Achievable Rate Performance of TDD Multi-cell Massive MIMO with Non-Orthogonal Pilots

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***Abstract*—Massive multiple-input multiple-output (MIMO) is a promising technology that provides high system capacity, and with excellent performance in energy efficiency and system robustness. However, pilot contamination restricts the system performance due to the limited length of orthogonal training sequences. In this paper, we propose optimization of non-orthogonal pilots for uplink training. The Grassmannian line packing method is applied in the design of pilots that leads to maximum chordal distances between training sequences which reduces pilot contamination. The analytical expression of the user equipments (UEs) signal-to-intereference-plus-noise ratio (SINR) in multi-cell massive MIMO system and the analytical SINR of UEs in Grassmannian manifold are presented. Simulation results show shown that non-orthogonal pilot sequences with the Grassmainn line packing method yield significant improvements of the system capacity.**

***Index Terms*—Massive MIMO, TDD, non-orthogonal pilots, Grassmannian line packing**

I. Introduction

Massive multiple-input-multiple-output (MIMO) system is an advanced wireless communication technology that has attracted great interest from both researchers and industrialists in recent years. The deployment of a large-scale array of antennas (tens or even hundreds of antennas) at the base station (BS) is desired to support increased numbers of user equipments (UEs) in the same time-frequency resource. It is shown that both spectrum and energy efficiency can be greatly improved through multiplexing gain and the pairwise orthogonal channel property [1]–[3]. Furthermore, the complexity of both detector and precoder designs can be much simplified.

However, the limited length of the coherence interval may cause *pilot contamination*, which seriously affects estimation of the channel state information (CSI) and consequently degrades the system performance. One solution is to reuse orthogonal pilots between UEs or cells and many novel reuse schemes have been proposed to reduce this effect. In [4] a precoding method was proposed for the multi-cell MIMO system to reduce the pilot contamination effect. An orthogonal pilot reuse scheme was proposed in [5] with reuse factor of seven. The pilot contamination was mitigated at the cost of increasing the sample duration of training sequences. In [6], the UEs are divided into cell-centre and cell-edge groups. Different pilot orthogonal pilots are assigned to these two groups which leads to a reduction of both the pilot contamination and the pilot sample duration.

In contrast with pilot reuse schemes, in this paper the focus is on the Grassmannian line packing (GLP) based pilot scheme. Although in [7], the GLP-based pilot sequence design is proposed to reduce contamination, the discussion relates to the estimation error performance and does not provide the theoretical analysis of pilot length on the throughput. The authors compare the throughput performance of different pilot schemes [8], but the study is limited to a single-cell scenario. In this paper we focus on evaluating the downlink achievable rate performance of a time-division-duplex (TDD) multi-cell massive MIMO system with the GLP based pilot scheme. The closed-form achievable sum rate expression for the multi-cell system is presented and comparisons are made with the conventional pilot reuse scheme. The numerical results show that the GLP-based pilot design significantly outperforms the pilot reuse scheme and it brings large gains in the system throughput.

The rest of the paper is organized as follows. In section II, an description of the multi-cell massive MIMO system model is given. In section III, we list the proposed analytical expressions and description of the Grassmannian line packing problem. The simulation results and discussions are given in section IV. Finally, the conclusion of the paper is given in section V.

Notation: The bold font notation is applied to represent matrix or vector. To a matrix $\mathbf{A}$, $\mathbf{A}^{\mathrm{T}}$ denotes its transpose, $\mathbf{A}^{\mathrm{H}}$ denotes its Hermitian transpose and $\mathbf{A}^{*}$ represents its conjugate transpose. $\mathcal{CN}(x, y)$ denotes the complex normal distribution with x as mean and y as variance, $\mathrm{diag}(\cdot)$ denotes the diagonal matrix, $\mathbb{E}[\cdot]$ and $\mathrm{Var}\{\cdot\}$ represents expection and variance, respectively. $\mathbf{I}_M$ is $\mathbf{M}$-dimensional identity matrix. $\otimes$ is Kronecker product.

