# Practical Aspects of Compress and Forward with BICM in the 3-Node Relay Channel

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***Abstract*—The classical relay channel is well investigated in literature. The most common relaying techniques are Decode-and-Forward (DF) and Compress-and-Forward (CF), whereby the achievable rates of these techniques outperform each other depending on the quality of the links between nodes. Based on the results from information theory assuming Gaussian codebooks, this paper focuses on practical aspects of the CF relay protocol which outperforms DF if the source-relay link becomes the bottleneck of the system. In practice, appropriate quantizers have to be found whose output can be exploited by real decoders. As the relay's receive signal contains also noise, maximum entropy quantizers are unrewarding. Therefore, the Information Bottleneck (IB) method is used to find the optimal quantizer for a specific scenario. Furthermore, it is a priori not clear whether signal processing before quantizing the received signal is useful considering coded modulation with iterative decoders. In a nutshell, the received signals may be either quantized directly, or after a few iterations of decoding. For either case, it is shown how the respective quantization indices are optimally exploited by a joint decoder at the destination. Results reveal that performing soft-output decoding at the relay prior to quantization can be slightly rewarding in some cases. In general however, the gain justifies not the additional decoding effort at the relay.**

## I. Introduction

The 3-node relay channel, introduced in [1], [2] by van der Meulen, consists besides source and destination node of only one relay node which can pursue different strategies [3]. The most common are Decode-and-Forward (DF), Compress-and-Forward (CF), and Amplify-and-Forward (AF), whereby the first two outperform the latter one [4]. Furthermore, it is shown that CF reaches the upper bound on the capacity for the relay being very close to the destination, whereby DF approaches the bound when the relay is close to the source [4], [5]. Due to practical issues, this work focuses on a half-duplex relay in a Time Division Multiple Access (TDMA) fashion with a broadcast and a subsequent Multiple Access (MAC) phase. The length of the two phases has to be optimized [5], [6]. Due to the half-duplex constrained, two different transmission strategies regarding the MAC phase are distinguished. In the orthogonal scheme, the source remains quiet in the second time slot. Contrarily, the source transmits jointly with the relay in the non-orthogonal scheme. While distributed beamforming is of most interest when source-relay link and source-destination link are equally good, mixing new information under the source's signal becomes better the more the relay moves to the destination. For CF, beamforming is of no interest because in scenarios, where it outperforms DF, the relay is located much closer to the destination than to the source. From information theory, non-orthgonal schemes promise a considerable gain against orthogonal schemes. On the contrary, taking the total consumed energy into account, there are cases where the additional effort due to a non-orthogonal scheme are not worthwhile [7], [8]. For this reason, both schemes are considered and will be compared in the results section.

Focusing on CF in this work, the most important question is how to do compression at the relay practically. The answer is given by use of the Information Bottleneck (IB) method originally introduced in [9] and extended for the relay channel in [10] which delivers the optimal quantizer as it maximizes the mutual information between the source's signal and the output of the quantizer given a specific achievable rate on the relay-destination-link. The quantizer delivers compression indices which are Wyner-Ziv-source encoded and forwarded to the destination. It will be shown how to obtain log-likelihood ratios (LLRs) from these indices which can be optimally exploited by any practical soft-input decoder. Furthermore, the setup is implemented by means of a Bit Interleaved Coded Modulation (BICM) scheme in conjunction with state of the art coding and Quadrature Amplitude Modulation (QAM) used in Universal Mobile Telecommunications System (UMTS) and Long Term Evolution (LTE). The simulations determine the respective quantizers by the iterative IB algorithm regarding receive symbols as well as soft-values of an additional decoder at the relay. Results are obtained from frame error rates (FERs) of the joint decoder at the destination. Finally, the advantage of optimal random quantization is compared to straightforward deterministic quantization.

The rest of the paper is organized as follows. Sec. II describes the system setup including channel model, CF relaying scheme, and Modulation and Coding Schemes (MCSs) used for practical coding. Then, Sec. III introduces the IB method to get the optimal quantizer for the given setup. Sec. IV describes how to exploit the quantizer output in an iterative decoder for different approaches. Finally, Sec. VI shows simulation results and Sec. VII concludes the paper. For the general mathematic notation: we use bold letters, e.g. $\mathbf{x}$, to denote vectors of realizations (lower case letters), e.g. $x$, of a random variable (uppercase letters), e.g. $X$, with a corresponding alphabet in calligraphic letters, e.g. $\mathcal{X}$.

## II. System Setup

### A. The classical (3-Node) Relay Channel

The 3-node relay channel, as depicted in Fig. 1, consists of one source node $S$, one relay node $R$, and one destination node $D$, where links between nodes are modeled as Additive White Gaussian Noise (AWGN) channels with $n_f \sim \mathcal{CN}(0,1)$ being the noise at receiving node $f \in \{R, D\}$. Furthermore, a path-loss is considered by channel coefficients $a_{ef} = d_{ef}^{(-\alpha/2)} \forall e \in \{S, R\}, \forall f \in \{R, D\}$, where $\alpha$ and $d_{ef}$ are path-loss exponent and distance between node $e$ and $f$, respectively.

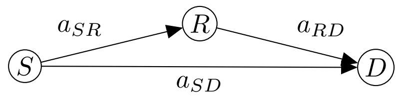


Fig. 1. The Classical Relay Channel

The transmission is organized in two time slots of length $\tau \in [0, 1]$ and $(1 - \tau)$ denoted as broadcast and MAC phase, respectively. First, $S$ transmits $\mathbf{x_{S1}}$ to $R$ and $D$

$$\mathbf{y_R} = a_{SR}\sqrt{P_{S1}}\mathbf{x_{S1}} + \mathbf{n_R} \quad (1)$$
$$\mathbf{y_{D1}} = a_{SD}\sqrt{P_{S1}}\mathbf{x_{S1}} + \mathbf{n_{D1}}, \quad (2)$$

where $P_{S1}$ denotes the transmit power of the related transmit vector $\mathbf{x_{S1}}$ whose elements are realizations of random variable $X_{S1}$ with $E\{|X_{S1}|^2\} = 1$.

