# Transmission of Analog Correlated Sources over MIMO Fading Channels

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***Abstract*—In this work we address the transmission of correlated Gaussian sources over Multiple Input Multiple Output fading channels using analog Joint Source Channel Coding (JSCC). The source symbols are first compressed using a continuous parametric mapping based on a sinusoidal function that exploits the source correlation. Given that the data at the encoder output is also correlated, the information corresponding to the covariance matrix is incorporated into the design of the linear transmit and receive filters. The results obtained from the simulations confirm the suitability of analog JSCC techniques for the considered scenario.**

## I. Introduction

The application of analog Joint Source Channel Coding (JSCC) techniques for the transmission of independent analog sources has been analyzed for different scenarios and communication models [1], [2], [3], [4]. These works confirm that this transmission strategy is a feasible alternative to traditional approaches based on the separation of the source and the channel coding operations. Analog JSCC has also been considered for the transmission of correlated sources, specially in the context of Wireless Sensor Networks [5], [6], [7]. Notice that the source-channel separation is suboptimal in scenarios such as the Multiple Access Channel (MAC) when the information is correlated, since the separate optimization of the source and channel encoders is not able to efficiently exploit the source correlation [5], [8], [9].

In the case of analog JSCC, the source symbols are directly encoded by using parametric space-filling curves. This encoding procedure significantly reduces the communication delay and the system complexity. In addition, analog JSCC schemes present graceful degradation for imprecise knowledge of the channel, and they can easily be adapted in time-varying environments without a complete redesign of the system. These appealing properties make analog JSCC a suitable strategy for the wireless communication of correlated data.

In this work, we address the transmission of discrete-time analog correlated symbols over fading channels using analog JSCC techniques. The utilization of multiple antennas at the transmit and receive side is considered to exploit the diversity of wireless channels and to increase the transmission rate. The main contributions of this work are summarized as follows:

- A parametric non-linear analog mapping is considered to exploit the correlation of two consecutive source symbols to produce one encoded symbol (bandwidth compression). This mapping is proposed in [10] for the transmission of correlated sources in Additive White Gaussian Noise (AWGN) channels. The advantage of parametric mappings with respect to non-parametric ones is the significant reduction of the computational cost in the coding and decoding operations. In addition, the utilization of parametric mappings enables the affordable optimization of the analog JSCC system by adapting the encoder parameters to the channel time variations.
- The proposed analog JSCC scheme for Multiple Input Multiple Output (MIMO) fading channels exhibits extremely low complexity and delay thanks to the system design based on a two-stage structure similar to the one proposed for uncorrelated sources in [11].
- The design of the transmit and receive linear filters incorporates the correlation information after the analog JSCC encoding. The transformation of the source symbols is assumed to be linear and, hence, the correlation between the symbols at the encoder output can be analytically calculated.
- The performance of the proposed analog JSCC system is evaluated over fading MIMO channels. Other well-known analog mappings are also considered to illustrate the suitability of the proposed mapping for this scenario. Finally, the obtained results are compared to the theoretical bounds given by the Optimum Performance Theoretically Attainable (OPTA).

In summary, we show that parametric analog mappings allow to efficiently exploit the correlation among the source symbols. The resulting analog JSCC system is also able to achieve high transmission rates due to the compression operation at the encoder, and the use of multiple antennas at the transmitter and the receiver. An additional advantage of this approach is the simplicity for the system optimization depending on the specific channel conditions.

## II. System Model

Let us assume a correlated analog source modeled as an autoregressive random process of order one, AR(1),

$$s_k = \rho s_{k-1} + e_k \tag{1}$$

where $\rho$ is a constant parameter, and $e_k$ is a zero-mean white Gaussian random process with variance $\sigma_\epsilon^2 = 1 - \rho^2$. In such model, the correlation between two arbitrary symbols

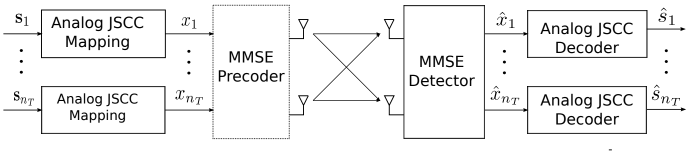


Fig. 1. Block diagram of the proposed analog JSCC system.

is $\mathbb{E}[s_i s_{i+n}] = \rho^n$. Hence, two consecutive source symbols, $\mathbf{s}_i = [s_{2i}, s_{2i+1}]^T$, follow a bivariate Gaussian distribution with zero-mean and covariance matrix

$$\mathbf{C}_\mathbf{s} = \mathbb{E}\left[\mathbf{s}_i \mathbf{s}_i^T\right] = \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}.$$