II. System model

We consider a multi-cell massive MIMO cellular network with L cells which is illustrated in Fig.1. Each cell includes a central BS equipped with M antennas serving K single-antenna UEs where $M \gg K$. We consider a block-fading channel and the channel vector from the k-th UE in the i-th cell to the BS in the j-th cell can be represented as

$$\mathbf{g}_{k,i,j} = \sqrt{\beta_{k,i,j}}\mathbf{h}_{k,i,j}, \tag{1}$$

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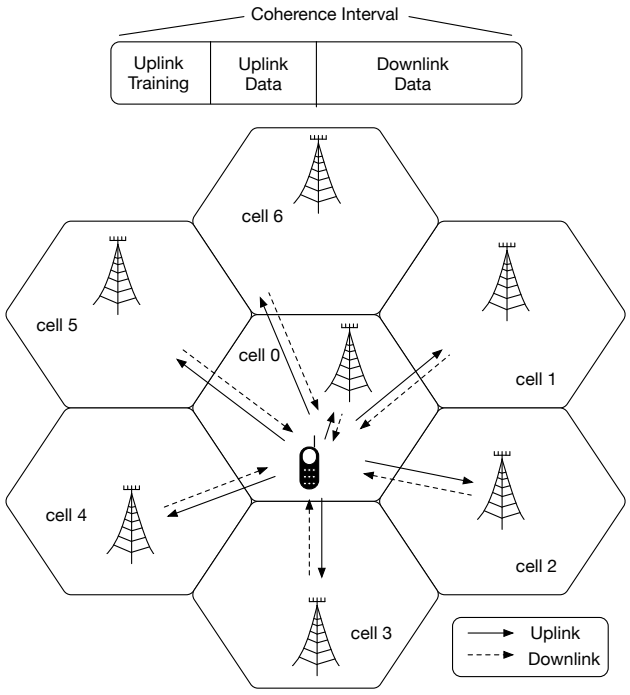

Fig. 1. A multi-cell massive MIMO system with signal antenna UE

where $\beta_{k,i,j}$ represents the large-scale fading coefficient that models the effect of path-loss and shadowing; $\mathbf{h}_{k,i,j}$ represents the small-scale fading and vector contains independent and identically distributed (i.i.d) random variables where $\mathbf{h}_{k,i,j} \in \mathcal{CN}(0, \mathbf{I}_M)$. The coherence interval is also illustrated in Fig.1, which can be divided into three parts: uplink training, uplink data transmission and downlink data transmission. With the coherence bandwidth B_c Hz and T_s seconds as the coherence time, the number of samples T_c equals to $T_c = T_s \times B_c$. It is assumed that the channel model $\sqrt{\beta_{k,i,j}}\mathbf{h}_{k,i,j}$ remains constant during one coherence interval.

A. Uplink Training

TDD operation is considered in this paper. The transmission starts with UEs sending their own training sequences to their BS simultaneously. To the UE i of the j-th cell, each UE will be assigned a pilot sequence vector which is denoted by $\mathbf{s}_{i,j}$. The $\mathbf{s}_{i,j} \in \mathbb{C}^{\tau \times 1}$ has a length of τ samples and $\mathbb{E}[|\mathbf{s}_{i,j}|^2] = 1$. The correlation coefficient of pilot sequences between the UE i and the UE k in the jth-cell is described as

$$\rho^2_{i,k,j} \triangleq |\mathbf{s}^{\mathrm{H}}_{i,j}\mathbf{s}_{k,j}|^2, \tag{2}$$

where $\rho^2_{i,k,j} \in [0, 1]$. To the BS in cell j, it is assumed to receive the uplink training sequences from the UEs in both cell j and the other $L-1$ cells, which can be represented as

