An Interference Management Scheme using Partial CSI for Partially Connected Cellular Networks

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***Abstract*—We consider a partially connected cellular network where amplify-and-forward relays are deployed in order to help the cell-edge users to suppress the inter-cell interferences in the downlink transmission. The one-way half-duplex relaying protocol is employed. An interference management scheme which achieves interference-free transmissions in the entire network using only partial channel state information is proposed. The feasibility conditions for the proposed scheme are discussed. Furthermore, the problem of how to maximize the achieved sum rate using the considered partial channel state information is addressed. Both a sum power constraint and individual power constraints are considered. The simulation results show that the proposed scheme is able to achieve an outstanding performance, especially if the network is sparse.**

I. Introduction

In future wireless communication networks, relays can be exploited not only for conventional purposes like coverage extension, but also for interference management [1]. In particular, deploying relays in cellular networks to suppress the inter-cell interference is of practical interest. To this purpose, a promising technique is relay-aided interference alignment (IA). Unlike IA without relays, it has been shown in literature that relay-aided IA requires only few antennas at the transmitters and the receivers, and that it has closed form solutions [2]. However, an obvious drawback of relay-aided IA, which is the same as for most of the conventional IA schemes, is the requirement of full channel state information (CSI). That is to say, every transmitter, every receiver, and every relay must be aware of all the channels in the entire network, which results in a severe signaling overhead. Finding an efficient and practical IA scheme without the full CSI requirement remains a challenging open topic in general. For IA without relays, a number of schemes without the full CSI requirement have been published, e.g., IA with topological knowledge of the network [3], IA with outdated CSI [4], or IA even with no CSI at all [5], [6]. Unfortunately, little work has been done in this direction for relay-aided IA.

In this work, we consider a cellular network where relays are deployed in order to help the cell-edge MSs to suppress the inter-cell interferences in the downlink transmission. The one-way half-duplex relaying protocol is employed. The cells in the entire network are divided up into several disjoint subsets such that each subset includes a few adjacent cells. A relay is deployed in each subset of cells. We assume that in each cell, a single mobile station (MS) is located close to the relay and a base station (BS) located at the cell center serves the MS. Due to path loss effect of wireless communications, such a cellular network can be considered as a partially connected network by neglecting some relatively weak links. In particular, we will employ the concept of subnetworks to describe the topology of the considered network. A subnetwork is formed by a subset of cells and the relay deployed therein. We assume that every subnetwork is fully connected, but the different subnetworks may be only partially connected to each other.

In our preliminary work [7], an ad-hoc network consisting of multiple partially connected subnetworks is considered. An IA scheme using partial CSI to perfectly suppress all the interferences in the entire network is proposed. However, the subnetworks in [7] are only connected by a few direct links between the source and destination nodes in different subnetworks. In the current work, we further assume that a relay may also receive interferences from a few BSs in other subnetworks. How to suppress these interferences, especially if full CSI is not available, is the new challenge. Extended from the IA scheme proposed in [7], we propose a scheme to nullify all the interferences in the considered cellular network using partial CSI.

The rest of this paper is organized as follows. In the next section, the considered network topology and the system model will be introduced. In Sec. III, we will introduce the considered partial CSI. Our scheme is proposed in Sec. IV and the feasibility conditions are discussed accordingly. In Sec. V, we will tackle the sum rate maximization problem in the considered networks using partial CSI. Followed by the simulation results shown in Sec. VI, we will conclude this paper.

II. System Model

We consider the downlink transmission in a cellular network consisting of K cells. Let the K cells be divided into Q disjoint subsets such that the q-th subset includes K_q adjacent cells. In the q-th subset of cells, a half-duplex amplify-and-forward relay equipped with N_q antennas is deployed to help the nearby MSs to suppress the inter-cell interferences. We assume that in each cell a single MS equipped with one antenna is located close to the relay and the BS uses a single antenna to serve the MS. We refer to the K_q BS-MS pairs along with the q-th relay as a subnetwork in the following of this paper.

Due to path losses, such a network typically can be considered as partially connected. In particular, we assume that each subnetwork is fully connected, i.e., the q-th relay is connected to all the K_q BSs and MSs in the q-th subnetwork and each one of the MSs directly receives signals from all the

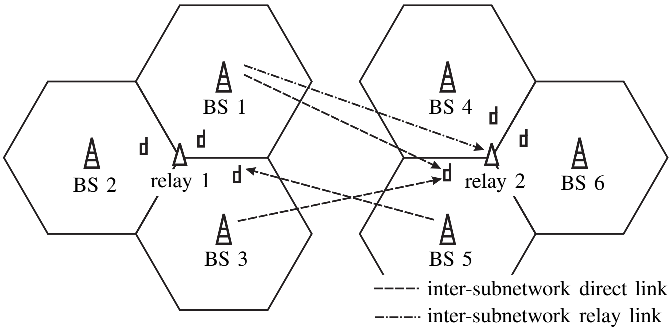

Fig. 1. An example of partially connected cellular networks. Three inter-subnetwork direct links and an inter-subnetwork relay link are present between the two subnetworks. The intra-subnetwork links are not depicted.

