Distributed Queue-Aware Beamforming in MISO Interference Channels

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***Abstract*—We study distributed queue-aware beamforming in a multiple input single output (MISO) interference channel. Instead of assuming full buffer traffic, we adopt a more realistic traffic model where the data arrival rates are finite, and the beamformers are adaptive to the data in the buffer. We propose a simple dynamic beamforming strategy which significantly improves the average sum rate compared to non-adaptive beamformers.**

I. Introduction

The MISO interference channel (IFC) is a well investigated model. Many linear beamforming strategies have been proposed with the objective to maximize the instantaneous sum of all user utilities [1], [2], [3], [4]. The beamforming strategies using a distributed optimization framework [1], [2] are more interesting due to lower requirements on network infrastructures, reduced complexity and latency. All the above work assumes a full buffer traffic model, i.e. all the transmitters always have infinite amount of data to transmit. However, in real networks, the data arriving at each transmitter is random, and the amount of data in the buffer changes over time.

In this study, we design distributed and dynamic beamformers which take into account the dynamic change of the data buffer. The transmitters update their transmission strategies based on locally available channel state information and exchange the buffer status parameters via the back haul. Our objective is to maximize the time average of the sum user utilities. To the best of our knowledge, there is not much work in the literature on multiple antenna transmission with random data arrivals. The most similar work is [5], where the authors perform a theoretical analysis on the stability optimal policy in the multiple antenna Multiple Access Channel (MAC). However the optimal policy assumes centralized control and non-linear precoding, so it can only serve as a theoretical upper-bound instead of a practical solution. Another work is [6], where the study is focused on user selection for MU-MIMO systems, and uses only random beamforming. In our work, we are mainly interested in providing simple queue-aware distributed beamforming solutions which brings big improvement compared to some of the existing queue-unaware simple distributed solutions.

II. System Model

We consider a data network which can be modeled as a MISO interference channel with K interfering links. Each link has a transmitter equipped with N antennas delivering data intended only for its own single-antenna receiver while causing interference to other links.

In the system, the design variables are the beamforming vectors at the transmitters. We denote the matrix formed by these beamforming vectors as $\mathbf{W}(t) = [\mathbf{w}_1(t)\ \mathbf{w}_2(t)\ ...\ \mathbf{w}_K(t)]^H$. The transmission power is limited at each transmitter, e.g. $\left\|\mathbf{w}_k^H(t)\right\|^2 \leq p_k$. We denote $\mathcal{W}$ as the set of all feasible beamformers.

Associated with each beamformers selection at time instant t is the instantaneous *achievable* user rate vector $\hat{\mathbf{r}}(t) = [\hat{r}_1(t)\ \hat{r}_2(t)\ ...\ \hat{r}_K(t)]^H$, with

$$\hat{r}_k(t) = \log\left(1 + \frac{\left|\mathbf{w}_k^H(t)\mathbf{h}_{kk}(t)\right|^2}{\sum_{l\neq k}\left|\mathbf{w}_l^H(t)\mathbf{h}_{kl}(t)\right|^2 + \sigma_k^2}\right),\ \mathbf{W}(t) \in \mathcal{W}$$

where $\mathbf{h}_{kl}(t)$, $k, l \in \{1, 2, ...K\}$ is the channel coefficient from transmitter l to receiver k at time instant t, this coefficient is locally available without involvement of information exchange among the transmitters.

The data arrival rate at each transmitter is modeled as a random process with a finite average arrival rate. We denote $\mathbf{a}(t) = [a_1(t)\ a_2(t)\ ...\ a_K(t)]^H$ is the *i.i.d.* random arrival process, and $\bar{\mathbf{a}} = [\bar{a}_1\ \bar{a}_2\ ...\ \bar{a}_K]^H$ is the *average* arrival rate vector, $\bar{\mathbf{a}} = E[\mathbf{a}(t)]$. When the arrived data can not be delivered immediately, they will form a queuing backlog and wait for the next transmission. The dynamic queue length (backlog size) vector is denoted as $\mathbf{s}(t) = [s_1(t)\ s_2(t)\ ...\ s_K(t)]^H$. We assume the channels change slowly and can be estimated accurately, and that the current queue lengths $\mathbf{s}(t)$ could be obtained for all the transmitters (for example through back haul communications).