$$\begin{aligned}
&\mathbf{y}_j \\
&= \underbrace{\sum_{k=1}^{K}\sqrt{p_u\beta_{k,j,j}}\mathbf{S}_{k,j,j}\mathbf{h}_{k,j,j}}_{\text{cell } j} + \underbrace{\sum_{l\neq j}^{L}\sum_{k=1}^{K}\sqrt{p_u\beta_{k,l,j}}\mathbf{S}_{k,l,j}\mathbf{h}_{k,l,j}}_{\text{other } L-1 \text{ cells}} + \mathbf{n}_j \\
&= \sum_{l=1}^{L}\sum_{k=1}^{K}\sqrt{p_u\beta_{k,l,j}}\mathbf{S}_{k,l,j}\mathbf{h}_{k,l,j} + \mathbf{n}_j,
\end{aligned} \tag{3}$$

where $\mathbf{y}_j \in \mathbb{C}^{\tau M \times 1}$ denotes the received pilot vector; p_u denotes the uplink transmission power; $\mathbf{n}_j \in \mathbb{C}^{\tau M \times 1}$ denotes matrix of additive white Gaussian noise (AWGN) at the BS with elements $n_{M,k,j} \in \mathcal{CN}(0, \sigma_n^2\mathbf{I}_M)$; $\mathbf{S}_{k,l,j}$ represents the matrix with pilot sequence and $\mathbf{S}_{k,l,j} = s_{k,l,j} \otimes \mathbf{I}_M$. Once the BS received the uplink training sequences, least squared (LS) algorithm is utilized for the channel estimation. The estimated CSI can be expressed as

$$\begin{aligned}
&\hat{\mathbf{h}}_{i,j,j} = \mathbf{S}^{\mathrm{T}}_{i,j,j}\mathbf{y}_j \\
&= \sqrt{\beta_{i,j,j}}\mathbf{h}_{i,j,j} + \sum_{k\neq i}^{K}\rho_{i,k,j}\mathbf{h}_{k,j,j} + \sum_{l\neq j}^{L}\sum_{k=1}^{K}\rho_{i,k,l}\mathbf{h}_{k,l,i} + \mathbf{S}^{\mathrm{T}}_{i,j,j}\mathbf{n}_j.
\end{aligned} \tag{4}$$

Equation (4) indicates that the correlation between pilots has a significant influence on the accuracy of the estimation. If both $\rho_{i,k,j}$ and $\rho_{i,k,l}$ are equal to 0, the estimated result will be the desired CSI corrupted only by noise. However, as mentioned above pilot contamination commonly exists in the massive MIMO systems. As a result, the interference between pilots might result in non-zero correlation coefficients which will reduce the estimation accuracy.

After uplink training, the UEs start the data transmission and the estimated CSI will be utilized in the BS for signal detection. The received uplink data at the j-th cell is given as

$$\mathbf{y}_{j,data} = \sum_{l=1}^{L}\sum_{k=1}^{K}\sqrt{p_u\beta_{k,l,j}}\mathbf{x}_{k,l,j}\mathbf{h}_{k,l,j} + \mathbf{n}_d, \tag{5}$$

where $\mathbf{y}_{j,data}$ denotes the received uplink data by the BS in the j-th cell; $\mathbf{x}_{k,l,j}$ represents the uplink transmit data; $\mathbf{n}_d$ denotes the additive noise. It is assumed that the power of data transmission is equal to that used for the uplink training.

B. Downlink Data Transmission

Recall that reciprocity in the TDD operation leads to the utilization of the estimated uplink CSI in the downlink data transmission. In addition, a linear precoding method is also adopted. We assume that the received noisy downlink signal by UE i in the j-th cell is denoted as $\mathbf{r}_{i,j}$, where it contains downlink data streams from both the BS in cell j and the other $L-1$ cells. It can be represented as

$$\mathbf{r}_{i,j} = \sum_{l=1}^{L}\sqrt{p^{DL}_{i,j,l}\beta_{i,j,l}}\mathbf{h}^{\mathrm{H}}_{i,j,l}\sum_{k=1}^{K}\mathbf{q}_{i,k,l}t_{i,k,l} + z_{i,j}, \tag{8}$$

