ratio of the EIRP limit EIRP_0 and the noise spectral density. This SNR is reached for isotropic radiation, where the EIRP constraint is fulfilled by equality in each direction.

In contrast to former publications elaborating on the topic of joint precoding and power control for EIRP-limited MIMO systems, in this contribution the three dimensional space is considered rather than a plane. Additionally, non-isotropic antenna characteristics generalize the formulations from earlier publications, which is necessary for the usage of physically realizable antennas. We include the characteristics $F^{n_\mathrm{T}}(\theta, \phi)$ of each antenna element n_T for both polarization directions. Please note that the characteristic $F^{n_\mathrm{T}}(\theta, \phi)$ is a angle-dependent measure at a constant distance of 1 m from the physical radiator. The unit of $F^{n_\mathrm{T}}(\theta, \phi)$ is volt [V]. In the information theory community it is commonly assumed that all antenna elements are positioned in a (typically linear) array and have an identical characteristic, which simply leads to a multiplication of the element characteristic with the array factor. Please note that for multi-mode antennas this simplification is not possible, because for each port the antenna characteristic is different.

The EIRP is a measure for the maximum directivity of the antenna plus array factor (AAF):

$$\mathrm{EIRP} = \frac{4\pi}{2Z_\mathrm{F0}} \max_{\theta,\phi} \left\{ \left|AAF(\theta, \phi, \boldsymbol{W})\right|^2 \right\}, \tag{4}$$

where Z_F0 is the free space impedance, which is approximated by $Z_\mathrm{F0} = 120\pi$. The unit of Z_F0 is ohm [Ω]. The absolute squared AAF is given by

$$\begin{aligned} &\left|AAF(\theta, \phi, \boldsymbol{W})\right|^2 = \\ &\quad \left| \sum_{n_\mathrm{T}} \sum_{n_\mathrm{S}} \boldsymbol{W}_{[n_\mathrm{T}, n_\mathrm{S}]} \cdot F_\vartheta^{n_\mathrm{T}}(\theta, \phi) \cdot \mathrm{e}^{\mathrm{j}2\pi \boldsymbol{k}(\theta,\phi)\cdot \boldsymbol{r}(n_\mathrm{T})} \right|^2 + \\ &\quad \left| \sum_{n_\mathrm{T}} \sum_{n_\mathrm{S}} \boldsymbol{W}_{[n_\mathrm{T}, n_\mathrm{S}]} \cdot F_\varphi^{n_\mathrm{T}}(\theta, \phi) \cdot \mathrm{e}^{\mathrm{j}2\pi \boldsymbol{k}(\theta,\phi)\cdot \boldsymbol{r}(n_\mathrm{T})} \right|^2, \end{aligned} \tag{5}$$

where ϑ and φ are horizontal and vertical polarization, respectively. It is obvious that the EIRP is a function of the current precoding and is not a static measure. The max-operator in (4) acts on the whole sphere, so any incident angle represented by the wave vector $\boldsymbol{k}(\theta, \phi)$ is considered. The sub-scripted brackets $(\cdot)_{[n,m]}$ denote the element in the nth row and the mth column of a matrix. In this general form arbitrary antenna positions can be specified via the positioning vectors $\boldsymbol{r}(n_\mathrm{T})$ for each element.

The constraint on the EIRP at the transmitter can be written as

$$\frac{4\pi}{2Z_\mathrm{F0}} \cdot \max_{\theta,\phi} \left\{ \left|AAF(\theta, \phi, \boldsymbol{W})\right|^2 \right\} \leq \mathrm{EIRP}_0 \tag{6}$$