Figure 1 shows the block diagram of the proposed analog JSCC system for the transmission of correlated source symbols over MIMO fading channels. As shown in the figure, the transmitter and the receiver are equipped with $n_T$ and $n_R$ antennas, respectively. At the $i$-th transmit antenna, two consecutive source symbols $\mathbf{s}_i$ are encoded into one symbol $x_i$ using a 2:1 analog JSCC mapping, i.e. $x_i = M_i(\mathbf{s}_i)$, where $M_i(\cdot) : \mathbb{R}^2 \to \mathbb{R}$ is the mapping function at the $i$-th antenna. The mapping $M_i(\mathbf{s}_i)$ is in general a non-linear function because it is based on the use of space-filling curves. As explained in Section IV, the analog JSCC mappings must be designed to exploit the correlation between the source symbols to be compressed.

After the encoding operation, the resulting vector of $n_T$ symbols $\mathbf{x} = [x_1, \ldots, x_{n_T}]^T$ is precoded and sent over a MIMO fading channel. The received signal is hence given by

$$\mathbf{y} = \mathbf{H}\mathbf{P}\mathbf{x} + \mathbf{n}, \quad (2)$$

where $\mathbf{H}$ is the MIMO channel response matrix, $\mathbf{P}$ is the precoding matrix and $\mathbf{n}$ is the AWGN with $\mathbf{n} \sim \mathcal{N}_\mathbb{C}(0, \sigma_n^2 \mathbf{I}_{n_R})$. The precoder $\mathbf{P}$ is designed to satisfy a total transmit power constraint $P_T$, hence the Signal-to-Noise Rate (SNR) is $\eta = P_T/\sigma_n^2$. For simplicity, along this paper the transmit power is assumed to be $P_T = 1$.

At the receiver, the vector of $n_R$ observed symbols is employed to calculate an estimate of the source symbols. MMSE decoding is optimum for analog JSCC given that it minimizes the distortion between source and decoded symbols. Nevertheless, the analog mapping involves non-linear transformations at the encoder and, hence, the calculation of the MMSE estimates requires the numerical computation of complicated integrals.

A low-complexity alternative is the concatenation of a linear MMSE filter and a Maximum Likelihood (ML) decoder, as proposed in [11] for the analog JSCC transmission of independent sources. In such case, a linear MMSE estimate of the transmitted symbols is obtained as follows

$$\hat{\mathbf{x}} = \mathbf{W}\mathbf{y} = \mathbf{W}\mathbf{H}\mathbf{P}\mathbf{x} + \mathbf{W}\mathbf{n}, \quad (3)$$

where $\mathbf{W}$ is the linear MMSE receive filter

$$\mathbf{W} = \mathbf{C}_x \mathbf{P}^H \mathbf{H}^H (\mathbf{H}\mathbf{P}\mathbf{C}_x \mathbf{P}^H \mathbf{H}^H + \sigma_n^2 \mathbf{I}_{n_R})^{-1}, \quad (4)$$

with $\mathbf{C}_x$ representing the covariance matrix of the encoded symbols. The covariance of the estimation error is

$$\mathbf{C}_e = \mathbf{C}_x - \mathbf{C}_x \mathbf{P}^H \mathbf{H}^H \left(\mathbf{H}\mathbf{P}\mathbf{C}_x \mathbf{P}^H \mathbf{H}^H + \sigma_n^2 \mathbf{I}_{n_R}\right)^{-1} \mathbf{H}\mathbf{P}\mathbf{C}_x. \quad (5)$$

If no Channel State Information (CSI) is available at the transmitter, the optimum precoder is $\mathbf{P}' = 1/\sqrt{n_T}\mathbf{I}_{n_T}$ and the linear MMSE detector simplifies to

$$\mathbf{W}' = (\mathbf{H}^H \mathbf{H} + n_T \sigma_n^2 \mathbf{C}_x^{-1})^{-1} \mathbf{H}^H, \quad (6)$$

An estimate of the source symbols $\hat{\mathbf{s}}_i$ is finally determined from the filtered symbols $\hat{\mathbf{x}}$ by using the corresponding ML decoder.

### A. Covariance of the Encoded Symbols

When the sources are correlated, the encoded symbols after the mapping operation are also correlated. This correlation specifically depends on the covariance matrix of the source symbols $\mathbf{C}_\mathbf{s}$, and the analog JSCC mapping employed at the encoding operation. The optimum transmit and receive filters can be hence designed to exploit the correlation between the encoded symbols with the aim of minimizing the expected distortion.