where $p^{DL}_{i,j,l}$ represents the downlink transmission power; $\sqrt{\beta_{i,j,l}}\mathbf{h}^{\mathrm{H}}_{i,j,l}$ denotes the downlink CSI from the BS in the

$$SINR_{i,j} = \frac{\beta_{i,j} p_{i,j}}{\beta_{i,j}\left(\sum\limits_{k\neq i}^{K} p_{k,j}\rho_{i,k,j}^2 + \sum\limits_{m=1}^{K} p_{m,j}\frac{\gamma_{m,j}}{M}\right) + \sum\limits_{l\neq j}^{L}\sum\limits_{\overline{k}=1}^{K}\beta_{i,l}P_{\overline{k},l}\frac{(M\rho_{i,\overline{k},l}^2+\gamma_{\overline{k},l})\gamma_{i,j}}{M\gamma_{\overline{k},l}} + \frac{\sigma_i^2\gamma_{i,j}}{M}} \tag{6}$$

$$\begin{aligned} SINR_{i,j} &= \frac{\beta_{i,j} p_{i,j}\frac{(\mathbb{E}[\mathbf{h}_{i,j}^H\hat{\mathbf{h}}_{i,j}])^2}{M\gamma_{k,j}}}{\beta_{i,j}\sum\limits_{k\neq i}^{K}\frac{p_{k,j}\mathbb{E}|\mathbf{h}_{i,j}^H\hat{\mathbf{h}}_{k,j}|^2}{M\gamma_{k,j}} + \beta_{i,j}p_{k,j}\frac{\mathbb{E}|(\mathbf{h}_{i,j}^H\hat{\mathbf{h}}_{i,j}-\mathbb{E}[\mathbf{h}_{i,j}^H\hat{\mathbf{h}}_{i,j}])|^2}{M\gamma_{i,j}} + \sum\limits_{l\neq j}^{L}\sum\limits_{\overline{k}=1}^{K}\frac{\beta_{\overline{k},j}p_{\overline{k},l}\mathbb{E}|\mathbf{h}_{i,l}^H\hat{\mathbf{h}}_{\overline{k},l}|^2}{M\gamma_{\overline{k},l}} + \sigma^2} \\ &= \frac{\beta_{i,j}p_{i,j}\frac{M^2}{M\gamma_{i,j}}}{\beta_{i,j}\sum\limits_{k\neq i}^{K}p_{k,j}\frac{M^2\rho_{i,k,j}^2+M\gamma_{k,j}}{M\gamma_{i,j}} + \beta_{i,j}p_{k,j}\frac{M\gamma_{i,j}}{M\gamma_{i,j}} + \sum\limits_{l\neq j}^{L}\sum\limits_{\overline{k}=1}^{K}\frac{\beta_{i,l}p_{\overline{k},l}(M^2\rho_{i,\overline{k},l}^2+M\gamma_{\overline{k},l})}{M\gamma_{\overline{k},l}} + \sigma^2} \\ &= \frac{\beta_{i,j}p_{i,j}}{\beta_{i,j}\sum\limits_{k\neq i}^{K}p_{k,j}\frac{(M\rho_{i,k,j}^2+\gamma_{k,j})}{M} + \beta_{i,j}p_{k,j}\frac{\gamma_{k,j}}{M} + \sum\limits_{l\neq j}^{L}\sum\limits_{\overline{k}=1}^{K}\beta_{i,l}p_{\overline{k},l}\frac{(M\rho_{i,\overline{k},l}^2+\gamma_{\overline{k},l})\gamma_{i,j}}{M\gamma_{\overline{k},l}} + \frac{\sigma^2\gamma_{i,j}}{M}} \\ &= \frac{\beta_{i,j}p_{i,j}}{\beta_{i,j}\left(\sum\limits_{k\neq i}^{K}p_{k,j}\rho_{i,k,j}^2 + \sum\limits_{m=1}^{K}p_{m,j}\frac{\gamma_{m,j}}{M}\right) + \sum\limits_{l\neq j}^{L}\sum\limits_{\overline{k}=1}^{K}\beta_{i,l}P_{\overline{k},l}\frac{(M\rho_{i,\overline{k},l}^2+\gamma_{\overline{k},l})\gamma_{i,j}}{M\gamma_{\overline{k},l}} + \frac{\sigma^2\gamma_{i,j}}{M}} \end{aligned} \tag{7}$$