In the MAC phase, $R$ transmits a compressed version of $\mathbf{y_R}$ and $S$ a new message which is separately encoded and modulated. Hence, $D$ receives a superposition of both.

$$\mathbf{y_{D2}} = a_{SD}\sqrt{P_{S2}}\mathbf{x_{S2}} + a_{RD}\sqrt{P_R}\mathbf{x_R} + \mathbf{n_{D2}} \quad (3)$$

For comparison, the source may also be quiet in the 2nd time slot to have orthogonal channel access, i.e., $P_{S2} = 0$.

### B. Compress and Forward

Ensuing from information theory, Wyner-Ziv coding [11] is applied to compress the received sequence $\mathbf{y_R}$ at the relay. The adjustment of the compression rate considers the amount of side information $\mathbf{y_{D1}}$ at the destination. First, $\mathbf{y_R}$ will be compressed following the distribution

$$\Pr\{z\} = \sum_{y_R} \Pr\{z|y_R\} \Pr\{y_R\}, \quad (4)$$

where $z$ denotes the compression index of the compressed received signal. The probabilities $\Pr\{z|y_R\}$ are obtained by the IB method and represent a random vector quantization $\mathbf{y_R} \rightarrow \mathbf{z}$. In a second step, the indices $\mathbf{z}$ will be source encoded with side information (binning) delivering indices $\mathbf{s}$ which are actually transmitted via $\mathbf{x_R}$.

By decoding of $\mathbf{y_{D2}}$ (treating $\mathbf{x_{S2}}$ as noise) the destination detects $\mathbf{s}$ and recovers $\mathbf{z}$. Then $\mathbf{z}$ and $\mathbf{y_{D1}}$ are used jointly to decode the message transmitted by $S$ in the broadcast phase. Please note that $S$ transmits a second message directly in the MAC phase. This can be detected from $\mathbf{y_{D2}}$ after subtracting the influence of $\mathbf{x_R}$.

As the focus of this investigation lies on exploiting $\mathbf{z}$ in an iterative decoder, the source coding step with side information to get $\mathbf{s}$ from $\mathbf{z}$ is omitted, i.e., $\mathbf{z}$ is assumed to be directly available at $D$ for the iterative decoding described in Sec. IV.

### C. Modulation and Coding Schemes

For practical coding, a set of 40 MCSs is available, whereby the practically relevant range is well covered. More precisely, the well known UMTS/LTE turbo code [12] and $M$-QAM with orders $m = \log_2 M, m \in \{2, 4, 6, 8, 10\}$ are used. The inherent code rate $R_c = 1/3$ of the turbo code is extended to a set of 8 code rates via puncturing as shown in Table I [13].

TABLE I. PUNCTURING PATTERNS (OCTAL)

| 4/5 | 2/3 | 4/7 | 1/2 | 4/9 | 2/5 | 4/11 | 1/3 |
|---|---|---|---|---|---|---|---|
| 100 | 101 | 101 | 121 | 125 | 125 | 335 | 377 |
| 001 | 021 | 261 | 263 | 363 | 377 | 377 | 377 |

For decoding, a turbo process with 8 iterations is implemented exchanging soft information between two log-map decoders (Bahl Cocke Jelinek Raviv (BCJR) [14]). The demapper is separated from the decoder by a random interleaver to split error bursts. Furthermore, it is not included in the turbo process known as BICM with parallel decoding [15].

## III. Information Bottleneck Method

This section introduces the IB method [9], [10], [16], [17] which finds the conditional distribution $\Pr\{z|y\}$ to quantize an observation $y$ of $x$ to $z$ forming a Markov chain $X \rightarrow Y \rightarrow Z$. As this problem is non-convex, the proposed iterative algorithm converges only to a local optimum. The algorithm finds a trade-off between the mutual information $I(X; Z)$ and source coding rate $I(Y; Z)$ defining the information-rate function

$$I(r) \triangleq \max_{\Pr\{z|y\}} I(X; Z) \qquad \text{s.t.} \qquad I(Y; Z) \leq r \quad (5)$$

for $0 < r \leq H(Y)$. Considering the 3-node relay channel, (5) can be extended to the trade-off between $I(X_{S1}; Z|Y_{D1})$ and $I(Y_R; Z|Y_{D1})$ exploiting the side information which is available at destination $D$ due to the broadcast phase. Following the derivation in [10], the extended information rate function given the joint distribution $\Pr\{x_{S1}, y_R, y_{D1}\}$ is defined as

$$I(r) \triangleq \max_{\Pr\{z|y_R\}} I(X_{S1}; Z|Y_{D1}) \quad \text{s.t.} \quad I(Y_R; Z|Y_{D1}) \leq r, \quad (6)$$

where $0 < r \leq H(Y_R|Y_{D1})$ denotes the rate after source coding with side information (Wyner-Ziv). Applying the method of Lagrangian multipliers, (6) can be solved by an iterative algorithm similar to the Blahut-Arimoto algorithm [9], [18]. Introducing the Lagrangian multiplier $\beta > 0$, (6) may be rewritten as [10]

$$\begin{aligned} I(r(\beta)) - \frac{1}{\beta} r(\beta) &= \max_{\Pr\{z|y_R\}} I(X_{S1}; Z|Y_{D1}) - \frac{1}{\beta} I(Y_R; Z|Y_{D1}) \\ &= \frac{1}{\beta} \min_{\Pr\{z|y_R\}} I(Y_R; Z|Y_{D1}) - \beta I(X; Z|Y_{D1}). \end{aligned} \quad (7)$$

From (7) the modified iterative IB algorithm, as shown in Algorithm 1, can be derived [10].