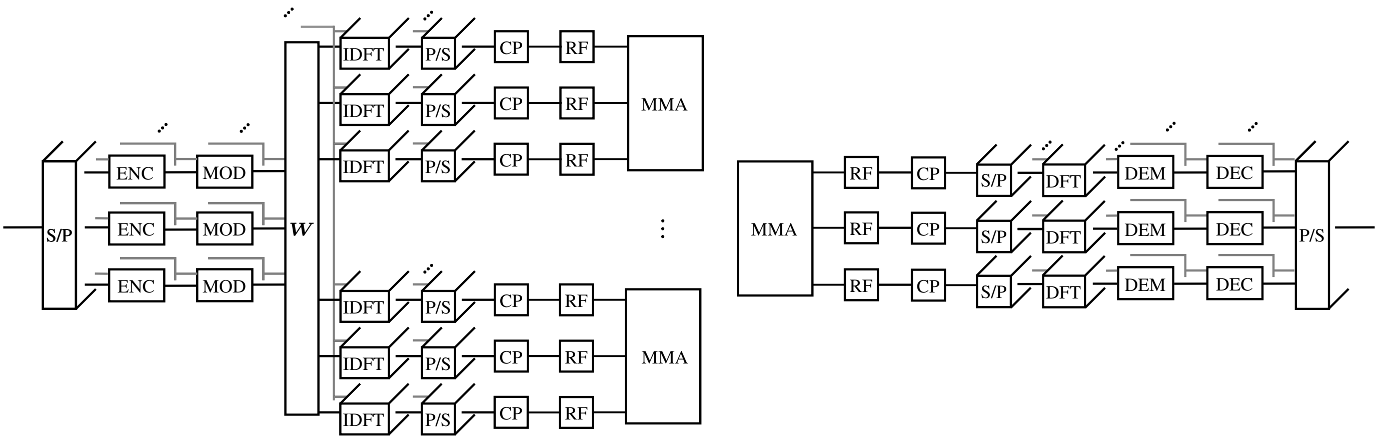


Figure 1. Block diagram of the downlink employing three streams that are encoded and modulated independently. An OFDM-like structure is used to create flat-fading channels, where each channel is beamformed separately, i.e. each frequency sub-channel has one matrix $\boldsymbol{W}$. Additional frequency bins are indicated in gray.

and the problem formulation for the maximization of the sum capacity given the constrained EIRP yields a conventional minimization problem with one inequality constraint:

$$\begin{aligned} \underset{\boldsymbol{W}\in\mathbb{C}^{N_\mathrm{T}\times N_\mathrm{R}}}{\text{minimize}} \quad & -\log_2\left[\det\left(\boldsymbol{I}_{N_\mathrm{R}} + \mathrm{SNR}_0\,\boldsymbol{H}\boldsymbol{W}\boldsymbol{W}^\mathrm{H}\boldsymbol{H}^\mathrm{H}\right)\right] \\ \text{subject to} \quad & \mathrm{EIRP}_0 - \frac{4\pi}{2Z_{\mathrm{F}0}}\max_{\theta,\phi}\left\{\left|AAF(\theta,\phi,\boldsymbol{W})\right|^2\right\} \geq 0. \end{aligned} \tag{7}$$

To our best knowledge, there is neither a known optimal value for the capacity, nor a known analytic way of solving the problem stated above to reach that optimal value. Besides the conventional formulation, the optimization problem bears severe problems for numerical optimization techniques.

The transmit power P_S is obtained by an integration over the whole sphere of the absolute squared AAF. Similarly, the EIRP in (4) is the integration over the maximum of the absolute squared AAF. Because the maximum of the absolute squared AAF is constant, the integration yields 4π. For both computations the sphere is assumed to have a radius of 1 m and the result is a measure of power with the unit watts [W]. The fraction $g = \mathrm{EIRP}/P_\mathrm{S}$ is the antenna gain. In a practical system, the simplest approach to comply with the EIRP limit and provide a sub-optimal solution to (7) is scaling down any precoding matrix $\boldsymbol{W}'$ by the square root of the gain factor, i.e.

$$\boldsymbol{W} = \frac{\boldsymbol{W}'}{\sqrt{g}}. \tag{8}$$

It is assumed that the theoretic maximum gain that an array can produce is in the "broadside" direction. The broadside is commonly agreed to be perpendicular to the two-dimensional plane that is spanned by the array. The assumption is justified because of the maximized virtual aperture that is seen from this direction. The precoding solution for this case is assumed to be a feeding with equal weights and phases to the antenna ports. The value of the gain for this case can be calculated offline and will be denoted as g_ANT. However, for a specific precoding solution the actual gain typically is than this maximum gain that the array could produce, due to a different main radiating direction than broadside. Therefore, the investigation in this contribution compares the capacities of the downlink for the case where the actual gain denoted by g_PRE for a specific precoding is used and the case where the maximum gain g_ANT of the antenna is used for the down-scaling of the precoding matrix.