K_q BSs in the subnetwork. Furthermore, the relay and some of the MSs in a subnetwork may also receive interference from a few BSs in the other subnetworks, which will be referred to as inter-subnetwork interferences. However, the relay in one subnetwork is assumed not to interfere with the MSs in the other subnetwork. Fig. 1 shows an example of such a partially connected cellular network, where relay 1 and the three cells close to it form a subnetwork, while the other three cells and relay 2 form another subnetwork. Three inter-subnetwork direct links and an inter-subnetwork relay link are depicted in the figure.

A synchronized two-hop transmission scheme is applied incorporating the idea of relay-aided IA. In the first time slot, each BS transmits a single data symbol to the connected relays and MSs. Every relay will then forward a linearly processed signal to the connected MSs in the second time slot while the BSs transmit again to the connected MSs. The channels are assumed to remain constant throughout the transmission. We must point out that in some special cases, the considered two-hop transmission scheme does not require the deployment of relays to achieve interference-free transmission. For instance, if a subnetwork includes only one or two cells and the MSs in the subnetwork do not receive any inter-subnetwork interference, the problem is trivial. To exclude these special cases, we assume that every subnetwork includes at least three cells.

Let $d^{(k)}$ denote the data symbol being transmitted by the k-th BS. The transmit filter at the k-th BS and the receive filter at the j-th MS are denoted by $(v_1^{(k)}, v_2^{(k)})^{\mathrm{T}}$ and $(u_1^{(j)}, u_2^{(j)})^{\mathrm{T}}$, respectively. The processing matrix at the q-th relay is denoted by the $N_q \times N_q$ matrix $\mathbf{G}^{(q)}$. Furthermore, the channel between the k-th BS and the j-th MS is denoted by the scalar $h_{\mathrm{MB}}^{(j,k)}$. The channel between the k-th MS and the q-th relay is denoted by the $N_q \times 1$ vector $\mathbf{h}_{\mathrm{RB}}^{(q,k)}$. Finally, the channel between the q-th relay and the j-th MS is denoted by the $1 \times N_q$ vector $\mathbf{h}_{\mathrm{MR}}^{(j,q)}$. The channel coefficients of the absent links are set to zero. The channel coefficients of the present links are assumed to be independently Gaussian distributed.

Using the notations introduced above, the signal received by the j-th MS in the first time slot and the signals received by the q-th relay can then be written as

$$r_{\mathrm{M},1}^{(j)} = \sum_{k=1}^{K} h_{\mathrm{MB}}^{(j,k)} v_1^{(k)} d^{(k)} + n_{\mathrm{M},1}^{(j)} \tag{1}$$

and

$$\mathbf{r}_{\mathrm{R}}^{(q)} = \sum_{k=1}^{K} \mathbf{h}_{\mathrm{RB}}^{(q,k)} v_1^{(k)} d^{(k)} + \mathbf{n}_{\mathrm{R}}^{(q)}, \tag{2}$$

where $n_{\mathrm{M},1}^{(j)}$ denotes the noise received by the j-th MS in the first time slot and $n_{\mathrm{R}}^{(q)}$ denotes the noise received by the q-th relay, both being independently identically distributed (i.i.d.) Gaussian noise with the variance σ_{M}^2 and σ_{R}^2, respectively. The signal received by the j-th MS in the second time slot can be written as

$$r_{\mathrm{M},2}^{(j)} = \mathbf{h}_{\mathrm{MR}}^{(j,q)} \mathbf{G}^{(q)} \mathbf{r}_{\mathrm{R}}^{(q)} + \sum_{k=1}^{K} h_{\mathrm{MB}}^{(j,k)} v_2^{(k)} d^{(k)} + n_{\mathrm{M},2}^{(j)}, \tag{3}$$

where $n_{\mathrm{M},2}^{(j)}$ represents the i.i.d. Gaussian noise with the variance σ_{M}^2 received by the j-th MS in the second time slot. In (3), the j-th MS is implicitly assumed, without loss of generality, to belong to the q-th subnetwork, and hance is only connected to the q-th relay. Afterwards, each MS linearly combines the signals it received in the two time slots and estimates the data symbol as

$$\hat{d}^{(j)} = u_1^{(j)*} r_{\mathrm{M},1}^{(j)} + u_2^{(j)*} r_{\mathrm{M},2}^{(j)}. \tag{4}$$

We aim at nullifying all the interferences in the entire network, i.e., the transmit filters at the BSs, the receive filters at the MSs, and the relay processing matrices shall be designed such that the interference-nulling (IN) condition

$$u_2^{(j)*} \mathbf{h}_{\mathrm{MR}}^{(j,q)} \mathbf{G}^{(q)} \mathbf{h}_{\mathrm{RB}}^{(q,k)} v_1^{(k)} + u_1^{(j)*} h_{\mathrm{MB}}^{(j,k)} v_1^{(k)} + u_2^{(j)*} h_{\mathrm{MB}}^{(j,k)} v_2^{(k)} = 0 \tag{5}$$

is satisfied for any $k \neq j$, where the j-th MS is assumed to belong to the q-th subnetwork. Equation (5) corresponds to an intra-subnetwork IN condition if the k-th BS also belongs to the q-th subnetwork, and an inter-subnetwork IN condition otherwise. In this paper, we focus on solving the IN conditions using only partial CSI. The specific required types of CSI will be discussed in the next section.

