Stochastic-Deterministic Multipath Model for Time-Delay Estimation

Friederike Wendler[1], Felix Antreich[1], Josef A. Nossek[2], A. Lee Swindlehurst[3]
German Aerospace Center (DLR), Inst. for Communications and Navigation, 82234 Wessling, Germany[1]
Inst. for Circuit Theory & Signal Processing, Technical University Munich (TUM), Germany[2]
Dept. of Electrical Engineering & Computer Science, The University of California, Irvine CA, U.S.A.[3]

***Abstract*—Line-of-sight (LOS) delay estimation in multipath scenarios is a central problem in global navigation satellite systems (GNSS). Deterministic channel models can be used to describe the multipath environment, but this usually requires the estimation of several nuisance parameters. In order to avoid this effort, stochastic channel models can be used. In this case the multipath statistics have to be estimated. The correlated path (CP) model combines the two approaches, dividing the multipath signal into a part correlated with the LOS signal, and the rest as uncorrelated multipath interference. In an earlier version of the CP model the multipath interference was modeled as temporally white noise. This is not an accurate assumption however, especially in the case when a bank of correlators is used for signal compression. In this paper we derive the temporal covariance matrix of the multipath interference and show how the temporal multipath correlation can incorporated into the maximum-likelihood (ML) estimator of the CP model. Simulation results show that this new approach leads to better LOS time-delay estimation performance.**

I. Introduction

GNSS are used in a wide variety of applications, whether for positioning or time synchronization. In these applications the estimation accuracy of the LOS signal time-delay estimate directly influences the quality of the service. Multipath, i.e. superimposed replicas of the LOS signal due to signal reflections and scattering in the propagation path, can severely degrade the LOS signal time-delay estimation performance [1].

In the past, different multipath mitigation techniques have been studied. The ML estimator which estimates the channel parameters of each multipath together with the LOS parameters is the optimum approach for solving the multipath problem [2]. However, the optimum ML estimator requires knowledge of the number of multipath rays and often. In order to avoid these problems advanced tracking loops [3] and multi-correlator-bank based approaches [4] have been proposed for single antenna receivers. If an array of multiple antennas is used, the spatial diversity can be exploited for multipath suppression. However, for multi-antenna receivers the computational complexity of the ML estimator increases due to the additional spatial dimension. Therefore, reduced complexity methods like the Space Alternating Generalized Expectation maximization (SAGE) algorithm [5] have been developed. While offering a significant reduction in computational effort in comparison to the exact ML estimator, the SAGE algorithm and its extensions [6], [7] still require the number of multipath rays. This can be avoided if a statistical multipath model [8], [9] is employed. All of the methods mentioned above perform best if LOS and multipath signals are temporally and spatially uncorrelated, i.e. sufficiently separated in time and space. Dual-polarization antenna arrays, i.e. antenna arrays with right-hand-circularly polarized (RHCP) and left-hand-circularly polarized (LHCP) outputs can offer an additional degree of freedom to identify and separate highly spatially and temporally correlated multipath signals from the LOS signal. In [10] a multipath mitigation approach based on dual-polarization arrays has been proposed. Additionally, the problem of model order estimation has been tackled by introducing the CP model, which divides the multipath signal into a signal correlated with the LOS signal and uncorrelated multipath interference. To achieve a simple ML estimator, the multipath interference is modeled as temporally white Gaussian noise in [10].

This white noise assumption is inaccurate when considering the properties of GNSS signals. Moreover, the computational complexity of [10] is high in comparison to using a multi-correlator bank to compress the signal. Therefore, we introduce the CP model for temporally correlated multipath interference and multi-correlator bank based compression. This leads to improved estimation performance for the LOS signal delay while still maintaining the benefits of the CP model:

- no model order estimation
- limited and constant number of parameters
- no selection of the LOS signal delay from the multipath signal delays

In order to incorporate the temporal correlation of the multipath interference, a spatio-temporal model is introduced. The spatial multipath and noise parameters are estimated in the same way as in [10], while the temporal multipath interference covariance matrix is taken into account for the estimation of the LOS signal time-delay. Finally, the performance of the improved CP model is shown for a dual-polarization global positioning system (GPS) receiver.

II. Multipath Signal Model

We consider a GNSS multipath scenario. One LOS signal with time delay $\tau_0 \in \mathbb{R}$ and L multipath signals with time delays $\tau_l \in \mathbb{R}$ for $l = 1, \cdots L$, are impinging on a dual-polarization antenna array composed of M antenna elements.