l-th cell to the UE i in the j-th cell; $\mathbf{q}_{i,k,l}$ is linear precoding vector and $t_{i,k,l}$ denotes the data symbols streams transmitted by the BS in the l-th cell; $z_{i,j}$ is the AWGN and $z_{i,j} \in \mathcal{CN}(0, \sigma_z^2\mathbf{I}_M)$. As a result, the UE's received signal can be derived in the form of the desired signal plus the effective noise indicating the inter- and intra-cell interference. Let $G_{i,j,j} = \sqrt{p_{i,j,j}^{DL}\beta_{i,j,j}}\mathbf{h}_{i,j,j}^H$, then the received signal can be represented as

$$\mathbf{r}_{i,j} = \mathbb{E}[G_{i,j,j}]\mathbf{q}_{i,j,j}t_{i,j,j} + z_{i,j}^{'}, \tag{9}$$

where $z_{i,j}^{'}$ represents the effective noise and can be expressed as

$$\begin{aligned} z_{i,j}^{'} = &(G_{i,j,j} - \mathbb{E}[G_{i,j,j}])\mathbf{q}_{i,j,j}t_{i,j,j} + G_{i,j,j}\left(\sum_{k\neq i}^{K}\mathbf{q}_{i,k,j}t_{i,k,j}\right) \\ &+ \sum_{l\neq j}^{L}\sum_{k=1}^{K}\sqrt{\beta_{i,j,l}}\mathbf{h}_{i,j,l}^H\mathbf{q}_{i,k,l}t_{i,k,l} + z_{i,j}. \end{aligned} \tag{10}$$

According to the descriptions in [8], the precoding vector of the UE i in the j-th cell, $\mathbf{q}_{i,j,j}$, is given as

$$\mathbf{q}_{i,j,j} = \frac{\hat{\mathbf{h}}_{i,j,j}}{\sqrt{M\left(\sum\limits_{\overline{k}=1}^{K}\rho_{i,\overline{k},j}^2 + \sigma_w^2\right)}}. \tag{11}$$

In addition, it is assumed that if the number of antennas at the BS increase to infinity, we can get the following law of large number or the asymptotic orthogonality between channels in massive MIMO system

$$\lim_{M\to\infty}\frac{1}{M}\mathbf{h}_{i,j,j}^{\mathrm{H}}\mathbf{h}_{k,j,j} = \begin{cases} 0, & \text{if } i \neq k \\ 1, & \text{if } i = k. \end{cases} \tag{12}$$

In Lemma 1 of [8], they only considered the SINR of single-cell scenario. The following proposition gives the achievable sum rate with the SINR of multi-cell scenario.

Proposition 1. *For a L cells TDD massive MIMO system serving K UEs with a training sequence length of τ, the achievable sum rate of the system can be given as*

$$R = \sum_{l=1}^{L}\sum_{k=1}^{K}\left(1 - \frac{\tau}{T}\right)\log_2(1 + \mathit{SINR}_{k,l}) \tag{13}$$

where the $\mathit{SINR}_{k,l}$ is given by equation (12) at the top of the page.