III. Interference Nulling and CSI Requirement

Equation (5) is in general a tri-linear equation of the transmit filters of the BSs, the receive filters of the MSs, and the processing matrices of the relays, which is difficult to solve. To tackle this issue, we introduce the quotients of the filter coefficients $v^{(k)} = v_2^{(k)}/v_1^{(k)}$ and $u^{(j)} = u_1^{(j)}/u_2^{(j)}$. In fact, the quotients $v^{(k)}$ and $u^{(j)}$ specify the one-dimensional transmit signal subspace at a BS and the one-dimensional receive signal subspace at a MS, respectively. Using $v^{(k)}$ and $u^{(j)}$, the IN condition of (5) can be linearized as

$$\mathbf{h}_{\mathrm{MR}}^{(j,q)} \mathbf{G}^{(q)} \mathbf{h}_{\mathrm{RB}}^{(q,k)} + h_{\mathrm{MB}}^{(j,k)} \left(v^{(k)} + u^{(j)*} \right) = 0, \tag{6}$$

where the quotients $v^{(k)}$, $u^{(j)*}$, and the elements of $\mathbf{G}^{(q)}$ are chosen to be the unknowns. A proper choice of the unknowns for all BSs, MSs, and relays which satisfies all the linearized IN conditions in the form of (6) in the entire network is referred to as an IN solution. Obviously, such a solution can be obtained by solving a system of linear equations, if full CSI is available either at a central control unit or at all the BSs, MSs, and the relays. However, this centralized approach does not exploit the partial connectivity in the considered networks to reduce the

CSI required for solving the IN problem. Basically, two types of interferences shall be nullified in the considered networks: the intra-subnetwork interferences and the inter-subnetwork interferences.

If the k-th BS also belongs to the q-th subnetwork, as the j-th MS does, (6) corresponds to a linearized intra-subnetwork IN condition. In this case, all the channel coefficients in (6) are almost surely non-zero. The intra-subnetwork IN problem in each individual subnetwork is essentially a relay-aided IA problem in fully connected networks [2], [8]. If the intra-subnetwork CSI is available, i.e., the channel realizations of all the intra-subnetwork links are known, an intra-subnetwork IN solution can be found by solving all the linearized intra-subnetwork IN conditions for each individual subnetwork. For instance, the q-th relay may act as a local control unit, which collects the intra-subnetwork CSI of the q-th subnetwork and computes the corresponding intra-subnetwork IN solution for the q-th subnetwork. However, the different subnetworks cannot choose their intra-subnetwork IN solutions independently, mainly due to the potential existence of inter-subnetwork links between them.

If the k-th BS does not belong to the q-th subnetwork, (6) corresponds to a linearized inter-subnetwork IN condition. Except that neither the inter-subnetwork direct link $h_{\mathrm{MB}}^{(j,k)}$ nor the inter-subnetwork relay link $\mathbf{h}_{\mathrm{RB}}^{(q,k)}$ is present, the j-th MS will receive inter-subnetwork interference from the k-th BS. Therefore, based on the network topology, the following three cases shall be distinguished and discussed separately.

Case 1. The k-th BS is connected to the q-th relay, but not connected to the j-th MS, i.e., $\mathbf{h}_{\mathrm{RB}}^{(q,k)} \neq 0$ and $h_{\mathrm{MB}}^{(j,k)} = 0$ hold. In this case, the j-th MS receives inter-subnetwork interference from the k-th BS only through the q-th relay. Therefore, the linearized inter-subnetwork IN condition of (6) can be simplified as

$$\mathbf{h}_{\mathrm{MR}}^{(j,q)} \mathbf{G}^{(q)} \mathbf{h}_{\mathrm{RB}}^{(q,k)} = 0. \tag{7}$$

It can be observed from (7) that neither the k-th BS nor the j-th MS is able to help in nullifying the inter-subnetwork interference. This is essentially an interference neutralization problem first discussed in [1]. Consequently, the q-th relay must forward the received inter-subnetwork interference in the null space of the channel vector $\mathbf{h}_{\mathrm{MR}}^{(j,q)}$ by properly designing its processing matrix. To this end, the q-th relay must have the receiver side CSI of the channel between itself and the k-th BS, in addition to the intra-subnetwork CSI of the q-th subnetwork. Otherwise the q-th relay is forced to be shut down and interference-free transmission is not achievable in the considered network.

Case 2. The k-th BS is directly connected to the j-th MS, but not connected to the q-th relay, i.e., $\mathbf{h}_{\mathrm{RB}}^{(q,k)} = 0$ and $h_{\mathrm{MB}}^{(j,k)} \neq 0$ hold. In this case, the j-th MS receives inter-subnetwork interference from the k-th BS only through the inter-subnetwork direct link between them. Therefore, the linearized inter-subnetwork IN condition of (6) can be simplified as

$$v^{(k)} + u^{(j)*} = 0. \tag{8}$$

In this case, the following three conclusions can be made. Firstly, the q-th relay is not able to help in nullifying the inter-subnetwork interference. Secondly, the channel realization $h_{\mathrm{MB}}^{(j,k)}$ of the inter-subnetwork direct link is irrelevant for nulling the inter-subnetwork interference. Instead, only the presence of the inter-subnetwork direct link needs to be known. Secondly, the one-dimensional transmit signal subspace at the k-th BS specified by $v^{(k)}$ and the one-dimensional receive signal subspace at the j-th MS specified by $u^{(j)}$ must be orthogonal. That is to say, either the k-th BS or the j-th MS is able to determine its own signal subspace if it knows the choice of the signal subspace of the other. For instance, the k-th BS can first choose its transmit signal subspace and then forward this information, i.e., $v^{(k)}$, to the j-th MS using pilots. Afterwards, the j-th MS can choose its receive signal subspace to nullify the inter-subnetwork interference.