This algorithm delivers $\Pr\{z|y_R\}$ given $\Pr\{x_{S1}, y_R, y_{D1}\}$ with respect to a specific trade-off $I(r(\beta))$ depending on Lagrangian multiplier $\beta > 0$, i.e., the algorithm takes $\beta$ as input parameter and outputs additionally the pair $(I(r(\beta)), r(\beta))$.

**Input**: $\Pr\{x_{S1}, y_R, y_{D1}\}$, $\mathcal{X}_{S1}$, $\mathcal{Y}_R$, $\mathcal{Y}_{D1}$, $\mathcal{Z}$, $\beta > 0$, $\epsilon > 0$

**Output**: $\Pr\{z|y_R\}$, $I(r(\beta))$, $r(\beta))$

initialize $\Pr\{z|y_R\}^{(0)}$ according to Maximum Output Entropy (MOE)

$k \leftarrow 1$

$\Pr\{z\}^{(0)} \leftarrow \sum_{y_R} \Pr\{y_R\} \Pr\{z|y_R\}^{(0)}$

$\Pr\{z, y_{D1}\}^{(0)} \leftarrow \sum_{y_R} \Pr\{y_R, y_{D1}\} \Pr\{z|y_R\}^{(0)}$

$\Pr\{z|y_{D1}\}^{(0)} \leftarrow \frac{\Pr\{z,y_{D1}\}^{(0)}}{\Pr\{y_{D1}\}}$

$\Pr\{x_{S1}|z, y_{D1}\}^{(0)} \leftarrow \frac{1}{\Pr\{z,y_{D1}\}^{(0)}} \sum_{y_R} \Pr\{x_{S1}, y_R, y_{D1}\} \Pr\{z|y_R\}^{(0)}$

$d^{(0)}(z, y_R) \leftarrow \beta \sum_{y_{D1}} \Pr\{y_{D1}|y_R\} D_{KL}\left(\Pr\{x_{S1}|y_R, y_{D1}\} || \Pr\{x_{S1}, z, y_{D1}^{(0)}\}\right) - \sum_{y_{D1}} \Pr\{y_{D1}|y_R\} \log_2\left(\Pr\{z|y_{D1}\}^{(0)}\right)$

$\Pr\{z|y_R\}^{(1)} \leftarrow 2^{-d^{(0)}(z,y_R)} / \sum_z 2^{-d^{(0)}(z,y_R)}$

**while** $|\Pr\{z|y_R\}^{(k)} - \Pr\{z|y_R\}^{(k-1)}|/(|\mathcal{Y}_R| \cdot |\mathcal{Z}|) \geq \epsilon$ **do**

- $\Pr\{z\}^{(k)} \leftarrow \sum_{y_R} \Pr\{y_R\} \Pr\{z|y_R\}^{(k)}$
- $\Pr\{z, y_{D1}\}^{(k)} \leftarrow \sum_{y_R} \Pr\{y_R, y_{D1}\} \Pr\{z|y_R\}^{(k)}$
- $\Pr\{z|y_{D1}\}^{(k)} \leftarrow \frac{\Pr\{z,y_{D1}\}^{(k)}}{\Pr\{y_{D1}\}}$
- $\Pr\{x_{S1}|z, y_{D1}\}^{(k)} \leftarrow \frac{1}{\Pr\{z,y_{D1}\}^{(k)}} \sum_{y_R} \Pr\{x_{S1}, y_R, y_{D1}\} \Pr\{z|y_R\}^{(0)}$
- $d^{(k)}(z, y_R) \leftarrow \beta \sum_{y_{D1}} \Pr\{y_{D1}|y_R\} D_{KL}\left(\Pr\{x_{S1}|y_R, y_{D1}\} || \Pr\{x_{S1}, z, y_{D1}^{(k)}\}\right) - \sum_{y_{D1}} \Pr\{y_{D1}|y_R\} \log_2\left(\Pr\{z|y_{D1}\}^{(k)}\right)$
- $\Pr\{z|y_R\}^{(k+1)} \leftarrow 2^{-d^{(k)}(z,y_R)} / \sum_z 2^{-d^{(k)}(z,y_R)}$
- $k \leftarrow k + 1$

**end**

$\Pr\{z|y_R\} \leftarrow \Pr\{z|y_R\}^{(k)}$

$r(\beta) \leftarrow \sum_{y_R,z} \Pr\{z|y_R\} \Pr\{y_R\} \log_2\left(\frac{\Pr\{z|y_R\}}{\Pr\{z\}}\right) - \sum_{y_{D1},z} \Pr\{z|y_{D1}\} \Pr\{y_{D1}\} \log_2\left(\frac{\Pr\{z|y_{D1}\}}{\Pr\{z\}}\right)$

$I(r(\beta)) \leftarrow \sum_{x_{S1},y_{D1},z} \Pr\{x_{S1}|z, y_{D1}\} \Pr\{z, y_{D1}\} \log_2\left(\frac{\Pr\{x_{S1}|z,y_{D1}}{\Pr\{x_{S1}|y_{D1}}\right)$

**Algorithm 1:** Iterative IB algorithm [10], where $D_{KL}$ denotes the Kullback-Leibler-Divergence.

To calculate the whole information rate curve, a range of $\beta$ is used, whereby the resulting quantizers are random except for $\beta \to \infty$. The maximum $\beta$ delivers the maximum rate $r = H(Z|Y_{D1})$ leading to a deterministic quantizer, i.e., $\Pr\{z|y_R\}$ equals either zero or one. According to the CF relay protocol, $r$ is restricted by the capacity of the relay destination link such that $\tau \cdot r \leq (1-\tau)I(X_R; Y_{D2})$ holds. As the algorithm is iterative, usually a random initialization for $\Pr\{z|y_R\}$ is needed. However, due to the non-convex nature of the problem, a random initialization needs several runs until a close to optimum value for $(I(r(\beta)), r(\beta))$ is obtained. To avoid repeatedly executions, MOE initialization is used in this work which is known to perform very well [19]. Please note that the input distribution $\Pr\{x_{S1}, y_R, y_{D1}\}$ to the IB algorithm is discrete whereas the relay channel delivers a