The evolution of the queue length is

$$\mathbf{s}(t+1) = \mathbf{s}(t) + \mathbf{a}(t+1) - \mathbf{r}(t+1). \tag{1}$$

Here, $\mathbf{r}(t+1) = [r_1(t)\ r_2(t)\ ...\ r_K(t)]^H$ denote the *actual* transmitted rate at time $t+1$, i.e.

$$\mathbf{r}(t) = \min\left[\mathbf{s}(t) + \mathbf{a}(t+1),\ \hat{\mathbf{r}}(t)\right]. \tag{2}$$

III. Average sum rate maximization

A. Lyapunov drift algorithm

Note that when the data arrival rate $\mathbf{a}(t)$ is an *i.i.d.* random arrival process, the queue length $\mathbf{s}(t)$ is an Markov process. This allows us to use the drift technique to minimize the Lyapunov function of $\mathbf{s}(t)$ as in [7]. The resulting dynamic rate allocation policy is stability-optimal, and is given by

$$\hat{\mathbf{r}}^*(t) = \underset{\hat{\mathbf{r}}(t)}{\operatorname{argmax}} \sum_{k=1}^{K} \hat{r}_k(t) s_k(t). \tag{3}$$

This is a weighted sum rate maximization problem and can be rewritten as

$$\begin{aligned} &\text{maximize} \quad \sum_{k=1}^{K} s_k(t)\cdot\log\left(1+\frac{\left|\mathbf{w}_k^H(t)\mathbf{h}_{kk}(t)\right|^2}{\sum_{l\neq k}\left|\mathbf{w}_l^H(t)\mathbf{h}_{kl}(t)\right|^2+\sigma_k^2}\right) \quad (4)\\ &\text{subject to} \quad \left\|\mathbf{w}_k^H(t)\right\|^2 \le p_k, \quad k=1,...,K. \end{aligned}$$

Problem (4) is NP-hard[8] but can for example be solved via the BRB algorithm in [3]. However, such a centralized solution is practically unfeasible in terms of computational complexity, back haul signaling, and scalability, and we are more interested in a low complexity distributed solution.

B. Distributed algorithm

The algorithm introduced in [2] could be used to reach a stationary point of (4). However, that algorithm still needs several iterations to converge. Since we are aiming at updating beamformers each time instant, it is preferred to have an even simpler algorithm. In [9] it is proven that all the Pareto optimal points in the achievable rate region are achieved by beamforming vectors which can be parameterized as

$$\mathbf{w}_k(t)=\frac{\left(\frac{\mu_k}{p_k}\mathbf{I}_{N_t}+\sum_{l\neq k}\frac{\lambda_l}{\sigma_l^2}\mathbf{h}_{lk}(t)\mathbf{h}_{lk}^H(t)\right)^{-1}\mathbf{h}_{kk}}{\left\|\left(\frac{\mu_k}{p_k}\mathbf{I}_{N_t}+\sum_{l\neq k}\frac{\lambda_l}{\sigma_l^2}\mathbf{h}_{lk}(t)\mathbf{h}_{lk}^H(t)\right)^{-1}\mathbf{h}_{kk}\right\|}, \quad k=1,2,...,K \quad (5)$$

where $\{\mu_k\}_{k=1}^K$ and $\{\lambda_l\}_{l=1}^K$ satisfy $\sum_{k=1}^K \mu_k=\sum_{l=1}^K \lambda_l = 1$. From (5) we can see that when μ_k is large compared to $\sum_{l\neq k}\lambda_l$, the beamformer acts more selfishly. On the other hand, when μ_k is small compared to $\sum_{l\neq k}\lambda_l$, the beamformer acts more altruistic. Intuitively, we want the links with long queues to act more selfish and the links with short queue lengths altruistic. Therefore, we can adjust the parameters μ_k and λ_k in (5) to be monotonically increasing with the queue length. Similar to the heuristic algorithm proposed in [10](4.36). We propose the following parametrization to λ and μ:

$$\mu_k=\lambda_k=\frac{s_k^\alpha}{\sum_{k=1}^K s_k^\alpha} \quad (6)$$

for some choice of $\alpha > 0$. Numerical experiments have shown that a good choice of α is 1.