A first estimation for the covariance matrix of the encoded symbols $\mathbf{C}_\mathbf{x} = \mathbb{E}[\mathbf{x}\mathbf{x}^H]$ is obtained by approximating the non-linear analog mappings $M_i(\cdot)$ to linear transformations of the form $x_i = k(s_{2i} + s_{2i+1})$, where $k$ is a factor to guarantee that $\mathbb{E}[x_i^2] = 1 \ \forall i$. Using this linear approximation, the resulting correlation between any two encoded symbols $x_i$ and $x_j$ is $[\mathbf{C}_\mathbf{x}]_{ij} = \mathbb{E}[x_i x_j] = 1/2(\rho^{2|i-j|-1}\rho^{2|i-j|})$. As an example, the correlation matrix $\mathbf{C}_\mathbf{x}$ for the case of $n_T = 4$ transmit antennas is

$$\mathbf{C}_\mathbf{x} = \frac{1}{2}\begin{bmatrix} 1 & \rho+\rho^2 & \rho^3+\rho^4 & \rho^5+\rho^6 \\ \rho+\rho^2 & 1 & \rho+\rho^2 & \rho^3+\rho^4 \\ \rho^3+\rho^4 & \rho+\rho^2 & 1 & \rho+\rho^2 \\ \rho^5+\rho^6 & \rho^3+\rho^4 & \rho+\rho^2 & 1 \end{bmatrix}.$$

From simulations we have observed that this approach provides accurate estimates of the actual correlation. In fact, the error observed between the linear approximations and the empirical correlation of the encoded symbols is about $10^{-2}$ for different values of $\rho$ in the range of considered SNRs. As an alternative, the Unscented Transform (UT) [12] has been employed to model the effect of the non-linear transformations of the analog mappings in the symbol correlation. The correlation values for $\mathbf{C}_\mathbf{x}$ and the error estimates obtained by using the UT approach are closely similar to that of the linear approximation.

## III. Linear MMSE Precoding

Let us now consider that the CSI is available at transmission and reception. In this case, CSI knowledge can be exploited to design a linear MMSE precoder to improve the system performance.

The linear transmit and receive filters are designed to minimize the MSE between the transmitted symbols $\mathbf{x}$ and the estimates $\hat{\mathbf{x}}$. The error vector is given by

$$\mathbf{e} = \mathbf{x} - \hat{\mathbf{x}} = \mathbf{x} - \mathbf{WHPx} + \mathbf{Wn}, \quad (7)$$

and, therefore, the transmit and receive filters $\mathbf{P}$ and $\mathbf{W}$ are calculated by solving

$$\arg\min_{\mathbf{P},\mathbf{W}} \mathbb{E}\Big[\mathrm{tr}\big(\mathbf{e}\mathbf{e}^H\big)\Big] \qquad \text{s.t.} \quad \mathrm{tr}\big(\mathbf{P}\mathbf{C}_x\mathbf{P}^H\big) = 1, \quad (8)$$

where $\mathrm{tr}\big(\cdot\big)$ represents the trace operator. This problem can be solved by differentiating this MSE expression with respect to $\mathbf{P}^H$ and $\mathbf{W}^H$. The resulting expressions can be used to obtain the filters $\mathbf{P}$ and $\mathbf{W}$ following an alternating approach.

Alternatively, a lower complexity solution can be found by following an approach similar to [13], [14] but incorporating the transmitted symbols correlation information into the derivation of the optimum filter expressions [15]. Let us consider the Single Value Decomposition (SVD) of the channel as $\mathbf{H} = \mathbf{U}_h\mathbf{\Sigma}_h\mathbf{V}_h^H$ and the SVD of the covariance matrix $\mathbf{C}_x = \mathbf{U}_x\mathbf{\Sigma}_x\mathbf{V}_x^H$. Assuming the optimum linear MMSE filters are of the form $\mathbf{P} = \mathbf{V}_h\mathbf{T}\mathbf{U}_x^H$ and $\mathbf{W} = \mathbf{U}_s\mathbf{D}\mathbf{U}_h^H$, where the matrices $\mathbf{T}$ and $\mathbf{D}$ are diagonal, the optimization problem (8) can be reformulated as follows

$$\begin{aligned} \arg\min_{\mathbf{D},\mathbf{T}} \quad & \mathrm{tr}\Big(\mathbf{\Sigma}_x + \mathbf{D}\mathbf{\Sigma}_h\mathbf{T}\mathbf{\Sigma}_x\mathbf{T}^H\mathbf{\Sigma}_h^H\mathbf{D}^H \\ & + \sigma_n^2\mathbf{D}\mathbf{D}^H - 2\Re\{\mathbf{D}\mathbf{\Sigma}_h\mathbf{T}\mathbf{\Sigma}_x\}\Big) \\ \text{s.t.} \quad & \mathrm{tr}\big(\mathbf{T}\mathbf{\Sigma}_x\mathbf{T}^H\big) = 1. \end{aligned} \quad (9)$$