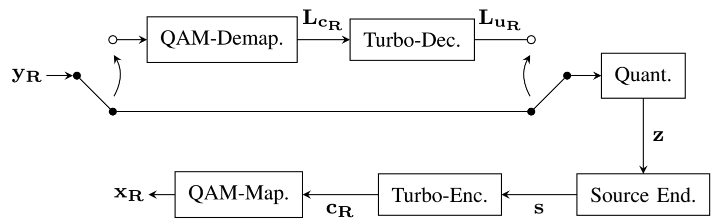


Fig. 2. Processing chain at the relay with either direct quantization or additional prior decoding.

continuous distribution $p_{X\tilde{Y}_R\tilde{Y}_{D1}}(x, \tilde{y_R}, \tilde{y}_{D1})$, that is, a pre-quantization of the channel outputs is necessary. In practice, this is mostly done in any case due to usual digital signal processing.

## IV. Joint Decoding

From an information theoretic perspective it is optimal to compress the received signal $\mathbf{y_R}$ and apply Wyner-Ziv coding exploiting $\mathbf{y_{D1}}$. In practice, however, two questions arise. First, the quantization obtained with the IB method is random and, thus, delivers no deterministic index for a received symbol. Hence, the question has to be answered how this index can be exploited in a practical soft-input decoder. Second, CF is usually applied when error free decoding is not possible. However, considering iterative decoding, the reliability of LLRs may be improved after a few iterations in an iterative decoder. Therefore, the approaches depicted in Fig. 2 will be distinguished:

1) Direct quantization of $\mathbf{y_R}$ to compression indices $\mathbf{z_y}$,
2) additional soft-output decoding to get LLRs $\mathbf{L_{u_R}} = L(\mathbf{u}|\mathbf{y_R})$ and subsequent quantization to $\mathbf{z_u}$.

A symbol-by-symbol Maximum-A-Posteriori (MAP) decoder delivers

$$L(\hat{u}_l) = \log \frac{\Pr\{u_l = 0, \mathbf{y_{D1}}, \mathbf{z}\}}{\Pr\{u_l = 1, \mathbf{y_{D1}}, \mathbf{z}\}} \tag{8}$$

for the final decision. As these joint distributions are not directly accessible, the set of all possible code words is divided into two subsets $\Gamma_l^{(1)}$ and $\Gamma_l^{(0)}$ containing code words $\mathbf{c}$ whose $l$th information bit is $u_l = 1$ and $u_l = 0$, respectively.

$$L(\hat{u}_l) = \log \frac{\sum_{\mathbf{c}\in\Gamma_l^{(0)}} \Pr\{\mathbf{c}, \mathbf{y_{D1}}, \mathbf{z}\}}{\sum_{\mathbf{c}\in\Gamma_l^{(1)}} \Pr\{\mathbf{c}, \mathbf{y_{D1}}, \mathbf{z}\}} \tag{9}$$

### A. Direct Quantization

For the direct quantization with no previous processing at the relay, $z$ corresponds to $z_y$. Then, (9) can be simplified to

$$
\begin{aligned}
L(\hat{u}_l) &= \log \frac{\sum_{\mathbf{c}\in\Gamma_l^{(0)}} \Pr\{\mathbf{y_{D1}},\mathbf{z}|\mathbf{c}\}\Pr\{\mathbf{c}\}}{\sum_{\mathbf{c}\in\Gamma_l^{(1)}} \Pr\{\mathbf{y_{D1}},\mathbf{z}|\mathbf{c}\}\Pr\{\mathbf{c}\}} \\
&= \log \frac{\sum_{\mathbf{c}\in\Gamma_l^{(0)}} \Pr\{\mathbf{y_{D1}}|\mathbf{c}\}\Pr\{\mathbf{z}|\mathbf{c}\}\Pr\{\mathbf{c}\}}{\sum_{\mathbf{c}\in\Gamma_l^{(1)}} \Pr\{\mathbf{y_{D1}}|\mathbf{c}\}\Pr\{\mathbf{z}|\mathbf{c}\}\Pr\{\mathbf{c}\}} \\
&= \log \frac{\sum\limits_{\mathbf{c}\in\Gamma_l^{(0)}} \prod\limits_{i=0}^{n-1} \Pr\{y_i|c_i\}\Pr\{z_i|c_i\} \prod\limits_{j=0}^{k-1} \Pr\{u_j\}}{\sum\limits_{\mathbf{c}\in\Gamma_l^{(1)}} \prod\limits_{i=0}^{n-1} \Pr\{y_i|c_i\}\Pr\{z_i|c_i\} \prod\limits_{j=0}^{k-1} \Pr\{u_j\}} \\
&= \log \frac{\sum\limits_{\mathbf{c}\in\Gamma_l^{(0)}} \prod\limits_{i=0}^{n-1} e^{-(L(y_i|c_i)+L(z_i|c_i))c_i} \prod\limits_{j=0}^{k-1} e^{-L(u_j)u_j}}{\sum\limits_{\mathbf{c}\in\Gamma_l^{(1)}} \prod\limits_{i=0}^{n-1} e^{-(L(y_i|c_i)+L(z_i|c_i))c_i} \prod\limits_{j=0}^{k-1} e^{-L(u_j)u_j}} .
\end{aligned}
\tag{10}
$$