IV. Numerical Results for Matched Filter Precoding

Massive MIMO research prefers linear precoding techniques like matched filter (MF) precoding, because their performance gets close to optimal precoding under certain propagation conditions [16]. Therefore, this section investigates the capacity for MF precoding under a constrained EIRP. Formally, unscaled MF precoding corresponds to

$$\boldsymbol{W}'_\mathrm{MF} = \boldsymbol{H}^\mathrm{H}. \tag{9}$$

As a channel model, the geometric, ray-based WINNER II channel model has been used [18]. It computes the effective channel coefficients between antenna pairs by a summation over clustered rays. The statistic distributions of the rays' angles, delays and powers are derived from excessive measurement campaigns. Despite the fact that the specified bandwidth of the chosen model slightly higher than the anticipated band considered in [11], it is assumed that the results will qualitatively hold.

Figure 2 shows the capacity for EIRP limited beamforming for a system with one mobile terminal. The terminal is equipped with a single multi-mode antenna with $N_\mathrm{R,eff} = 4$ ports and the base station (BS) is equipped a variable number of multi-mode antennas in a two-dimensional square array. The precoding matrix is scaled down according to (8) with g_ANT to yield the capacity C_ANT and with g_PRE to yield the capacity C_PRE, respectively.

For an increasing number of effective antenna ports in the BS the variable down-scaling scheme outperforms the

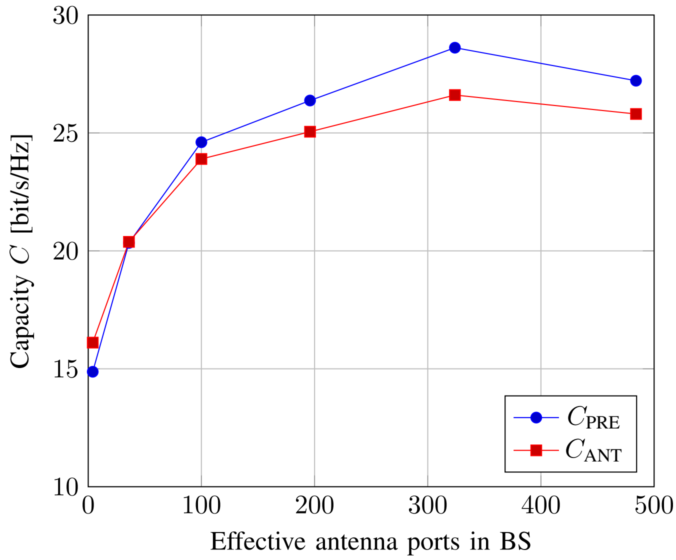
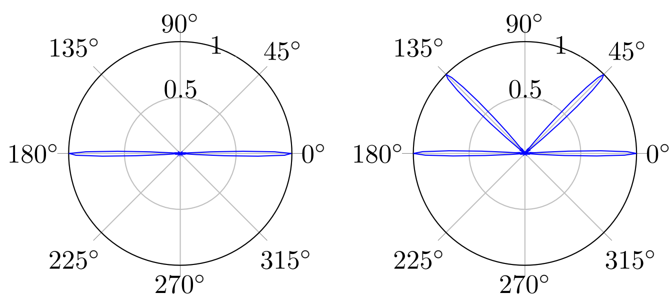


Figure 2. Ergodic capacity of channel realizations over number of effective base station ports for multi-mode system with $N_{\text{R,eff}} = 4$. C_{ANT} corresponds to the capacity, where the theoretical maximum gain of the antenna g_{ANT} is used for down-scaling, and C_{PRE} corresponds to the capacity, where the actual gain of the respective precoding solution g_{PRE} is used for the down-scaling as explained in the text. The SNR_0 is 30 dB.