Case 3. The k-th BS is connected to both the j-th MS and the q-th relay, thus all channel coefficients in (6) are almost surely non-zero. In this case, the j-th MS receives inter-subnetwork interference from the k-th BS through both the inter-subnetwork direct link between them and the q-th relay. In order to nullify the inter-subnetwork interference at the j-th MS, two strategies are possible. The first option is to jointly design the transmit filters of the k-th BS, the receive filters of the j-th MS, and the processing matrix of the q-th relay in order to the satisfy the inter-subnetwork IN condition. The second option is to decompose the inter-subnetwork IN condition as

$$\begin{cases} \mathbf{h}_{\mathrm{MR}}^{(j,q)} \mathbf{G}^{(q)} \mathbf{h}_{\mathrm{RB}}^{(q,k)} = 0 \\ v^{(k)} + u^{(j)*} = 0 \end{cases} \tag{9}$$

in a suboptimal way. This suggests that the inter-subnetwork interferences propagating through the inter-subnetwork direct link and through the q-th relay are nullified separately as discussed in the previous two cases. The advantage of doing so is to avoid exchanging CSI between the two subnetworks. More specifically, the q-th subnetwork only needs to have the receiver side CSI of the inter-subnetwork relay link and to know the presence of the inter-subnetwork direct link. Since we mainly focus on partial CSI in this work, only the second option will be considered in the following. A detailed comparison of the two options is beyond the scope of this paper.

Based on the previous discussions in this section, we conclude that it is possible to find an IN solution in the considered partially connected networks without using full CSI. Specifically, only the following four types of information are required. Firstly, each subnetwork shall have its intra-subnetwork CSI to nullify the intra-subnetwork interferences. Secondly, if the relay in one subnetwork receives interferences from the BSs in other subnetworks, the receiver side CSI of the inter-subnetwork relay link shall be available at the relay. Thirdly, each subnetwork shall know the network topology, i.e., the presence of the inter-subnetwork direct links. Finally, in order to nullify the interference propagating through an inter-subnetwork direct link, the choice of the signal subspace at one end of the link shall be known by the other one. We will refer to this as side information in the following of the paper. The side information can be considered as a compressed version of the CSI of the other subnetworks, which contains the relevant information for inter-subnetwork IN. In the next section, a scheme which only uses the above four types of information to nullify all the interferences in the considered network will be proposed.

IV. Proposed Scheme and Feasibility Conditions

The scheme proposed in this section is extended from the IA scheme using partial CSI proposed in our previous work [7]. We first introduce a few terms which will be used later. The intra-subnetwork IN solution space of the q-th subnetwork $\mathbb{W}_q$ is the solution space of the system of linear equations consisting of all the linearized intra-subnetwork IN conditions (6) in the q-th subnetwork. Each subnetwork only needs the intra-subnetwork CSI to obtain its intra-subnetwork IN solution space. As discussed in Sec. III, not all the intra-subnetwork IN solutions in $\mathbb{W}_q$ are able to nullify the potential inter-subnetwork interferences in the rest of the network. Arising from the inter-subnetwork IN conditions, some other constraints shall also be considered by the q-th subnetwork in addition to its intra-subnetwork IN conditions. Two types of additional constraints shall be considered by each subnetwork in principle.

The first type of additional constraints results from nullifying the inter-subnetwork interferences which propagate through the inter-subnetwork direct links. We refer to this type of constraints as the external constraints. They have been defined and discussed in our previous works [9] and [7]. For reasons of completeness, the definition is given again as follows.

Definition 1 (External Constraints): A path consisting of present inter-subnetwork links results in an external constraint between the end nodes of the path, which only depends on the types of the end nodes of the path. Specifically, if such a path exists between the k-th BS and the j-th MS, the external constraint $v^{(k)} + u^{(j)*} = 0$ follows. If such a path exists between two BSs or between two MSs, the corresponding external constraint is $v^{(k)} = v^{(j)}$ or $u^{(k)} = u^{(j)}$, respectively.

For instance in the network shown in Fig. 1, both BS 1 and BS 3 are connected to a common MS in subnetwork 2. To nullify both inter-subnetwork interferences at the MS simultaneously, subnetwork 1 has to choose an intra-subnetwork IN solution which also satisfies the external constraint

$$v^{(1)} = v^{(3)}. \tag{10}$$

Equation (10) implies that the transmit signal subspaces at BS 1 and BS 3 are parallel. From an IA perspective, it is the only way to align the two inter-subnetwork interferences in a one dimensional subspace at the MS. Since subnetwork 1 knows the network topology, (10) shall be, and can be considered by subnetwork 1 in addition to its intra-subnetwork IN conditions.

The second type of additional constraints results from nulling the inter-subnetwork interferences which propagate through the relays. In the same example, the signals transmitted by BS 1 will also be received by relay 2. Therefore, relay 2 must forward these signals in such a way that they are nullified at all the MSs in subnetwork 2 according to (7). These constraints shall be considered by subnetwork 2 in addition to its intra-subnetwork IN conditions. This is achievable since relay 2 has the receiver side CSI of the inter-subnetwork relay link and the intra-subnetwork CSI.