Since the problem is expressed as the product of diagonal matrices, the Lagrangian cost function can be written as

$$\mathcal{L} = \sum_{i=1}^{L} \lambda_{x,i}(d_i t_i \lambda_{h,i} - 1)^2 + \sigma_n^2 d_i^2 + \Delta(\sum_{i=1}^{L} t_i^2\lambda_{x,i} - 1), \quad (10)$$

where $d_i$ and $t_i$ are the diagonal elements of $\mathbf{D}$ and $\mathbf{T}$, respectively; $\Delta \geq 0$ is a Lagrange multiplier; and $\lambda_{x,i}$ and $\lambda_{h,i}$ are the eigenvalues of the source covariance matrix and the channel, respectively. Thus, $\mathbf{\Sigma}_x = \{\lambda_{x,1}, \lambda_{x,2}, \ldots, \lambda_{x,n_T}\}$ and $\mathbf{\Sigma}_h = \{\lambda_{h,1}, \lambda_{h,2}, \ldots, \lambda_{h,L}\}$, with $L$ the number of non-zero channel eigenvalues. The solutions for $d_i$ and $t_i$ are given by

$$d_i^2 = \frac{1}{\lambda_{h,i}^2}\left[\lambda_{h,i}\sqrt{\frac{\lambda_{x,i}\Delta}{\sigma_n^2}} - \Delta\right]^+ \quad (11)$$

$$t_i^2 = \frac{1}{\lambda_{h,i}^2}\left[\lambda_{h,i}\sqrt{\frac{\sigma_n^2}{\lambda_{x,i}\Delta}} - \frac{\sigma_n^2}{\lambda_{x,i}}\right]^+, \quad (12)$$

The operator $[\cdot]^+$ takes the positive arguments and sets negative arguments to zero.

Substituting (12) into the power constraint, the following value is obtained for the Lagrange multiplier

$$\Delta = \frac{1}{\sigma_n^2}\left(\frac{\sum_{k=1}^{L^*}\frac{\sqrt{\lambda_{x,k}}}{\lambda_{h,k}}}{\eta + \sum_{k=1}^{L^*}\frac{1}{\lambda_{h,1}^2}}\right)^2. \quad (13)$$

Finally, substituting this value for $\Delta$ into (11) and (12), we find the following solution for the diagonal matrices $\mathbf{T}$ and $\mathbf{D}$

$$d_i = \sqrt{\frac{1}{\sigma_n^2}A_i\left[\sqrt{\lambda_{x_i}} - A_i\right]^+} \quad (14)$$

$$t_i = \sqrt{\frac{\sigma_n^2}{\lambda_{h,i}^2}\left[\frac{1}{A_i\lambda_{x,i}} - \frac{1}{\lambda_{x_i}}\right]^+} \quad (15)$$

where

$$A_i = \frac{\frac{1}{\lambda_{h,i}}\sum_{k=1}^{L^*}\frac{\lambda_{x,k}}{\lambda_{h,k}}}{\eta + \sum_{k=1}^{L^*}\frac{1}{\lambda_{h,k}^2}}. \quad (16)$$

The number $L^* \leq L$ refers to the number of singular values whose corresponding expressions for $d_i$ or $t_i$ are non-zero. The solution previously described resembles that obtained for the case of uncorrelated inputs in [14], but including the eigenvalues of the source covariance matrix. Equivalently to [14], it can also be observed that the obtained solution resembles the traditional waterfilling algorithm in the sense that it provides the optimal distribution of the transmit power among the data streams that minimizes the MSE.

## IV. Analog JSCC Mapping

Let us focus on the 2:1 compression of the source information. For this scenario, we propose parametric non-linear analog mappings based on a sinusoidal function. This type of analog JSCC mappings is chosen after examining the optimal non-parametric mappings obtained by following an approach similar to [16] for the case of correlated sources. The use of sinusoidal mappings is also supported by the results obtained in [17], where the design of Power Constrained Channel Optimized Vector Quantizers is addressed for the same scenario. Notice that this type of encoders can be interpreted as a discrete version of the optimal analog mappings.