static scheme. In fact the performance gap increases with an increasing number of ports. Besides the relative performance gap, the capacities reach a maximum for $N_{\text{T,eff}} = 324$ and decrease for a larger array. This effect is due to the inability of the matched filter precoding to use the spatial channel effectively. As an illustrative, purely academic example, in Fig. 3 one can see the radiated pattern for a BS with $N_{\text{T,eff}} = 24$ antenna ports, while a MS with three ports is considered. The channel model is a reduced complexity, ray-based channel



(a) MF

(b) Phased Array

Figure 3. Radiated antenna plus array pattern for under EIRP constraint, given a two-ray channel model and a BS with $N_{\text{T,eff}} = 24$ ports for a linear antenna array with omni-directional antennas. Case (a) refers to the MF solution ($\boldsymbol{W}_{\text{MF}}$) and case (b) refers to phased array beamforming, when the two rays are perfectly known and each ray is used by one stream (i. e. each stream forms one lobe). The capacities at 30 dB is given by $C_{\text{MF}} = 19.07$ bps/Hz and $C_{\text{phasedarray}} = 20.97$ bps/Hz.

with two rays with equal power going out of the BS in the directions $\Phi_1 = 0°$ and $\Phi_2 = 45°$. Intuitively, one would expect lobes in the ray directions with equal power, because of the equal powers of the rays. In this example, however, in Fig. 3(a) the MF solution creates only one of the lobes. In other words, for the given example the MF beamforming technique tries to focus all available power in one direction. Under a constrained sum-power this might lead to the same performance as forming a beam in the direction of the other ray, or to distribute the available power among both ray directions. For EIRP-constrained systems, however, a single narrow lobe turns out to be sub-optimal, because *each* lobe has to fulfill the constraint. An additional lobe in the other ray direction will therefore increase the sum-power budget of the transmitter, but does not alter the fact that the other lobe still complies with the EIRP limit. At the same time the instantaneous sum-capacity increases because of the additional lobe and the system benefits from the transmission of a second stream over the other lobe. In Fig. 3(b) classical phased array beamforming has been applied, where a fixed phase difference between the signals of the elements leads to a specific main radiation direction at a given frequency.

V. Conclusion

In the evolution of wireless communication standards, multi-stream processing is becoming more important in multi-user scenarios as well as in point-to-point links. A crucial performance parameter is the EIRP limitation. However, most researchers use a normalized sum-power constraint to optimize and compare different systems. In contrast, in this contribution the problem formulation for a multi-stream point-to-point downlink in conjunction with an EIRP constraint is developed. Down-scaling of the precoding matrix is proposed as a sub-optimal solution to the stated optimization problem. The numerical results take a massive MIMO scenario under consideration. It is shown that it is crucial to perform the down-scaling with the knowledge of the actual precoding matrix used instead of an offline computed value. Additionally, under an EIRP constraint simple algorithms like MF precoding appear to be too simple to benefit from massive MIMO arrays with vast degrees of freedom. A growth of the number of elements in the array even leads to a slight decrease of the channel capacity after reaching a maximum.

Acknowledgement

This work is supported by the Deutsche Forschungsgemeinschaft (DFG) within the priority program SPP 1655 under grant HO 2226/14-1.

References

[1] P. Zetterberg, M. Bengtsson, D. McNamara, P. Karlsson, and M. A. Beach, "Downlink beamforming with delayed channel estimates under total power, element power and equivalent isotropic radiated power (EIRP) constraints," in *Proc. IEEE Vehicular Technology Conf. (VTC Fall)*, vol. 1, 2001, pp. 516–520.

[2] ——, "Performance of multiple-receive multiple-transmit beamforming in WLAN-type systems under power or EIRP constraints with delayed channel estimates," in *Proc. IEEE Vehicular Technology Conf. (VTC Spring)*, vol. 4, 2002, pp. 1906–1910.