The first exponential term in (10) represents information about the code bits from the channels ($S \to D$, $S \to R$) and the second exponential term a priori knowledge about the information bits. Both will be given as input to an appropriate decoder like the BCJR [14]. It becomes clear that the decoder itself needs not to be modified. Solely, the LLRs $L(z_i|c_i)$ have to be found and added to the LLRs $L(y_{D_i}|c_i)$ which are delivered by the demapper. The LLR of interest for each $i$ is then

$$
\begin{aligned}
L(z_y|c) &= \log \frac{\Pr\{z_y|c=0\}}{\Pr\{z_y|c=1\}} = \log \frac{\sum\limits_{y_R\in\mathcal{Y}_R} \Pr\{z_y,y_R|c=0\}}{\sum\limits_{y_R\in\mathcal{Y}_R} \Pr\{z_y,y_R|c=1\}} \\
&= \log \frac{\sum\limits_{y_R\in\mathcal{Y}_R} \Pr\{z_y|y_R,c=0\}\Pr\{y_R|c=0\}}{\sum\limits_{y_R\in\mathcal{Y}_R} \Pr\{z_y|y_R,c=1\}\Pr\{y_R|c=1\}} \\
&= \log \frac{\sum\limits_{y_R\in\mathcal{Y}_R} \Pr\{z_y|y_R\}\Pr\{y_R|c=0\}}{\sum\limits_{y_R\in\mathcal{Y}_R} \Pr\{z_y|y_R\}\Pr\{y_R|c=1\}},
\end{aligned}
\tag{11}
$$

where the condition on $c$ in the first term cancels due to Markov property $C \to X \to Y_R \to Z$. The first distribution $\Pr\{z_y|y_R\}$ is known from IB method. The second distribution $\Pr\{y_R|c\}$ can be considered as the demapper output $L(y_R|c)$ for all possible $y_R \in \mathcal{Y}_\mathcal{R}$. In this context (11) may be seen as a virtual relay demapper delivering an average LLR.

### B. Quantization of Soft-Decoder-Output

When the relay applies additional decoding prior to quantization, the IB method delivers $\Pr\{z_u|L_{u_R}\}$ given the distribution $\Pr\{u, L_{u_R}, Lu_{D1}\}$ which is obtained numerically. As $z_u$ represents knowledge about the information bit, (9) is simplified in another way exploiting $\Pr\{\mathbf{z}|\mathbf{c}\} = \Pr\{\mathbf{z}|\mathbf{u}\}$.

$$
\begin{aligned}
L(\hat{u}_l) &= \log \frac{\sum_{\mathbf{c}\in\Gamma_l^{(0)}} \Pr\{\mathbf{y_{D1}}|\mathbf{c}\}\Pr\{\mathbf{z}|\mathbf{u}\}\Pr\{\mathbf{u}\}}{\sum_{\mathbf{c}\in\Gamma_l^{(1)}} \Pr\{\mathbf{y_{D1}}|\mathbf{c}\}\Pr\{\mathbf{z}|\mathbf{u}\}\Pr\{\mathbf{u}\}} \\
&= \log \frac{\sum\limits_{\mathbf{c}\in\Gamma_l^{(0)}} \prod\limits_{i=0}^{n-1} \Pr\{y_i|c_i\} \prod\limits_{j=0}^{k-1} \Pr\{z_j|u_j\} \prod\limits_{j=0}^{k-1} \Pr\{u_j\}}{\sum\limits_{\mathbf{c}\in\Gamma_l^{(1)}} \prod\limits_{i=0}^{n-1} \Pr\{y_i|c_i\} \prod\limits_{j=0}^{k-1} \Pr\{z_j|u_j\} \prod\limits_{j=0}^{k-1} \Pr\{u_j\}} \\
&= \log \frac{\sum\limits_{\mathbf{c}\in\Gamma_l^{(0)}} \prod\limits_{i=0}^{n-1} e^{-L(y_i|c_i)c_i} \prod\limits_{j=0}^{k-1} e^{-(L(u_j)+L(z_j|u_j))u_j}}{\sum\limits_{\mathbf{c}\in\Gamma_l^{(1)}} \prod\limits_{i=0}^{n-1} e^{-L(y_i|c_i)c_i} \prod\limits_{j=0}^{k-1} e^{-(L(u_j)+L(z_j|u_j))u_j}},
\end{aligned}
\tag{12}
$$

where $L(z_j|u_j)$ is now added to the a priori information $L(u_j)$ that is fed into a practical decoder. Similarly as before, the LLR of interest for each $i$ is

$$
\begin{aligned}
L(z_u|u) &= \log \frac{\Pr\{z_u|u=0\}}{\Pr\{z_u|u=1\}} = \log \frac{\sum\limits_{L_{u_R}\in\mathcal{L}_R^u} \Pr\{z_u,L_{u_R}|u=0\}}{\sum\limits_{L_{u_R}\in\mathcal{L}_R^u} \Pr\{z_u,L_{u_R}|u=1\}} \\
&= \log \frac{\sum\limits_{L_{u_R}\in\mathcal{L}_R^u} \Pr\{z_u|L_{u_R}\}\Pr\{L_{u_R}|u=0\}}{\sum\limits_{L_{u_R}\in\mathcal{L}_R^u} \Pr\{z_u|L_{u_R}\}\Pr\{L_{u_R}|u=1\}},
\end{aligned}
\tag{13}
$$

where $\Pr\{z_u|L_{u_R}\}$ is the distribution of the quantizer known from the IB method. Furthermore, the distribution $\Pr\{L_{u_R}|u=0\}$ is given by $\Pr\{u, L_{u_R}, Lu_{D1}\}$ which is an input of the IB method.