The analog JSCC mapping directly transforms two correlated source symbols $\mathbf{s}_i = [s_{2i}, s_{(2i+1)}]^T$ into one encoded symbol $x_i$. Let $\mathbf{C}_s = \mathbf{U}^H\mathbf{\Sigma}\mathbf{U}$ be the eigendecomposition of the source covariance matrix. The proposed mapping is based on the space-filling curves defined by the following parametric expression:

$$\mathbf{K}_i(t) = \mathbf{U}\mathbf{\Sigma}\begin{bmatrix} t - \frac{1}{2\alpha_i}\sin(\alpha_i t) \\ \Delta_i\sin(\alpha_i t) \end{bmatrix}, \quad (17)$$

where $\mathbf{K}_i(t)$ represents a point into the bidimensional source space given a parameter $t$ in the one-dimensional channel space. The parameters $\alpha_i$ and $\Delta_i$ represent the frequency and the amplitude of the sinusoidal function, respectively, for the $i$-th transmit antenna. The optimal values for these parameters depend on the source correlation and the SNR value. An

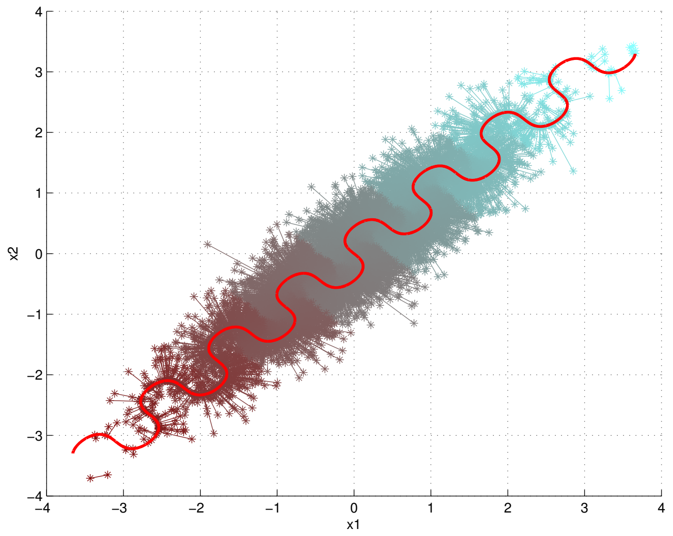


Fig. 2. Proposed 2:1 analog JSCC mapping for SNR = 25 dB for $\rho = 0.9$.

adequate optimization of $\alpha_i$ and $\Delta_i$ is important to closely approach the optimal cost-distortion tradeoff.

Besides the parametric curve given by (17), it is necessary to define a function $M_i(\mathbf{s}_i)$ that specifies the mapping of the points in the source space into the corresponding point in the parametric curve. In this case, the mapping function is

$$x_i = M_i(\mathbf{s}_i) = \arg\min_t \int_{-\infty}^{\infty} \|\mathbf{s}_i - \mathbf{K}_i(u)\|^2 p_n(u-t)\mathrm{d}u, \tag{18}$$

where $p_n(n)$ represents the probability density function of the noise. If the noise distribution is disregarded, i.e. $p_n(n) = \delta(n)$, the mapping function reduces to the minimum Euclidean distance.

Figure 2 shows the specific analog JSCC mapping for $\rho = 0.9$ and $\mathrm{SNR} = 25\,\mathrm{dB}$. As observed, the red curve corresponds to the sinusoidal function given by (17) with the optimal parameters $\alpha_i$ and $\Delta_i$ for that SNR and source correlation. The point cloud around the curve represent the bivariate source symbols $\mathbf{s}_i$. The figure also shows how the correlated Gaussian symbols are mapped to the corresponding point on the curve according to (18). Finally, the different colour-schemes represents the variation of the encoded values given by the curve parameter $t$.

At the receiver, an estimate of the source symbols transmitted over the $i$-th antenna is computed using a two-stage decoder [11]. First, a linear MMSE estimate of the transmitted symbols is obtained with (4) or (6), and then the Maximum Likelihood (ML) decoder is applied to the resulting estimates, thus

$$\hat{\mathbf{s}}_i = \mathbf{K}_i(\hat{x}_i) = \mathbf{K}_i([\mathbf{W}\mathbf{y}]_i).$$

As already mentioned, the value of the parameters $\alpha_i$ and $\Delta_i$ can be optimized depending on the SNR and $\rho$ values to improve the system performance. In the case of fading channels, the effective SNRs should be estimated at the receiver and fed back to the transmitter. Thereby, the encoder may adapt the mapping parameters to the channel fluctuations. The effective SNRs are estimated by using the covariance matrix of the error. Hence, the estimation of the SNRs per antenna can be obtained from (5) as

$$\hat{\boldsymbol{\eta}} = \mathrm{diag}\left(\mathbf{C}_e^{-1}\right), \tag{19}$$

where the operator $\mathrm{diag}(\cdot)$ provides a vector with the diagonal elements of the input matrix.

## V. OPTA Calculation

The performance of analog communications is measured in terms of the Signal-to-Distortion Rate (SDR) with respect to the SNR. The SDR is defined as

$$\mathrm{SDR[dB]} = 10\log_{10}(\sigma_s^2/\xi),$$

where the term $\xi = \frac{1}{M}\sum_{i=1}^{M}\mathbb{E}\left[\|\hat{s}_i - s_i\|^2\right]$ represents the MSE between the source and the estimated symbols, and $\sigma_s^2$ is the source variance.