[3] A. M. Kuzminskiy, "EIRP-restricted downlink beamforming in WLAN OFDM systems," in *Proc. IEEE Signal Processing Workshop on Signal Processing Advances in Wireless Communications (SPAWC)*, Jul. 2006, pp. 1–5.

[4] C. M. Vithanage, Y. Wang, and J. P. Coon, "Spatial PAPR reduction based beamforming scheme for EIRP constrained systems," in *Proc. IEEE Global Communications Conf. (GLOBECOM)*, Nov. 2008, pp. 1–5.

[5] C. M. Vithanage, J. P. Coon, and S. C. J. Parker, "On capacity-optimal precoding for multiple antenna systems subject to EIRP restrictions," *IEEE Trans. Wireless Commun.*, vol. 7, no. 12, pp. 5182–5187, Dec. 2008.

[6] C. M. Vithanage, Y. Wang, and J. P. Coon, "Transmit beamforming methods for improved received signal-to-noise ratio in equivalent isotropic radiated power-constrained systems," *IET Communications*, vol. 3, no. 1, pp. 38–47, Jan. 2009.

[7] C. M. Vithanage, M. Sandell, J. P. Coon, and Y. Wang, "Precoding in OFDM-based multi-antenna ultra-wideband systems," *IEEE Commun. Mag.*, vol. 47, no. 1, pp. 41–47, Jan. 2009.

[8] T. M. Kim, A. Ghaderipoor, and A. Paulraj, "Transmit beamforming for EIRP-limited MIMO systems based on Golay sequence," in *Proc. IEEE Global Communications Conf. (GLOBECOM)*, Dec. 2012, pp. 4798–4803.

[9] FCC, *Code of Federal Regulations, Title 47, Part 15, Subpart F*, Federal Communications Commission, Feb. 2002.

[10] ETSI, *European Standard EN 302 065 V1.3.1*, European Telecommunications Standards Institute, Apr. 2014.

[11] N. Doose and P. A. Hoeher, "Massive MIMO ultra-wideband communications using multi-mode antennas," in *Proc. Int. ITG Conf. on Systems, Communications and Coding (SCC)*, Feb. 2015, pp. 1–6.

[12] P. A. Hoeher and N. Doose, "A massive MIMO terminal concept based on small-size multi-mode antennas," *Trans. Emerging Telecommunications Technologies*, Mar. 2015. [Online]. Available: http://dx.doi.org/10.1002/ett.2934

[13] R. Martens and D. Manteuffel, "Systematic design method of a mobile multiple antenna system using the theory of characteristic modes," *IET Microwaves, Antennas & Propagation*, vol. 8, pp. 887–893, Sep. 2014.

[14] D. Manteuffel and R. Martens, "Compact multi mode multi element antenna for indoor UWB massive MIMO," *submitted to IEEE Trans. Antennas and Propagation*, 2015.

[15] T. L. Marzetta, "Noncooperative cellular wireless with unlimited numbers of base station antennas," *IEEE Trans. Wireless Commun.*, vol. 9, no. 11, pp. 3590–3600, Nov. 2010.

[16] F. Rusek, D. Persson, B. K. Lau, E. G. Larsson, T. L. Marzetta, O. Edfors, and F. Tufvesson, "Scaling up MIMO: Opportunities and challenges with very large arrays," *IEEE Signal Processing Mag.*, vol. 30, no. 1, pp. 40–60, Jan. 2013.

[17] E. G. Larsson, F. Tufvesson, O. Edfors, and T. L. Marzetta, "Massive MIMO for next generation wireless systems," *IEEE Commun. Mag.*, vol. 52, no. 2, pp. 186–195, Feb. 2014.

[18] P. Kyösti, J. Meinilä, L. Hentilä, X. Zhao *et al.*, "IST-WINNER D1.1.2: WINNER II channel models," Technical Report (ver. 1.1), Sep. 2007. [Online]. Available: http://www.ist-winner.org/WINNER2-Deliverables/D1.1.2v1.1.pdf