IV. Simulation Results

A. Channel model

Here we generate the simulation results considering a simple MISO interference channel scenario. We consider three transmitter-receiver pairs, each transmitter equipped with three antennas. The channel vector from transmitter l to receiver k, $\mathbf{h}_{kl}$ is i.i.d and modeled as complex Gaussian

$$\mathbf{h}_{lk} \sim \begin{cases}\mathcal{CN}(0,\mathbf{I}_K), & \text{when } k=l\\ \mathcal{CN}(0,\frac{1}{4}\mathbf{I}_K), & \text{when } k\neq l\end{cases}, \quad (7)$$

where $\forall k,l \in \{1,2,3\}$, $K=3$. In the simulations, we will generate 20 channel realizations randomly and study the average performance over these channel realizations.

The system SNR is defined as

$$SNR=\frac{P}{\sigma^2},$$

where P is the constant power constraint at each transmitter, and σ^2 is the constant noise power at each receiver.

For each channel realization and SNR combination, we select a number of points on the convex hull of the Pareto boundary. To do this, we refer to the *robust fairness-profile optimization* algorithm in [3]. We randomly choose a certain number of starting points and the same number of directions, then apply the *robust fairness-profile optimization* algorithm to find the corresponding points on the Pareto boundary of the rate region. Then the convex hull of these boundary points is found by applying a triangulation technique such as '*convhull*' in MATLAB. In our simulations, for each channel with a certain SNR, we find 10 points on the convex hull of the Pareto boundary, and these points will be used to find a proper traffic load to simulate, which will be explained in details in the following section. Notice that in practice when we use the beamformers, the rate region and the Pareto boundary information is not needed. We compute these points here only to facilitate our choices of the average data arrival rates for the simulations. Another issue to mention is that since we only generate a limited number of points on the boundary, it is not enough to calculate the convex hull with high precision.

B. Traffic model

We assume independent Poisson data arrival process for each user, and the average data arrival rates $\bar{\mathbf{a}}$ are selected as

$$\bar{\mathbf{a}}=\rho\cdot\mathbf{r}, \quad \mathbf{r}\in\mathbf{R}^{\mathbf{hull}} \quad (8)$$

where $\mathbf{R}^{\mathbf{hull}}$ is the collection of the points on the convex hull of the rate region which is described in the previous subsection. And ρ is a parameter indicating how much traffic loaded in the system. For example, if $\rho > 1$, the data arrives to the system at a rate larger than the maximum rates that can be transmitted. Thus the queue(s) will accumulate and the traffic model is more like a full buffer model as time goes by. But when $\rho < 1$, it is possible in theory to find a transmission strategy to keep the queue lengths for all users stable and do not grow monotonically over time [11]. The beamforming strategy that can keep the queue lengths for all users stable as long as $\rho < 1$ is defined as stability optimal beamforming strategy. And the beamforming strategy which satisfy (3) is stable optimal. We will change the value of ρ in our simulations to test the queue length stability performance of different beamforming schemes.

Here we explain how to choose $\mathbf{r}$ from $\mathbf{R}^{\mathbf{hull}}$ in (8) to determine the average arrival rate. We test two kinds of traffic: balanced traffic and unbalanced traffic.

Balanced traffic: choose a point $\mathbf{r}$ such that all the users have similar rates. Thus, all users have similar average data arrival rates and traffic.

Unbalanced traffic: choose a corner point from the convex hull, which means one user has a much larger rate and the remaining users have very small rates. This results in an unbalanced traffic among users with one user with a large traffic, and the other two users have very small traffic. We think this model is more relevant when we want to capture the extreme behavior of a dynamic data arrival system, where the queue lengths are very different for different users. To model unbalanced traffic, we choose the rate point that corresponds to user 1 applying MRT beamformer and user 2 and 3 applying zero forcing beamformers.

Later in the simulations we can see that the proposed beamformer gives larger performance gain under unbalanced traffic.

C. Reference beamforming strategies

In this subsection, we will describe a few beamformers that will be used to compare performance with the proposed beamformer. First we introduce a queue-aware beamformer:

a) WMMSE (weighted MMSE): This beamforming strategy is proposed in [2], which is an iterative algorithm maximizing the weighted sum rate of the system. At each time instance, we set the weight to each user as the current queue length of the user as in (3), then the WMMSE beamformer gives the optimal performance over time in the sense of maximizing average sum rate and minimizing average queue lengths. Therefore, WMMSE beamformer can serve as a performance upper bound for queue-aware beamformers.