Proof. As shown in Lemma 1 [8] , $\mathbb{E}[|\mathbf{h}_{i,j}^H\hat{\mathbf{h}}_{k,j}|] = M^2\rho_{i,k,j}^2 + M\gamma_{k,j}$ where $\gamma_{k,j} = \sum\limits_{z=1}^{k}\rho_{k,z,j}^2 + \sigma^2$, $\mathbb{E}[|\mathbf{h}_{i,j}^H\hat{h}_{i,j}|] = M^2$. By substituting into equation (6), we can get the proposed SINR and consequently the achievable sum rate. □

The proposed achievable sum rate in Proposition 1 has clearly shown how the varying length of pilots τ and the correlations between pilots $\rho_{i,k,j}$ influences the performance of the system. Assuming the training sequences are limited in length, different methods have been proposed to reduce the interference caused by pilot reuse in the denominator of the SINR [6], [7]. In the next section, the pilots design utilizing the GLP method will be introduced.

III. Grassmannian Line Packing based Pilot Design

The GLP design problem is to find the optimal packing of one-dimensional vectors in a vector space which has already been utilized for beamforming codebook design under the limited feedback in a multi-user MIMO system [9], [10]. In this section, we will introduce the GLP problem and investigate the application to pilot sequence design.

Consider two 1-dimensional unit vectors $\mathbf{w}_1$ and $\mathbf{w}_2$ all belong to a matrix Φ_m with $\mathbf{w}_1 \equiv \mathbf{w}_2$. The equivalence of two vectors can be seen as two lines with same length in a complex vector space $\mathbb{C}^m$. The two lines can also be seen as two one-dimensional subspaces in $\mathbb{C}^m$. The set of all the one-dimensional subspaces in $\mathbb{C}^m$ is denoted as $\mathcal{G}(m,1)$, named the Grassmann Manifold. In the Grassmann manifold, the distance between the two subspaces $\mathbf{w}_1$ and $\mathbf{w}_2$ is defined as the sine of the angle $\theta_{\mathbf{w}_1,\mathbf{w}_2}$ between these two subspaces, which is formulated as

$$d_c(\mathbf{w}_1,\mathbf{w}_2) = \sin\theta_{\mathbf{w}_1,\mathbf{w}_2} = \sqrt{1-|\mathbf{w}_1^{\mathrm{H}}\mathbf{w}_2|^2}. \quad (14)$$

Assume that there are N one-dimensional subspaces in the Grassmann manifold $\mathcal{G}(m,1)$, the GLP problem is to optimally pack these N subspaces, i.e. the N subspaces are equally separated in space with the largest possible adjacent distances.

The GLP method can also be utilized in pilot sequence design. If each UE is assigned a training sequence with length τ and there are totally K UEs in the L cells, it forms a $K \times \tau$ matrix with the pilot sequence as the row vector. Due to the limited length of τ, it is not possible to assign each UE with orthogonal pilots in a TDD massive MIMO system. As a result, in the pilot reuse scheme [6], only some of the UEs are assigned with orthogonal pilots and the rest of UEs reuse these pilots. However, the pilot sequences can be treated as packing K one-dimensional subspaces in a $\mathcal{G}(\tau,1)$ Grassmann manifold. With maximum adjacent distances between pilots, the correlations between pilot sequences will be minimized. The correlation between pilot sequence $\mathbf{s}_1$ and $\mathbf{s}_2$ can be written as

$$\rho^2_{\mathbf{s}_1,\mathbf{s}_2} = |\mathbf{s}_1^{\mathrm{H}}\mathbf{s}_2|^2 = 1 - d_c^2(\mathbf{s}_1,\mathbf{s}_2). \quad (16)$$

In [11], [12] is proposed an upper bound of the distance in the Grassmannian manifold. To pack K subspaces (ie. K UEs) in the $\mathcal{G}(\tau,1)$ Grassmann manifold, the bound can be expressed as

$$d_c^2 \le \begin{cases} \frac{(\tau-1)}{\tau}\cdot\frac{K}{K-1}, & \text{if } K \le \tau(\tau+1)/2 \\ \frac{(\tau-1)}{\tau}, & \text{if } K > \tau(\tau+1)/2. \end{cases} \quad (17)$$

The above upper bound of distance can be utilized in bounding the correlations of the pilot sequences scheme with GLP. Consequently, we can estimate the achievable sum rate with the proposed pilot sequence design.