## V. Rate Allocation for Practical Codes

This section describes the allocation of a discrete rate $R_b = m \cdot R_c$ (ensuing from the MCSs presented in Sec. II-C) to a specific link which is defined by its signal to noise ratio (SNR) or its signal to interference plus noise ratio (SINR). Therefore, Monte Carlo Simulations are applied to a simple AWGN channel to find FER vs. SNR for all MCS. From these curves, one can find a threshold SNR for each rate $R_b$ such that a target FER of $10^{-2}$ is reached (cf. Fig. 3). Then, Fig. 3 is used to determine the MAC rates $R_b^R$ and $R_b^{S2}$ of $\mathbf{x_R}$ and $\mathbf{x_{S2}}$, respectively.

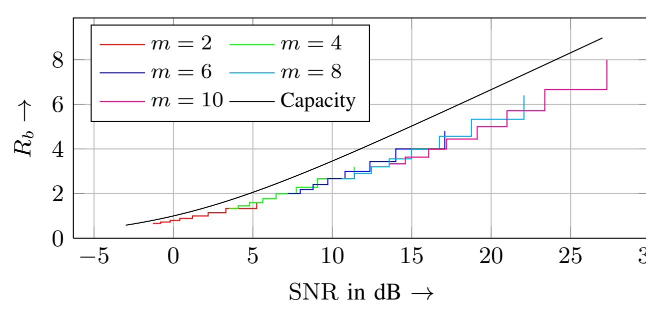


Fig. 3. Achievable rates $R_b$ versus SNR for FER $= 10^{-2}$ on direct link.

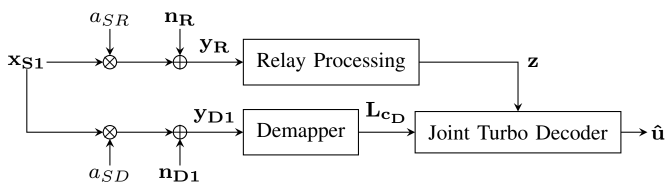


Fig. 4. Simulation Setup excluding Wyner-Ziv coding.

As the influence of the indices $z$ on the destination decoder cannot be analytically described, the modified Monte Carlo simulation depicted in Fig. 4 is applied to determine the broadcast rate $R_b^{S1}$ of $\mathbf{x_{S1}}$. Following the description in Sec. IV, this simulation includes optimal quantization regarding $\mathbf{Y_R}$ and $\mathbf{L_{u_R}}$ and extended decoding exploiting the compression indices $\mathbf{z_y}$ and $\mathbf{z_u}$ in the form of (11) and (13), respectively.[1] The optimal quantizers are determined on the fly for $|\mathcal{Z}| = 16$ and for different values of the parameter $\beta \in [1, 5, 25, 100, 200, 500]$ delivering a set of compression rates $r(\beta)$ as the optimal $r$ is a priori unknown. Moreover the quantizers are stored in a Look-Up-Table (LUT) so that they are calculated only once. Regarding the compression of $\mathbf{L_{u_R}}$ additional decoding prior to quantization is simulated for 1 to 4 iterations.

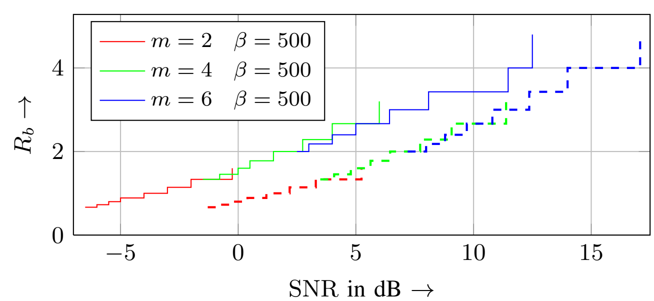


Fig. 5. Achievable rates $R_b$ for FER $= 10^{-2}$ versus SNR on direct link exploiting compression indices $\mathbf{z_y}$ ($\beta = 500$) received through the relay. Additionally dashed lines correspond to the rates of Fig. 3.

To visualize the gain in the destination's decoder due to the compression indices from the relay, Fig. 5 shows exemplary achievable rates $R_b^{S1}$ (direct quantization) in the broadcast phase versus the SNR of the direct link for a typical CF scenario with $\beta = 500$ for all quantizers. Such rates $R_b^{S1}(\beta)$ are obtained for all $\beta \in [1, 5, 25, 100, 200, 500]$ in combination with 4 turbo iterations at the relay for the second approach. Hence, $R_b^{S1}(\beta)$ can be straightforward determined given the SNR

$$\mathrm{SNR} = a_{SD}^2 \cdot P_{S1}. \tag{14}$$

In the following, the total CF-rates $R_{CF}$ will be determined by means of Figures 3 and 5 as well as the corresponding SNRs of the specific links. Regarding the SNR or SINR, respectively, of the MAC phase (3), non-orthogonal and orthogonal channel access need to be distinguished.

[1]Please remember that the Wyner-Ziv coding and decoding as well as the transmission $R \rightarrow D$ are not included into the simulation rather the compression indices $\mathbf{z} \in \{\mathbf{z_y}, \mathbf{z_u}\}$ are assumed to be available at the destination's decoder.

### A. Orthogonal Channel Access

For the orthogonal scheme, $P_{S2} = 0$ holds. Hence, the SNR of (3) to determine $R_b^R$ is given by

$$\mathrm{SNR} = a_{RD}^2 \cdot P_R. \tag{15}$$

The total throughput, where $R_b^{S1}(\beta)$ is determined with (14) as depicted in Fig. 5, is

$$R_{CF}^{orth} = \max_{\tau} \{\tau R_b^{S1}(\beta)\} \text{ s.t. } \tau \cdot R_s(\beta) \leq (1-\tau) R_b^R, \tag{16}$$

where $R_s(\beta) = 2 \cdot r_y(\beta)$, or $R_s(\beta) = R_b^{S1}(\beta) \cdot r_u(\beta)$ depending on the processing strategy of the relay introduced in Sec. IV.[2] Solving the condition in (16) with equality gives $\tau$ depending on a specific source coding rate $r(\beta) \in \{r_y(\beta), r_u(\beta)\}$ since $R_b^R$ and $R_b^{S1}(\beta)$ are given due to (15) and (14), respectively. Hence, the maximizing rate is chosen from all available rates $r(\beta)$.