It is interesting to compare the performance of an analog communication system to the corresponding optimal cost-distortion tradeoff, referred to as the Optimum Performance Theoretically Attainable (OPTA). In general, this bound is calculated by equating the rate distortion of the source and the channel capacity [18].

For multivariate Gaussian sources and the MSE as the distortion criterion, the rate distortion function can be represented parametrically as [19]

$$D(\theta) = \frac{1}{M}\sum_{i=1}^{M}\min[\theta, \lambda_{s,i}],$$
$$R(\theta) = \frac{1}{M}\sum_{i=1}^{M}\max\left[0, \frac{1}{2}\log\left(\frac{\lambda_{s,i}}{\theta}\right)\right], \tag{20}$$

where $D(\theta)$ is the distortion function, $\lambda_i$ represent the eigenvalues of the covariance matrix and $M$ is the source dimension. Notice that the analog JSCC system transmits $2n_T$ source symbols per channel use, hence $M$ is actually $2n_T$. In this case, the covariance matrix for $2n_T$ consecutive symbols generated by an AR(1) process is

$$\tilde{\mathbf{C}}_\mathbf{s} = \begin{bmatrix} 1 & \rho & \rho^2 & \cdots & \rho^{2n_T-1} \\ \rho & 1 & \rho & \cdots & \rho^{2n_T-2} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \rho^{2n_T-1} & \rho^{2n_T-2} & \cdots & \rho & 1 \end{bmatrix}.$$

On the other hand, the capacity of an $n_T \times n_R$ MIMO system is [20]

$$\mathrm{C}(\mathbf{H}) = \log\det\left(\mathbf{I}_{n_R} + \frac{1}{\sigma_n^2}\mathbf{H}\mathbf{P}\mathbf{C}_\mathbf{x}\mathbf{P}^H\mathbf{H}^H\right). \tag{21}$$

Notice that the capacity given by (21) is maximized when the precoder is designed according to the waterfilling solution. When the channel is unknown at the transmitter, the optimal power allocation consists in distributing the available power

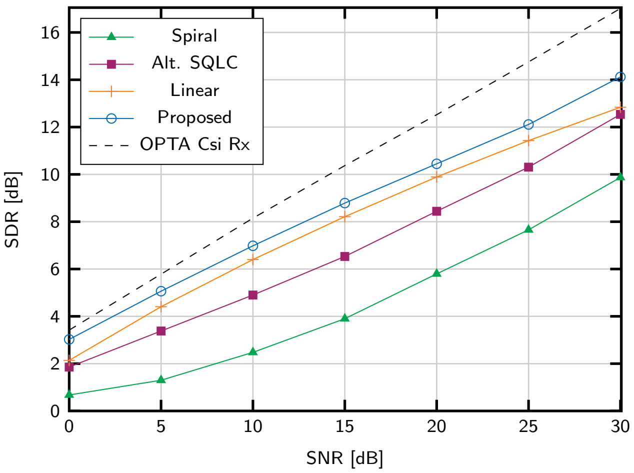


Fig. 3. Performance of different analog JSCC mappings for $2 \times 2$ MIMO channels with CSI available at the receiver and $\rho = 0.9$.

uniformly among the transmit antennas and, hence, equation (21) is also applicable with $\mathbf{P} = 1/\sqrt{n_T}\mathbf{I}_{n_T}$.

Equating (20) and (21), solving for the distortion function $D(\theta)$ and, finally, calculating the mathematical expectation of the resulting expression, we determine the expected minimum achievable distortion or, equivalently, the optimal performance depending on the considered SNR.

## VI. Simulation Results

In this section, the results of several computer simulations are presented to illustrate the performance of the proposed analog JSCC system for the transmission of correlated information over MIMO channels. In particular, we focus on $n_T \times n_R$ spatially white Rayleigh fading channels $\mathbf{H}$, such that $\mathbb{E}\left[\text{tr}\left(\mathbf{H}\mathbf{H}^H\right)\right] = n_R n_T$. At the transmitter, each pair of source symbols is encoded by using the parametric analog mapping described in Section IV. The optimal values of the encoder parameters $\alpha_i$ and $\Delta_i$ are determined for each transmit antenna $i$ from the covariance matrix of the source and the estimate of the per-antenna effective SNRs. At the receiver, an estimate of the source symbols is obtained by using the two-stage receiver based on the concatenation of the linear MMSE filter and the ML decoder. Finally, the average distortion between the source and decoded symbols is computed.