Corollary 1. *For a L cells TDD massive MIMO system serving K UEs with the length of training sequence equal to τ, the achievable sum rate of the system with the proposed pilot sequence design can be expressed as*

$$R_G = \sum_{l=1}^{L}\sum_{k=1}^{K}\left(1-\frac{\tau}{T}\right)\log_2\left(1+SINR_{k,l}^G\right) \quad (18)$$

where $SINR_{k,l}^G$ is given by equation (19) and ρ_G^2 is represented as

$$\rho_G^2 = \begin{cases} 1-\left(\frac{\tau-1}{\tau}\cdot\frac{K}{K-1}\right), & \text{if } K \le \tau(\tau+1)/2 \\ 1-\left(\frac{\tau-1}{\tau}\right), & \text{if } K > \tau(\tau+1)/2, \end{cases}$$

TABLE I
TABLE OF SYSTEM PARAMETERS

Number of cells L	7
Number of antennas at BS M	64-128
Cell radius	250 m
Minimum distance between UE and BS	35 m
Average transmit power at BS	40 W
System bandwidth	200 kHz
Carrier frequency	2 GHz
Velocity of mobile	10 km/h

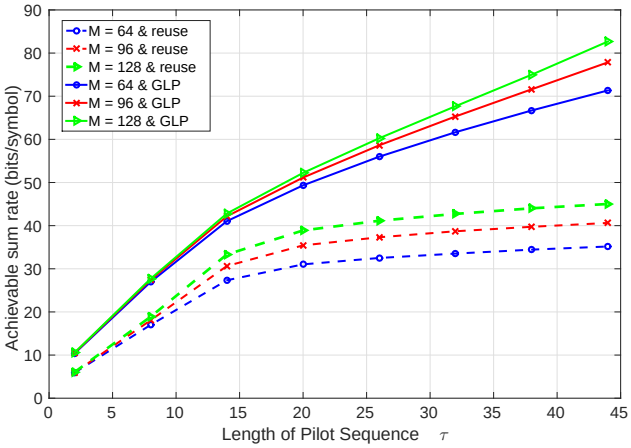

Fig. 2. Comparison of the achievable sum rate with pilot reuse scheme and non-orthogonal pilot scheme in the pilot contamination regions

Proof. Based on equations (16) and (17), we can get a lower bound on the correlation values. And by applying the GLP in pilot design, the correlation between pilots is same and by applying the bound of ρ in Proposition 1, it provides an upper bound on the achievable sum rate of the system. □

IV. Simulation Results and Discussions

A. Simulations results

The performance of the proposed GLP based pilot design is evaluated by Monte-Carlo simulations. Consider a massive MIMO system with 7 equal size hexagon shaped cells with a radius of 250 meters In [13], it pointed out that massive MIMO systems are likely to be installed in high-capacity hot-spot regions to serve a large number of UEs, such as in football stadia or shopping malls. As a result, the non-uniform distribution of UEs in a multi-cell massive MIMO system is considered in the simulation. The simulation includes seven cells with an equal number of antennas at each BS but the central cell is considered to serve more UEs than the neighbouring six cells. The UEs are randomly distributed in each cell with minimum 35 meters distance from BS. Furthermore, the velocity of each mobile is set to a small value (10 km/h). The calculation of path-loss follows the model $35.3 + 37.6\log_{10} d_{U,B}$ where $d_{U,B}$ denotes the distance (with unit meter) between UE and BS. The downlink transmit power is assumed to be equally distributed to the UEs in each cell.