### B. Non-orthogonal Channel Access

For the non-orthogonal scheme in (3), the SINR[3]

$$\mathrm{SINR} = \frac{a_{RD}^2 \cdot P_R}{a_{SD}^2 P_{S2} + 1} \tag{17}$$

is used to determine $R_b^R$. Hence, the interference $a_{SD}\sqrt{P_{S2}}\mathbf{x_{S2}}$ is treated as noise. After interference cancellation, the SNR with respect to $\mathbf{x_{S2}}$, to determine the rate $R_b^{S2}$, is given as

$$\mathrm{SNR} = a_{SD}^2 \cdot P_{S2}. \tag{18}$$

Similarly as above, the total throughput is

$$R_{CF}^{non} = \max_{\tau} \{\tau R_b^{S1}(\beta) + (1-\tau) R_b^{S2}\} \text{ s.t. } \tau \cdot R_s(\beta) \leq (1-\tau) R_b^R, \tag{19}$$

where $(1-\tau)R_b^{S2}$ depicts the non-orthogonal amount of information.

## VI. Results

For the results in this section, all nodes are placed on a line with $S$, $D$ and $R$ at positions 0, 1, and $d$, respectively. Its transmit powers are set equally to $P_{S1} = P_{S2} = P_R = P$. To ensure a fair comparison the total consumed energy per transmission will be considered as follows

$$E_{total}^{non} = \tau_{non} P + (1-\tau_{non}) 2P \quad = (2-\tau_{non})P \tag{20a}$$
$$E_{total}^{orth} = \tau_{orth} P + (1-\tau_{orth}) P \quad = P, \tag{20b}$$

whereby the total transmission time is normalized to 1. Fig. 6 shows results for $R$ being at $d = 0.8$. As expected, CF outperforms DF (Results taken from [7]) almost in the whole depicted range due to $S \rightarrow R$ being the bottleneck of the system. Anyhow for some SNRs in the low range, DF is superior or equally good. Moving the relay closer to the destination at $d = 0.9$ as in Fig. 7, DF is clearly outperformed by CF in the whole range.

[2]Here, the factors in front of the specific source coding rates $r_y$, and $r_u$ (given by the IB method) denote the rate change due the specific pre-processing at the relay. The factor 2 related to $r_y$ is necessary because inphase and quadrature component are quantized independently.

[3]Please remember that the noise power is normalized to $\sigma_N^2 = 1$

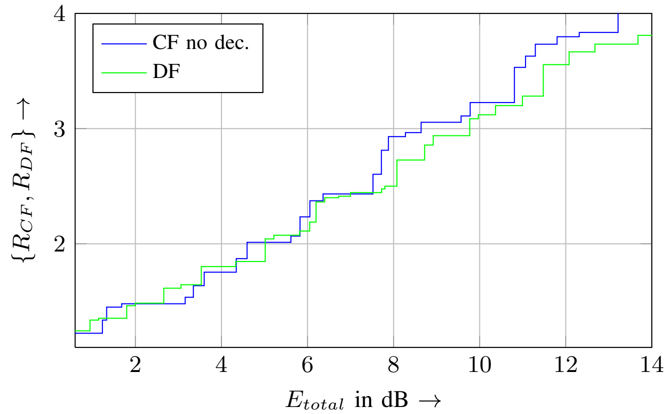


Fig. 6. Total achievable rates for CF and DF versus SNR at $d = 0.8$ for non-orthogonal channel access in the MAC phase. Non-orthogonal scheme considered.

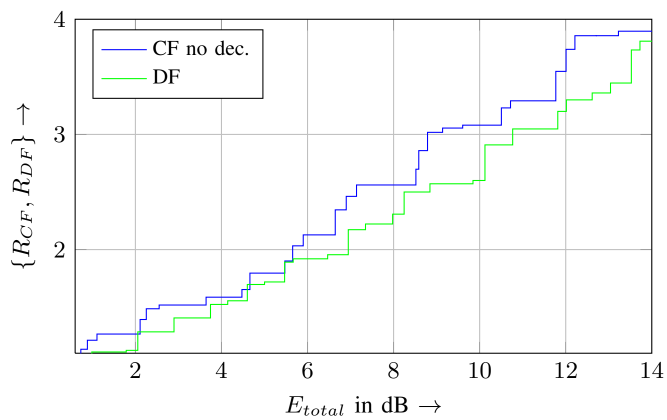


Fig. 7. Total achievable rates for CF and DF versus SNR at $d = 0.9$ for non-orthogonal channel access in the MAC phase. Non-orthogonal scheme considered.

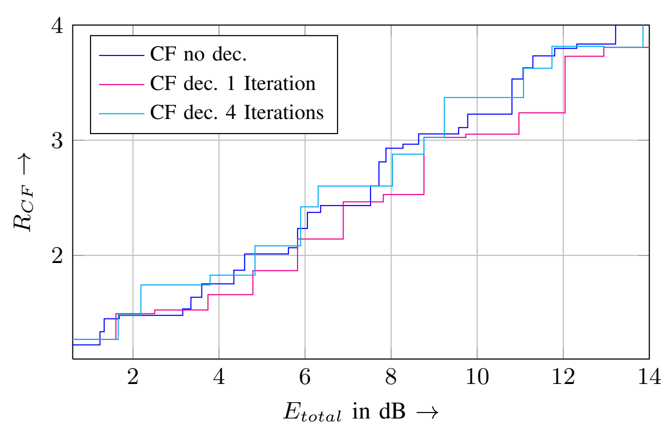


Fig. 8. Total achievable rates for CF versus SNR at $d = 0.8$ for strategies, where the relay performs either direct quantization (blue) or additional soft-output decoding (red). Non-orthogonal scheme considered

Regarding the different processing strategies (cf. Fig. 8) at the relay, it becomes visible that soft-output decoding with 4 iterations at the relay prior to quantization can be rewarding for some SNRs. Generally however, there is no significant advantage (sometimes even a loss) compared to direct quantization. It has to be noted that direct quantization affects modulated symbols while the quantization of the decoder output (bit level) may affect considerably more values depending on the spectral efficiency. For less iterations, e.g., only one, decoding at the relay becomes even worse. In a nutshell, the additional effort due to decoding does not pay off.