In general, all the intra-subnetwork IN conditions for the q-th subnetwork, the external constraints for the q-th subnetwork, and the constraints due to the inter-subnetwork relay links between the q-th relay and the BSs in other subnetworks together define an IN solution space $\mathbb{W}'_q$ of the q-th subnetwork. Clearly, $\mathbb{W}'_q$ is a subspace of $\mathbb{W}_q$. Any intra-subnetwork IN solution for the q-th subnetwork which is not in $\mathbb{W}'_q$ would not be able to nullify all the interferences in the entire network for sure. Furthermore, $\mathbb{W}'_q$ can be fully determined by the q-th subnetwork using the partial CSI which is available at the q-th subnetwork. Therefore, solving the IN problem in the entire network is decomposed to properly choosing an IN solution for each subnetwork in its IN solution space $\mathbb{W}'_q$. The main difference between the IN scheme proposed in this paper and the IA scheme proposed in [7] is the way to obtain the IN solution space $\mathbb{W}'_q$ of each subnetwork. In [7], only the first type of additional constraints, i.e., the external constraints, need to be considered. However, in the networks considered in this paper, the relays may also receive interferences from some of the BSs in other subnetworks. Therefore, the idea of interference neutralization is exploited to nullify these interferences. Consequently, the second type of additional constraints shall also be considered in order to find $\mathbb{W}'_q$. Once the IN solution space $\mathbb{W}'_q$ of each subnetwork is obtained, the following procedure will be similar to the scheme proposed in [7]. Therefore, it will only be briefly described as follows. A more detailed description can be found in [7].

For a general network consisting of Q subnetworks, the proposed scheme needs Q steps. In each step, one of the Q subnetworks chooses a solution from its IN solution space $\mathbb{W}'_q$. To facilitate the description, let the subnetworks be indexed such that the q-th subnetwork will choose its IN solution in the q-th step in the following part of this section. The q-th subnetwork shall be connected to at least one of the $1, 2, \ldots, q-1$ subnetworks by external constraints. The IN solution of the q-th subnetwork shall be chosen in such a way that all the external constraints between the q-th subnetwork and the subnetworks $1, 2, \ldots, q-1$ are nullified using the side information obtained from those subnetworks. Once the IN solution for the q-th subnetwork is determined, it then forwards the side information to those remaining subnetworks which are connected to the q-th subnetwork by external constraints. Specially, the 1st subnetwork can be any arbitrary one. In practical networks, we can assume that the inter-subnetwork direct links as well as the external constraints only exist between two neighboring subnetworks due to the power constraints of the BSs. Exploiting this, the above scheme can be modified to allow several subnetworks which are not connected by external constraints to choose their IN solutions simultaneously. This would not lead to any conflict since the subnetworks which are not connected by external constraints do not need to exchange side information between each other.

In the following, we will derive the feasibility conditions for the proposed scheme. It is clear that the feasibility conditions depend on the topology of the considered network, i.e., the present inter-subnetwork direct and relay links. First of all, suppose the q-th relay is connected to M_q BSs in other subnetworks. Nullifying all these inter-subnetwork interferences at every MS in the q-th subnetwork results in $M_q K_q$ additional constraints in the form of (7). Secondly, the number of linearly independent external constraints for the q-th subnetwork can be found using the graphical method proposed in [9]. Let a set of subnetworks S be the subnetworks being indexed by the elements of $S \subseteq \{1, \ldots, Q\}$. The external constraints with both ends belonging to the set of subnetworks S can be represented by the edges of a graph G_S. It has been shown in [9] that the rank of the incidence matrix of the graph, which is simply denoted by $\operatorname{rank}(G_S)$, represents the number of linearly independent external constraints for S. Therefore, $\operatorname{rank}\left(G_{\{q\}}\right)$ is the number of linearly independent external constraints for the q-th subnetwork. Since all channel coefficients are assumed to be independent, we can conclude, from an engineering point of view, that the additional constraints for the q-th subnetwork are almost surely linearly independent from the intra-subnetwork IN conditions of the q-th subnetwork if the dimension of its intra-subnetwork IN solution space $\mathbb{W}_q$ is sufficiently large. Hence, the dimension of the IN solution space $\mathbb{W}'_q$ of the q-th subnetwork is given by

$$\dim \mathbb{W}'_q = N_q^2 - K_q(K_q - 3) - M_q K_q - \operatorname{rank}\left(G_{\{q\}}\right). \quad (11)$$

Note that the IN solution space $\mathbb{W}'_q$ has to be at least two dimensional, otherwise all the useful signals in the q-th subnetwork will be aligned with the interferences, see [10]. Therefore, the condition

$$K_q(K_q - 3) + M_q K_q + \operatorname{rank}\left(G_{\{q\}}\right) \leq N_q^2 - 2 \quad (12)$$

has to be satisfied. Furthermore, the dimension of $\mathbb{W}'_q$ has to be large enough such that there exists at least one IN solution in $\mathbb{W}'_q$, which also fulfills the external constraints between the q-th subnetwork and the subnetwork $1, \ldots, q-1$. Thus the dimension of $\mathbb{W}'_q$ needs to be at least $\operatorname{rank}\left(G_{\{1,\ldots,q\}}\right) - \operatorname{rank}\left(G_{\{1,\ldots,q-1\}}\right) - \operatorname{rank}\left(G_{\{q\}}\right)$ as shown in [7]. Therefore, the condition