Fig.2 shows the simulation results of achievable sum rate with increasing pilot length τ for different numbers of antennas at BS where $M = 64, 96$ or 128. The comparison

$$SINR_{i,j} = \frac{\beta_{i,j} p_{i,j}}{\beta_{i,j}\left(\rho_G^2 \sum_{k\neq i}^{K} p_{k,j} + \frac{K\rho_G^2+\sigma_w^2}{M} \sum_{m=1}^{K} p_{m,j}\right) + \sum_{l\neq j}^{L}\sum_{\overline{k}=1}^{K} \beta_{i,l} p_{\overline{k},l} \frac{(M+K)\rho_G^2+\sigma_w^2}{M} + \frac{K\rho_G^2+\sigma_w^2}{M}\sigma_i^2} \tag{15}$$

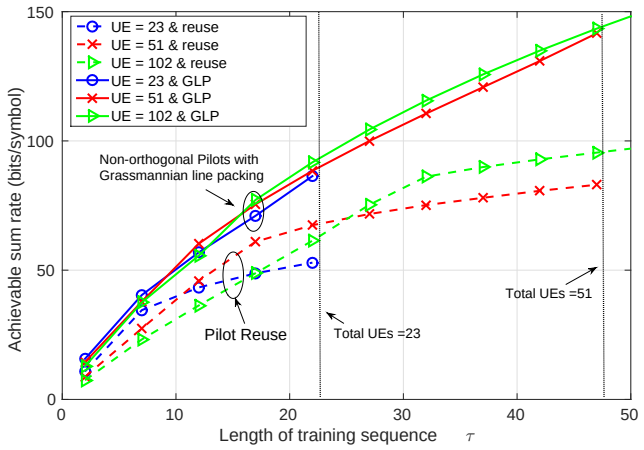

Fig. 3. Comparison of the achievable sum rate with pilot reuse scheme and non-orthogonal pilot scheme in the pilot contamination regions

TABLE II
TABLE OF UE GROUPS

	Central cell	Neighbouring 6 cells	Total UEs
Group 1	5 UEs	3 UEs	23 UEs
Group 2	15 UEs	6 UEs	51 UEs
Group 3	30 UEs	12 UEs	102 UEs

is made between pilot reuse scheme and the non-orthogonal pilot scheme with pilot contamination where $\tau < K$. The central cell contains 15 UEs while the neighbouring 6 cells contain 5 UEs. It can be observed that all the achievable sum rates linearly increase with the number of antennas at the BS. This is consistent with the proposition in [8] that the transmission rate is fundamentally limited by the length of the pilot sequences. When $\tau = 44$, the sum rate of the GLP based training sequence is doubled over the pilot reuse scheme. Furthermore, the GLP based pilot scheme has achieved significant improvements in transmission rate over the pilot reuse scheme for all the antenna configurations. This result is due to the small τ value leading to limited reuse of possibilities for the pilot signals so that the pilot contamination degrades the throughput performance significantly. Nevertheless, the GLP-based pilot design is able to mitigate the pilot contamination even with a small τ value. As a result, it has achieved much higher sum rate.

Fig.3 shows the achievable sum rate versus the varying number of UEs. Table II lists the three different groups of UE distributions that used in the simulation with $M = 256$. Again the GLP-based pilot design outperforms the pilot reuse schemes among the three UE distributions. The performance gaps between these two pilot schemes increase with the length of the training sequence. The GLP based pilot sequence brings approximately 70% increase of sum rate over the pilot reuse scheme. In addition, simulation results indicate the GLP-based pilot design is desired to be applied for a large number of UEs. For example, when τ is equal to 50, the gap of two schemes for $K = 51$ is about 60 bits/symbol and for $K = 102$ it is 80 bits/symbol.

V. CONCLUSIONS

The achievable sum rate performance of a massive MIMO system with a Grassmannian line packing based pilot scheme is investigated in this paper. A new closed-form achievable sum rate is proposed and simulations has been made with the TDD multi-cell massive MIMO systems with different configurations of the number of BS antennas, UE configurations and pilot sequence length. The results showed that the application of Grassmannian line packing in pilot design has significantly improved the throughput performance by at least 70%. In addition, it has shown that the Grassmannian line packing based pilot scheme is suitable to serve cells with large number of UEs. Future work will be focused on the joint optimization of capacity and energy efficiency.

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