We start considering a $2\times2$ MIMO system with a correlation factor $\rho = 0.9$ for the source symbols. The MIMO channel is assumed to be known at the receiver, but it is not available at the transmitter. In this scenario, the performance obtained for the proposed analog mapping based on sinusoidal curves is compared to the performance of other three analog JSCC mappings that can also be applied to this communication model:

- Linear encoding: the source symbols are linearly transformed as $x_i = \kappa(s_{2i} + s_{2i+1})$, where $\kappa$ is the normalization factor. For normalized bivariate Gaussian sources, $\kappa = 1/\sqrt{2(1+\rho)}$.
- Spiral-like mappings based on a doubly intertwined Archimedean spiral [1], [21]. This analog mapping is traditionally employed for the 2:1 compression of independent sources. In the case of correlated sources, the source samples are first decorrelated and then encoded with this mapping.
- Alternating Scalar Quantizer Linear Coder (SQLC) that was proposed in [22] for the transmission of correlated sources in Gaussian Broadcast Channels (BC). This mapping extends the SQLC [6], exploiting the fact that the source symbols are jointly mapped at the transmitter. In this scenario, it is required to define a projection matrix $\mathbf{H}$, which has been assumed to be $\mathbf{H} = \mathbf{U}\mathbf{\Sigma}$.

Figure 3 shows the SDR curves for the proposed mapping and for the three alternative analog mappings. The optimal distortion-cost tradeoff given by the corresponding OPTA is also included in the figure. As observed, the best performance corresponds to the proposed sinusoidal mapping for all range of SNRs. The performance gain of the proposed mapping ranges from 0.5 dB to 1.5 dB with respect to the linear encoding, and from 1.2 dB to 2.2 dB in the case of the alternating SQLC. As expected, the worst performance is obtained with the Archimedean spiral, because it does not exploit the correlation between the source symbols.

In addition, the sinusoidal mapping closely approaches the OPTA bound for low SNRs, although the gap between both curves grows as the SNR is larger (almost 3 dB for SNR = 30 dB). A possible cause of this performance loss is the sub-optimality of the two-stage receiver, since it exploits the correlation of the $2n_T$ transmitted source symbols only partially. Notice that the impact of the individual ML decoding of each pair of source symbols is larger as the noise level is lower and the number of transmit antennas increases. This problem can be solved by using the optimal MMSE estimator to jointly decode the $2n_T$ source symbols . However, this strategy requires a discretization of the $2n_T$-dimensional source space to numerically compute the associated integrals. Hence the practical implementation of an MMSE decoder that provides accurate estimations is unaffordable even for small values of $n_T$.

Figure 4 shows the performance of the proposed analog JSCC system (sinusoidal mapping) for a $2 \times 2$ MIMO system, source correlation $\rho = 0.9$, and three different filtering strategies: 1) a linear MMSE receive filter without exploiting the correlation of the encoded symbols, that is, assuming $\rho = 0$, 2) the receive filter exploits such a correlation, and 3) a linear MMSE precoder at transmission. The OPTA curves corresponding to the cases of CSI only available at reception and full CSI are also plotted in the figure. As expected, the worst performance corresponds to the case of linear MMSE receive filtering for uncorrelated sources. The exploitation of the correlation into the design of the receive filters improves the system performance, specially for low and medium SNRs. In the high SNR region, the correlation factor present in (6) is less significant because it is weighted by the noise variance. The linear MMSE precoder described in Section III

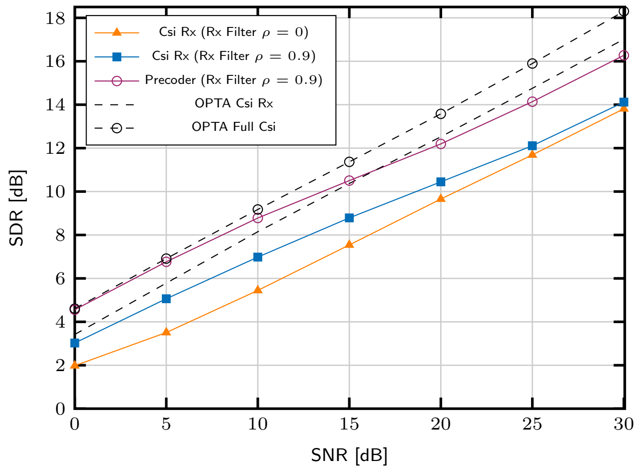


Fig. 4. Performance of the proposed analog JSCC systems for $2 \times 2$ MIMO channels with CSI at the receiver and full CSI, and $\rho = 0.9$.