$$K_q(K_q - 3) + M_q K_q + \operatorname{rank}\left(G_{\{1,\ldots,q\}}\right) - \operatorname{rank}\left(G_{\{1,\ldots,q-1\}}\right) \leq N_q^2 \quad (13)$$

must also be satisfied. To summarize, the proposed IN scheme is almost surely feasible if and only if (12) and (13) are both satisfied for the q-th subnetwork, where $q = 1, \ldots, Q$. Note that the feasibility conditions for the proposed scheme depend on the order in which the individual subnetworks choose their IN solutions. For instance in the network shown in Fig. 1, if subnetwork 1 wants to choose its IN solution first, the number of antennas at relay 1 shall satisfy $N_1^2 \geq 3$ and the number of antennas at relay 2 shall satisfy $N_2^2 \geq 6$. However, if subnetwork 2 wants to choose its IN solution first, the number of antennas at relay 2 shall satisfy $N_2^2 \geq 5$ and the number of antennas at relay 1 shall satisfy $N_1^2 \geq 4$. If the number of antennas at relay 1 satisfies $N_1^2 \geq 4$ and the number of antennas at relay 2 satisfies $N_2^2 \geq 6$, either of the two subnetworks can choose its IN solution first. This is illustrated in Fig. 2.

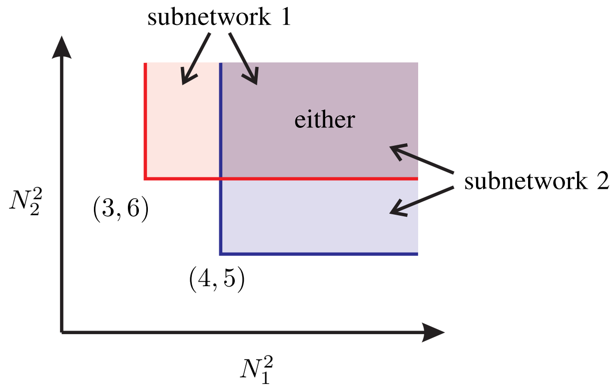

Fig. 2. Which subnetwork can choose its IN solution first depends on the numbers of antennas at the relays.

V. Reconstruction of the Filters

Suppose an IN solution for the entire network has been obtained using the scheme proposed in Sec. IV, which consists of the relay processing matrices $\mathbf{G}^{(q)}$, the quotients of the transmit filter coefficients of the BSs $v^{(k)}$, and the quotients of the receive filter coefficients of the MSs $u^{(j)}$. Given this IN solution, the remaining problem is to reconstruct the transmit filters of the BSs and the receive filters of the MSs. In this work, the filters, especially the transmit filters of the BSs, will be reconstructed aiming at maximizing the sum rate achieved in each individual subnetwork using the considered partial CSI. For this problem, two kinds of power constraints will be considered: 1) an average power constraint for each individual BS and relay, and 2) an average sum power constraint for each subnetwork.

We assume that all the transmitted data symbols $d^{(k)}$ have unit variance, i.e., $\mathrm{E}\left\{\left|d^{(k)}\right|^2\right\} = 1$ holds for all k. Let the average sum power of the k-th BS, which can be considered as the total energy consumed by the BS in two time slots divided by the duration of a single time slot, be denoted by $p_{\mathrm{B}}^{(k)}$. Therefore, the average powers of the k-th BS in the first and in the second time slot can be represented by

$$p_{\mathrm{B},1}^{(k)} = \left|v_1^{(k)}\right|^2 = \frac{1}{1 + \left|v^{(k)}\right|^2} p_{\mathrm{B}}^{(k)} \quad (14)$$

and

$$p_{\mathrm{B},2}^{(k)} = \left|v_2^{(k)}\right|^2 = \frac{\left|v^{(k)}\right|^2}{1 + \left|v^{(k)}\right|^2} p_{\mathrm{B}}^{(k)}, \quad (15)$$

respectively. To facilitate our discussions, let the BS-MS pairs belonging to the q-th subnetwork be indexed by $1, 2, \ldots, K_q$. Then the average power of the q-th relay can be written as

$$\begin{aligned} p_{\mathrm{R}}^{(q)} &= \operatorname{tr}\left(\mathbf{G}^{(q)} \mathbf{r}_{\mathrm{R}}^{(q)} \mathbf{r}_{\mathrm{R}}^{(q)*\mathrm{T}} \mathbf{G}^{(q)*\mathrm{T}}\right) \\ &= \sum_{k=1}^{K_q} \frac{a^{(q,k)}}{1 + \left|v^{(k)}\right|^2} p_{\mathrm{B}}^{(k)} + \operatorname{tr}\left(\mathbf{G}^{(q)} \mathbf{G}^{(q)*\mathrm{T}}\right) \sigma_{\mathrm{R}}^2 \\ &\quad + \sum_{k=K_q+1}^{K} \frac{a^{(q,k)}}{1 + \left|v^{(k)}\right|^2} p_{\mathrm{B}}^{(k)}, \end{aligned} \quad (16)$$

where $a^{(q,k)}$ is defined to be

$$a^{(q,k)} = \mathbf{h}_{\mathrm{RB}}^{(q,k)*\mathrm{T}} \mathbf{G}^{(q)*\mathrm{T}} \mathbf{G}^{(q)} \mathbf{h}_{\mathrm{RB}}^{(q,k)} \quad (17)$$