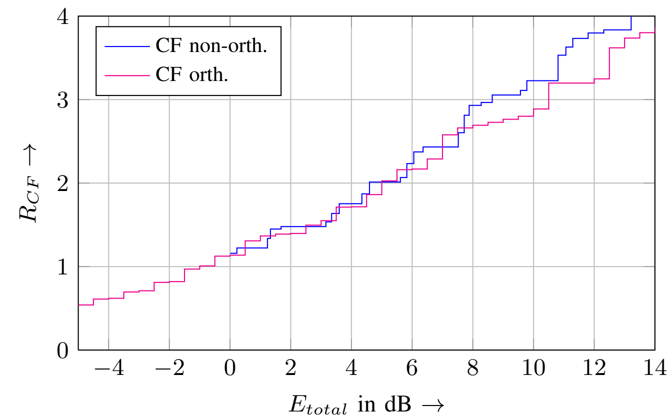


Fig. 9. Total achievable rates for CF versus SNR at $d = 0.8$ comparing orthogonal to non-orthogonal channel access in the MAC phase for direct quantization at the relay.

Let us next compare the non-orthogonal to the orthogonal scheme. Fig. 9 shows corresponding rates for direct quantization at $d = 0.8$. In the low SNR-range up to 0 dB, no MCS supports the transmission of new information over the direct link, i.e. the non-orthogonal scheme brings no benefit. For moderate SNRs both schemes perform similarly, that is, the additional effort to separate the signals received by source and relay in the MAC phase is not worthwhile. In fact, this statement holds only for fixed total energy consumption. Without the energy constraint (20), the non-orthogonal scheme would be slightly advantageous. In the high SNRs-range however, the non-orthogonal scheme clearly outperforms the orthogonal even with the total energy constraint. The reason lies in the limited range of available MCSs in practice. Please note that the highest considered MCS of 8Bit/s corresponds to a 1024-QAM with a code rate of $4/5$ which is even beyond state of the art implementations. Due to this limit, the $R \rightarrow D$ link saturates for $d \rightarrow 1$ and, thus, its capacity cannot be exploited. Consequently, more time is needed in the MAC phase to transmit the information from $R$ to $D$. Hypothetically, if the capacity of the link could be exploited, the total throughput would be greater due to more available time in the first time slot. This effect degrades the orthogonal more than the non-orthogonal scheme because the non-orthogonal scheme reveals a huge gain due to the direct link in the second time slot.

Another interesting question is whether the loss due to suboptimal deterministic quantization compared to optimal random quantization carries weight regarding the limited amount of discrete rates $R_b$. Thus, Fig. 10 compares optimum random with deterministic quantization which may also be determined by the IB method setting $\beta \to \infty$. In the whole range, the scheme using deterministic quantization is outperformed by the one using optimal random quantization, and is consequently not a recommendable alternative.

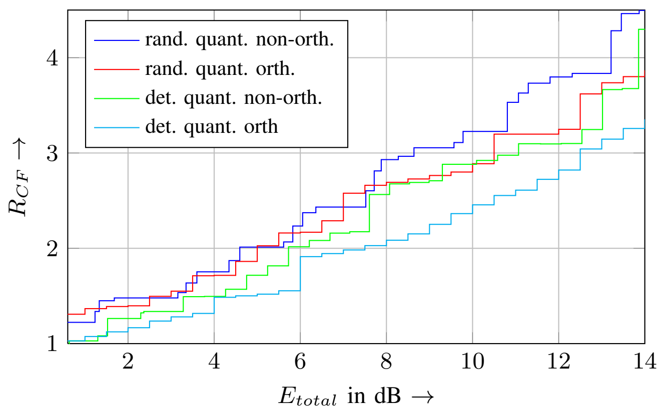


Fig. 10. Total achievable rates for CF versus SNR at $d = 0.8$ comparing random and deterministic quantization at the relay. Direct quantization without decoding at the relay is considered.

## VII. Conclusion

In this paper, practical aspects of the CF relay protocol were investigated. The major emphasis was the analysis of optimum practical quantization obtained by the iterative IB algorithm and the optimal integration of outcoming quantization indices into a joint decoder at the destination. As a result achievable rates of a system with state of the art coding and modulation were obtained by the help of Monte Carlo simulations. Therefore, the available MCSs provide rates $R_b \in [2/3, 8]$ Bit/s which cover the practically relevant range. Furthermore, it was investigated if additional soft-output-decoding at the relay prior to quantization improve the performance of the overall system. Results reveal that there is small gain for some SNRs. Considering the additional effort however, it is not worthwhile. In comparison to the DF-protocol, CF is mostly superior or at least similarly good, and, therefore preferable for relay positions close to the destination. Results regarding non-orthogonal channel access in the MAC phase expose on the one hand an advantage in the high SNR-range against straightforward orthogonal access. On the other hand, the additional effort of the non-orthogonal scheme to separate the superimposed signals of source and relay does not pay off in the moderate SNR-range. For low SNRs non-orthogonal channel access cannot be established due to the in practice limited range of available MCSs. Finally, it was shown that the use of more complex random quantizers leads to a high gain in performance compared to the use of deterministic quantizers which need a much higher compression rate.

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