The unstructured base-band representation of the signal is [10]

$$\boldsymbol{y}(t) = \boldsymbol{b}_0 c(t-\tau_0) + \sum_{l=1}^{L} \boldsymbol{b}_l c(t-\tau_l) + \boldsymbol{\eta}(t)\,, \tag{1}$$

where $c(t) \in \mathbb{R}$ is the GNSS transmit signal with single-sided bandwidth $B \in \mathbb{R}$ and $\boldsymbol{b}_l \in \mathbb{C}^{2M}$ denotes the signal's spatial and polarization signature. A spatially structured model for $\boldsymbol{b}_l$ is given in [10]. In the following, $\boldsymbol{\eta}(t) \in \mathbb{C}^{2M}$ is assumed to be temporally and spatially white Gaussian noise, i.e. $\boldsymbol{\eta}(t) \sim \mathcal{CN}\big(0, \sigma_{\boldsymbol{\eta}}^2 \boldsymbol{I}_{2M}\big)$. After collecting N time samples of (1) at sampling rate $f_s = 2B$, the discrete time representation is

$$\boldsymbol{Y} = \boldsymbol{b}_0 \boldsymbol{c}(\tau_0)^{\mathrm{T}} + \sum_{l=1}^{L} \boldsymbol{b}_l \boldsymbol{c}(\tau_l)^{\mathrm{T}} + \boldsymbol{E}, \tag{2}$$

where $T_s = 1/f_s$ and

$$\boldsymbol{Y} = [\boldsymbol{y}[T_s]\ \boldsymbol{y}[2T_s]\ \ldots\ \boldsymbol{y}[NT_s]] \in \mathbb{C}^{2M\times N} \tag{3}$$

$$\boldsymbol{c}(\tau_l) = [c[T_s-\tau_l]\ c[2T_s-\tau_l]\ \ldots\ c[NT_s-\tau_l]]^{\mathrm{T}} \in \mathbb{C}^{N} \tag{4}$$

$$\boldsymbol{E} = [\boldsymbol{\eta}[T_s]\ \boldsymbol{\eta}[2T_s]\ \ldots\ \boldsymbol{\eta}[NT_s]] \in \mathbb{C}^{2M\times N}. \tag{5}$$

The noise covariance matrix is given by

$$\mathrm{E}\Big[\mathrm{vec}(\boldsymbol{E})\,\mathrm{vec}(\boldsymbol{E})^{\mathrm{H}}\Big] = \sigma_{\boldsymbol{\eta}}^2 \boldsymbol{I}_N \otimes \boldsymbol{I}_{2M}, \tag{6}$$

where vec(•) vectorizes a matrix by stacking its columns, E[•] denotes the expected value and $\otimes$ denotes the Kronecker product [11].

A. Compression with a Multi-Correlator Bank

Since the number of samples N is often large, the received signal $\boldsymbol{Y}$ is compressed to a signal $\boldsymbol{Z} \in \mathbb{R}^{2M\times Q}$ with lower temporal dimension $Q < N$ using a multi-correlator bank. The compression can be represented by a multiplication of $\boldsymbol{Y}$ with the compression matrix $\boldsymbol{Q} \in \mathbb{R}^{N\times Q}$ from the right hand side:

$$\boldsymbol{Z} = \boldsymbol{b}_0 \boldsymbol{c}(\tau_0)^{\mathrm{T}} \boldsymbol{Q} + \sum_{l=1}^{L} \boldsymbol{b}_l \boldsymbol{c}(\tau_l)^{\mathrm{T}} \boldsymbol{Q} + \boldsymbol{E}\boldsymbol{Q} \tag{7}$$

$$= \boldsymbol{b}_0 \boldsymbol{q}(\tau_0)^{\mathrm{T}} + \sum_{l=1}^{L} \boldsymbol{b}_l \boldsymbol{q}(\tau_l)^{\mathrm{T}} + \boldsymbol{E}_{\boldsymbol{Q}}. \tag{8}$$

Eq. (8) can be parameterized by

$$\boldsymbol{\xi} = \left[\tau_0, \ldots, \tau_L, \boldsymbol{b}_0^{\mathrm{T}}, \ldots, \boldsymbol{b}_L^{\mathrm{T}}, \sigma_{\boldsymbol{\eta}}^2\right]^{\mathrm{T}} \in \mathbb{C}^{(L+1)(2M+1)+1}. \tag{9}$$

For compression set $\boldsymbol{Q} = \boldsymbol{U}$, where U are the left singular vectors of

$$\boldsymbol{U}\boldsymbol{\Sigma}\boldsymbol{V}^{\mathrm{H}} = [\boldsymbol{c}(\kappa_1), \boldsymbol{c}(\kappa_2), \ldots, \boldsymbol{c}(\kappa_Q)]^{\mathrm{T}}. \tag{10}$$

Eq. (10) realizes the canonical component (CC) method [12]. The CC method is based on correlating the sampled received signal $\boldsymbol{Y}$ with Q replicas of $\boldsymbol{c}(\tau)$ with different delays κ_q and minimizes the Fisher information loss due to compression [12], as well as maintaining the multiple access properties of the direct sequence code division multiple access (DS-CDMA) system used in GNSS. Using the unitary matrix $\boldsymbol{U}$ ensures that the noise $\boldsymbol{E}_Q$ after correlation is still white Gaussian noise

$$\mathrm{E}\Big[\mathrm{vec}(\boldsymbol{E}_{\boldsymbol{Q}})\,\mathrm{vec}(\boldsymbol{E}_{\boldsymbol{Q}})^{\mathrm{H}}\Big] = \mathrm{Cov}[\mathrm{vec}(\boldsymbol{E}_{\boldsymbol{Q}})] \tag{11}$$

$$= \sigma_{\boldsymbol{\eta}}^2 \boldsymbol{I}_Q \otimes \boldsymbol{I}_{2M}, \tag{12}$$

where Cov[•] denotes the covariance matrix operator.