We would also like to compare performance with some queue-unaware beamformers: SLNR, ZF and MRT. These beamformers do not take the queue lengths into consideration, but only use channel information.

b) MRT (Maximum Ratio Transmission): The MRT beamformer aims to maximize its own SNR at the receiver, the beamformer can be expressed as

$$\mathbf{w}_k^{MRT} = \frac{\mathbf{h}_{kk}}{\|\mathbf{h}_{kk}\|},$$

where $\mathbf{h}_{kk}(t)$ is the channel coefficient from transmitter k to receiver k.

c) ZF (Zero Forcing): The ZF beamformer aims at completely eliminate interference causing to others, and beamformer is given by

$$\mathbf{w}_k^{ZF} = \frac{\Pi_{kk}^{\perp}\mathbf{h}_{kk}}{\left\|\Pi_{kk}^{\perp}\mathbf{h}_{kk}\right\|},$$

where $\Pi_{kk}^{\perp}$ is the projection matrix onto the null space of $\mathbf{h}_{kk}(t)$.

Note that the ZF beamforming above is valid since we have the number of transmission antennas equal to the number of receivers. If the number of antennas at the transmitter smaller than the number of users, completely removing interference is not possible.

d) SLNR (maximum Signal to Leakage and Noise Ratio): The SLNR beamformer aims at maximizing the ratio between its own received signal strength and the interference leakage to unintended receivers. Expressed in formula:

$$\mathbf{w}_k^{SLNR} = \frac{\left(\frac{1}{p_k}\mathbf{I}_{N_t} + \sum \frac{1}{\sigma_l^2}\mathbf{h}_{lk}(t)\mathbf{h}_{lk}^H(t)\right)^{-1}\mathbf{h}_{kk}}{\left\|\left(\frac{1}{p_k}\mathbf{I}_{N_t} + \sum \frac{1}{\sigma_l^2}\mathbf{h}_{lk}(t)\mathbf{h}_{lk}^H(t)\right)^{-1}\mathbf{h}_{kk}\right\|}.$$

D. Performance measures

In this study, we use average sum rate and average queue length of the system as two figure of merits. The average sum rate measures the transmission rate and the average queue length measures the queuing stability.

To calculate the average sum rate, we first generate 20 channel realizations, for each channel realization, we generate a data arrival sequence for each user for 10^4 channel uses. At each time instance, different beamforming strategies are applied to calculate the actual transmitted rates as in (2). The average sum rate is then calculated as the average of the instantaneous sum rate over 10^4 channel uses and then over the 20 channel realizations.

Similarly, the average queue length is the instantaneous queue lengths (1) averaged over different users, different channel uses and different channel realizations.

E. Sum Rate Performance

In Figure 1 and 2, we compare the average sum rate for different beamformers: The weighted MMSE (WMMSE) [2]; the proposed beamformer (Proposed) (5); the maximum Signal to Leakage and Noise Ratio (SLNR), the Zero Forcing (ZF) and the maximum Ratio Transmission (MRT) beamformers[9]. The traffic load indicator ρ is 0.99. Figure 1 is generated under balanced traffic and figure 2 is generated under unbalanced traffic.

We can see that the beamformer we proposed in this paper performs equally well as the weighted MMSE beamformer, and much better than the other beamformers under unbalanced traffic. But if we set $\alpha = 0$ for the proposed beamformer, which means the proposed beamformer do not adapt with the queue lengths, then it performs worse than SLNR beamformer and even ZF for high SNR scenario. So we can conclude that the sum rate gain brought by the proposed beamforming is mainly due to its adaptive nature. Since the unbalanced traffic is more relevant and shows more gain, we will focus our later simulations for unbalanced traffic.