to simplify the notations. In (16), the first summand represents the total power used to forward the signals which are transmitted by the BSs in the q-th subnetwork, the second summand represents the power used to forward the received noise, and the third summand represents the total power used to forward the intra-subnetwork interferences. For those BSs which are not connected to the q-th relay, the corresponding coefficients $a^{(q,k)}$ are zero. Assuming that the k-th MS belongs to the q-th subnetwork, the signal-to-noise ratio (SNR) achieved at the k-th MS is computed as

$$\gamma^{(k)} = \frac{b^{(k)} p_{\text{B}}^{(k)}}{c^{(k)}}, \tag{18}$$

where

$$b^{(k)} = \frac{\left| u^{(k)} h_{\text{MB}}^{(k,k)} + h_{\text{MB}}^{(k,k)} v^{(k)} + \mathbf{h}_{\text{MR}}^{(k,q)} \mathbf{G}^{(q)} \mathbf{h}_{\text{RB}}^{(q,k)} \right|^2}{1 + \left| v^{(k)} \right|^2} \tag{19}$$

represents the gain of the virtual channel resulting from the obtained IN solution, and

$$c^{(k)} = \left(\left| u^{(j)} \right|^2 + 1 \right) \sigma_{\text{M}}^2 + \left(\mathbf{h}_{\text{MR}}^{(j,q)} \mathbf{G}^{(q)} \mathbf{G}^{(q)*\text{T}} \mathbf{h}_{\text{MR}}^{(j,q)*\text{T}} \right) \sigma_{\text{R}}^2 \tag{20}$$

represents the effective noise variance.

First assume that the maximum sum power of the k-th BS is $P_{\text{B,max}}^{(k)}$ and the maximum power of the q-th relay is $P_{\text{R,max}}^{(q)}$. Then the sum rate maximization problem in the q-th network can be formulated as

$$\underset{p_{\text{B}}^{(k)}, k \in \{1,\ldots,K_q\}}{\arg\max} \left\{ \sum_{k=1}^{K_q} \text{ld} \left(1 + \frac{b^{(k)} p_{\text{B}}^{(k)}}{c^{(k)}} \right) \right\}, \tag{21}$$

subject to the individual power constraints

$$0 < p_{\text{B}}^{(k)} \leq P_{\text{B,max}}^{(k)} \qquad \forall\ k = 1, \ldots, K_q, \tag{22}$$

$$0 < p_{\text{R}}^{(q)} \leq P_{\text{R,max}}^{(q)}, \tag{23}$$

where $p_{\text{R}}^{(q)}$ is given by (16). Due to the power constraint for the relay (23), solving the above optimization problem requires that the q-th subnetwork knows the quotients of the transmit filter coefficients $v^{(k)}$ of those BSs which are connected to the q-th relay. However, it may happend that this information has not been forwarded to the q-th subnetwork as side information according to the proposed scheme, and thus is not available at the q-th subnetwork. One way to address this issue is to modify the proposed scheme. For instance, every BS could forward the corresponding $v^{(k)}$ to all the connected relays after the IN solutions have been determined. However, this causes additional signaling overhead and delay. From a practical point of view, we can assume that the maximum sum powers of the BSs $P_{\text{B,max}}^{(k)}$ are independent of the channel realization and are known by the entire network prior to the transmission. Hence we propose to relax the relay power constraint (23) as

$$\begin{aligned} 0 < \sum_{k=1}^{K_q} \frac{a^{(q,k)}}{1 + \left| v^{(k)} \right|^2} p_{\text{B}}^{(k)} + \text{tr} \left(\mathbf{G}^{(q)} \mathbf{G}^{(q)*\text{T}} \right) \sigma_{\text{R}}^2 \\ + \sum_{k=K_q+1}^{K} a^{(q,k)} P_{\text{B,max}}^{(k)} \leq P_{\text{R,max}}^{(q)}, \end{aligned} \tag{24}$$

where $a^{(q,k)}$ is defined in (17). Consequently, the q-th subnetwork is able to solve the sum rate maximization problem using only the considered partial CSI. The sum rate maximization problem (21) subject to the individual power constraints for the BSs (22) and the relaxed power constraints for the relay (24) is a convex optimization problem and can be readily solved using convex programming [11].

Besides the individual power constraints for each BS and relay, a sum power constraint for each subnetwork can also be considered. Assuming that the maximum sum power of the q-th subnetwork is $P_{\text{sum,max}}^{(q)}$, the sum power constraint for the q-th subnetwork can be formulated as

$$0 < \sum_{k=1}^{K_q} p_{\text{B}}^{(k)} + p_{\text{R}}^{(q)} \leq P_{\text{sum,max}}^{(q)}, \tag{25}$$

where $p_{\text{R}}^{(q)}$ is given by (16). Note that the sum power constraint of (25) can be relaxed in a similar way as (24) such that the sum rate maximization problem can be solved only using the considered partial CSI.