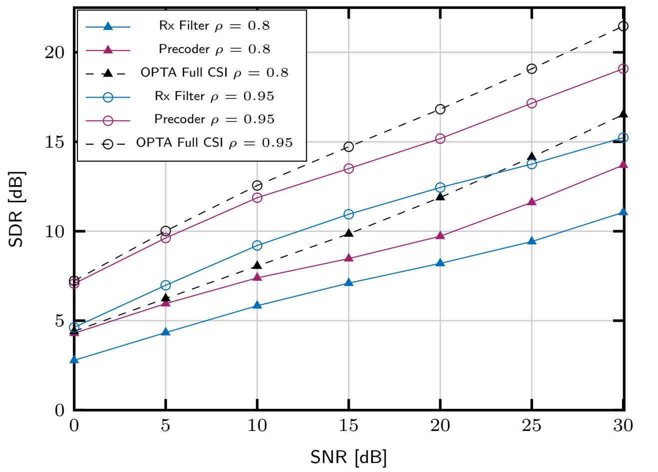


Fig. 6. Performance of the analog JSCC scheme for $\rho = 0.8$ and $\rho = 0.95$ in MIMO $4 \times 4$.

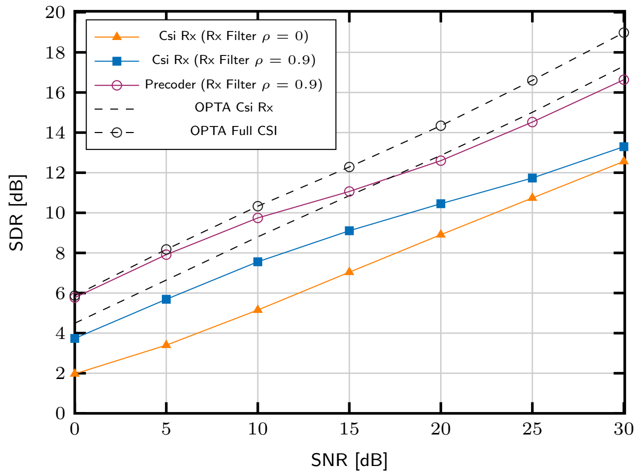


Fig. 5. Performance of the proposed analog JSCC systems for $4 \times 4$ MIMO channels with CSI at the receiver and full CSI, and $\rho = 0.9$.

significantly outperforms the two previous strategies thanks to exploiting the source correlation together with the channel information at the transmitter. In addition, the gap between the performance of the analog JSCC system with precoding and the OPTA corresponding to the case of full CSI is smaller than that of only CSI at reception (2 dB and 3 dB, respectively).

Figure 5 shows the same performance curves for $4 \times 4$ MIMO channels and identical source correlation $\rho = 0.9$. As observed, conclusions similar to those of the $2 \times 2$ case can be drawn. As previously pointed out, increasing the number of transmit antennas penalizes the performance of analog JSCC with linear receive filter for high SNRs because the correlation of the $2n_T$ symbols is not completely exploited by the two-stage receiver. As expected, this degradation is more noticeable when using the linear MMSE filter for uncorrelated sources. In both cases, the gap with the precoding approach increases to 3 dB and 4 dB, respectively. Moreover, the difference between the system performance with precoding and the OPTA remains about 2 dB for high SNRs, like in $2 \times 2$ MIMO channels. Thus, the utilization of a linear MMSE precoder mitigates the impact of using the sub-optimal two-stage decoding strategy.

The influence of the source correlation on the performance of the analog JSCC system is illustrated in Figure 6. In particular, we consider two different values for the source correlation: $\rho = 0.8$ and $\rho = 0.95$. The performance gain with linear precoding is greater for $\rho = 0.95$, specially for medium and high SNRs. In particular, at 30 dB the gain is 4 dB for $\rho = 0.95$ and about 2.2 dB for $\rho = 0.85$. Similarly, the gap between the performance of the analog system with linear precoding and the OPTA bound is reduced as the correlation factor increases.

According to these results, it is reasonable to conclude that the combination of sinusoidal mappings and the proposed linear MMSE precoder is able to efficiently exploit the source correlation to improve the system performance, even for a large number of transmit antennas. This idea is specially interesting for the practical implementation of analog JSCC systems that achieve high transmission rates with affordable complexity and low delay when transmitting correlated sources over MIMO channels.

## VII. Conclusions

In this work, we have addressed the transmission of correlated Gaussian sources over MIMO fading channels using analog JSCC. We considered the 2:1 compression of correlated sources, using an analog mapping based on sinusoidal functions. The utilization of multiple antennas at both transmission and reception allows to increase the system throughput. The structured design of the proposed system preserves the main advantages of analog JSCC communications, namely, low complexity, negligible latency and robustness against time variations of fading channels. According to this idea, we have designed those linear transmit and receive filters that minimize the signal distortion considering the specific correlation at the encoded symbol vectors to be transmitted. The results obtained in the simulations confirm the utility of analog JSCC techniques to achieve a satisfactory performance with low complexity and delay in the considered scenario.

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