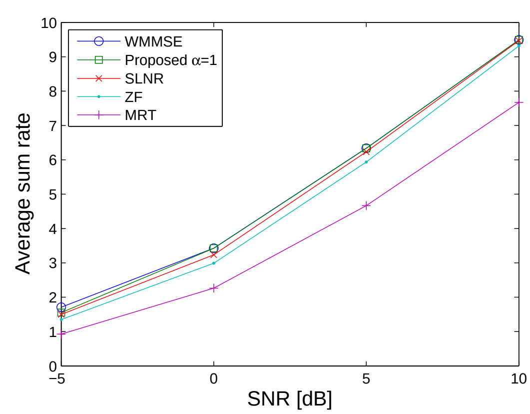

Figure 1. Average sum rate achieved by different beamformers under balanced traffic

In Figure 3, we study the impact of the traffic load on the beamformer performance. The SNR is set to 5dB and the average sum rate for each beamformer is calculated

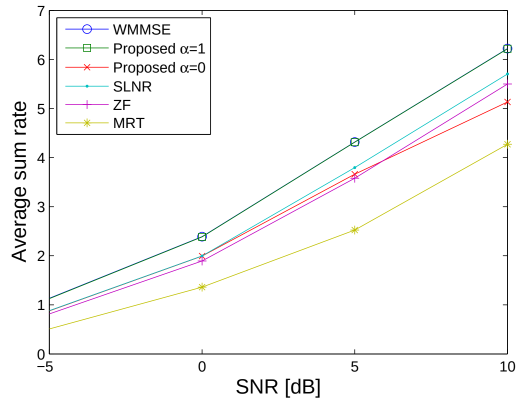

Figure 2. Average sum rate achieved by different beamformers under unbalanced traffic

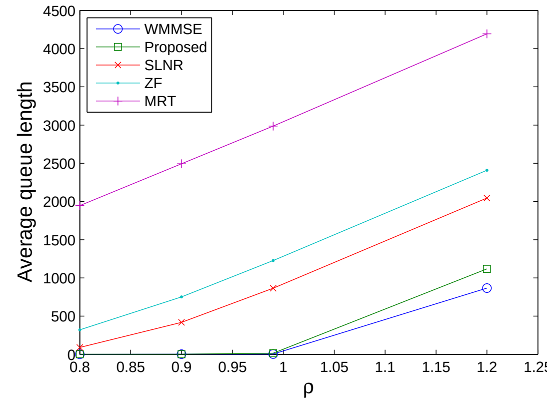

Figure 4. Average queue length achieved by different beamforming strategies

as the average of instantaneous system sum rate over 10^4 channel uses and 20 channel realizations. We can see from the figure that the sum rate gain brought by the queue-aware beamformers increases with the traffic load until the traffic become overloaded ($\rho > 1$). The proposed beamformer in the simulation corresponds to $\alpha = 1$.

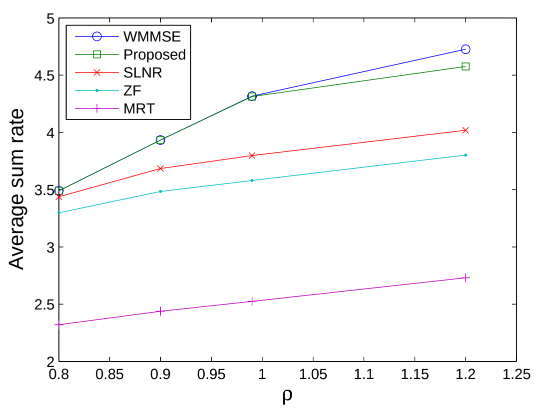

Figure 3. Average sum rate achieved for different traffic load

F. Queue length stability

In Figure 4, we plot the average queue length for different beamformers respective to different traffic load (ρ). The SNR is set to 5dB. We notice from the figure that when $\rho < 1$, the queue-aware beamformers (WMMSE and Proposed) are good at maintaining a stable queue length. That is, the average queue length for the queue-aware beamformers do not increase linearly with the traffic load. However, the queue lengths to the queue-unaware beamformers (SLNR, ZF and MRT) increase almost linearly with the traffic load, thus making the corresponding queues unstable. When $\rho > 1$, all beamforming strategies fail to hold the system queue stable. This is reasonable, since when $\rho > 1$, the average data arrival rates are outside the convex hull of the Pareto boundary of the system, so it is impossible to find a transmission strategy that prevents the queues from increasing over time.

V. Conclusion

In this study, we have proposed simple and distributed beamforming strategy considering random data arrival process. The proposed strategy dynamically changes beamforming according to the queue length. Simulation results show that the proposed beamformer gives near optimal performance in the sense of maximizing sum rate and minimizing average queue length under dynamic traffic.

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