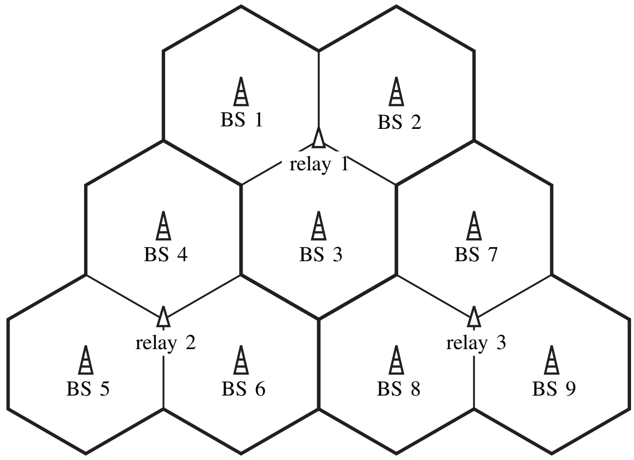

Fig. 3. Setup of the simulations. A cellular network consisting of 9 hexagonal cells and 3 relays is considered.

VI. Simulations

In the following simulations, a cellular network consisting of 9 hexagonal cells is considered. Three relays are deployed in the network. Each relay and the three cells adjacent to the relay form a subnetwork, as depicted in Fig. 3. In each cell, a BS which is located at the cell center serves a single MS which is located close to the relay in the subnetwork. The relays are supposed to help in nullifying the inter-cell interferences received by those MSs using the IN scheme proposed in this paper.

Let the distance between a relay and the nearest BS be R. The distances between a relay and the nearby MSs are assumed to be $R/4$. The distance between a BS and a MS is approximated by the distance between the BS and the nearest relay of the MS. The channels are generated as follows. First of all, the channel coefficients of all the links in the network are independently drawn from a Gaussian distribution. The average channel gain of each link is is assumed to be influenced by the distance d between the corresponding nodes due to the path loss effect. Specifically, the average channel gain is assumed to be proportional to d^{-4}. To ensure that the network topology matches the partially connected networks considered in this paper, we furthermore assume that all the intra-subnetwork

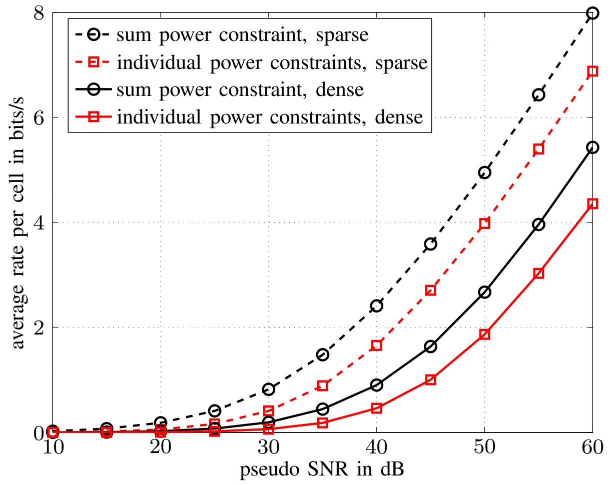

Fig. 4. Average sum rate per cell as a function of pseudo SNR

links are present and the links between a relay and the MSs in the other two subnetworks are absent. An inter-subnetwork direct or relay link is assumed to be present if its channel gain is larger than a threshold chosen in advance. In our simulations, two cases will be considered. In the first case, the threshold is chosen to be the average channel gain of the link between BS 4 and relay 1. In the second case, the threshold is chosen to be the average channel gain of the link between BS 5 and relay 1. The former case in fact represents a sparser network than the latter. Finally, the channel coefficients of the present links are normalized such that channels between a relay and the nearby MSs have unit average gain and the noises received by all the MSs and relays are assumed to be independently Gaussian noise with a common variance of σ^2.

An IN solution is first obtained using the scheme proposed in Sec. IV. To ensure that the proposed scheme is always feasible, each relay is assumed to have 5 antennas. Afterwards, each subnetwork maximizes the achieved sum rate in the subnetwork based on the chosen IN solution as discussed in Sec. V. Both kinds of power constraints investigated in Sec. V will be considered. If an average power constraint for each individual BS and relay is considered, the maximum sum power of each BS is assumed to be P while the maximum power of each relay is assumed to be $3P$. If an average sum power constraint for each subnetwork is considered, the maximum sum power of each subnetwork is assumed to be $6P$ so that the achieved sum rate under the two kinds of power constraints can be fairly compared. The achieved rate per cell is depicted in Fig. 4 as a function of the pseudo SNR, which is defined to be P/σ^2. The results are averaged over a large number of channel realizations. Firstly, it can be observed from Fig. 4 that higher rates can be achieved under a sum power constraint for each subnetwork, as expected. Secondly, higher rates can be achieved in sparse networks. This can be explained as follows. In a denser network, more inter-subnetwork direct and relay links result in more constraints for each subnetwork. Therefore, the IN solution space $\mathbb{W}'_q$ of each subnetwork has a smaller dimension. Consequently, it is more difficult for each subnetwork to choose an IN solution which yields good performance.

VII. Conclusions

In this work, downlink transmissions in partially connected cellular networks are considered, where relays are deployed to help in nullifying the inter-cell interferences received by the nearby MSs. An IN scheme is proposed to achieve interference-free transmissions in the entire network using only partial CSI. The feasibility conditions for the proposed scheme are discussed. Furthermore, the sum rate maximization problems under different kinds of power constraints are addressed. The sum rate achieved in each subnetwork can be maximized using the considered partial CSI.

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