III. Correlated Path Model

The optimum estimator for τ_0 in (8) is the ML estimator which estimates all parameters in (9) [2]. However, this estimator requires knowledge of the number of multipaths L. Additionally, it must determine the actual LOS delay from all other multipath delays, which can be difficult if LOS signal and multipath signal are highly temporally correlated. Moreover, this estimator has to cope with a number of nuisance parameters. To avoid these problems we employ the CP model proposed in [10]. Let

$$\rho_l = \boldsymbol{q}(\tau_0)^{\mathrm{T}} \boldsymbol{q}(\tau_l) \tag{13}$$

denote the temporal correlation between the LOS and l-th multipath signal. In the case of only one multipath signal, i.e. $L = 1$ the multipath signal can then be decomposed as

$$\boldsymbol{b}_1 \boldsymbol{q}(\tau_1)^{\mathrm{T}} = \rho_1 \boldsymbol{b}_1 \boldsymbol{q}(\tau_0)^{\mathrm{T}} + \sqrt{1-\rho_1^2}\,\boldsymbol{b}_1 \boldsymbol{u}^{\mathrm{T}}, \tag{14}$$

where the multipath interference $\boldsymbol{u} \in \mathbb{R}^{Q}$ is uncorrelated with the LOS signal $\boldsymbol{q}(\tau_0)$, i.e.

$$\mathrm{E}\big[\boldsymbol{u}^{\mathrm{H}} \boldsymbol{q}(\tau_0)\big] = 0 \tag{15}$$

and has a temporal covariance matrix

$$\mathrm{Cov}[\boldsymbol{u}] = \boldsymbol{R}_{\boldsymbol{u}} \in \mathbb{R}^{Q\times Q}. \tag{16}$$

In [10] it is assumed that $\boldsymbol{u}$ is temporally white Gaussian noise and therefore $\boldsymbol{R}_{\boldsymbol{u}}$ is an identity matrix. Due to the properties of the signal $\boldsymbol{q}(\tau_l)$ this is not the case.
The multipath space-time covariance matrix is

$$\mathrm{Cov}\Big[\mathrm{vec}\Big(\boldsymbol{b}_1 \boldsymbol{q}(\tau_1)^{\mathrm{T}}\Big)\Big] = (\boldsymbol{I}_Q \otimes \boldsymbol{b}_1)\,\boldsymbol{R}_{\boldsymbol{q}_1}\,(\boldsymbol{I}_Q \otimes \boldsymbol{b}_1)^{\mathrm{H}} \tag{17}$$

with $\boldsymbol{R}_{\boldsymbol{q}_1} = \mathrm{Cov}[\boldsymbol{q}(\tau_1)]$, while

$$\begin{aligned}\mathrm{Cov}\Big[\mathrm{vec}\Big(\rho_1 \boldsymbol{b}_1 \boldsymbol{q}(\tau_0)^{\mathrm{T}} + \sqrt{1-\rho_1^2}\,\boldsymbol{b}_1 \boldsymbol{u}^{\mathrm{T}}\Big)\Big] &= \\ \rho_1^2\,(\boldsymbol{I}_Q \otimes \boldsymbol{b}_1)\,\boldsymbol{R}_{\boldsymbol{q}_0}\,(\boldsymbol{I}_Q \otimes \boldsymbol{b}_1)^{\mathrm{H}} & \\ + \big(1-\rho_1^2\big)\,(\boldsymbol{I}_Q \otimes \boldsymbol{b}_1)\,\boldsymbol{R}_{\boldsymbol{u}}\,(\boldsymbol{I}_Q \otimes \boldsymbol{b}_1)^{\mathrm{H}} &\end{aligned} \tag{18}$$

is the correlated path model space-time covariance matrix with $\boldsymbol{R}_{\boldsymbol{q}_0} = \mathrm{Cov}[\boldsymbol{q}(\tau_0)]$. Assuming that the second moment of the multipath signal is maintained, (17) and (18) are equal, i.e.

$$\begin{aligned}(\boldsymbol{I}_Q \otimes \boldsymbol{b}_1)\,\boldsymbol{R}_{\boldsymbol{q}_1}\,(\boldsymbol{I}_Q \otimes \boldsymbol{b}_1)^{\mathrm{H}} &= \rho_1^2\,(\boldsymbol{I}_Q \otimes \boldsymbol{b}_1)\,\boldsymbol{R}_{\boldsymbol{q}_0}\,(\boldsymbol{I}_Q \otimes \boldsymbol{b}_1)^{\mathrm{H}} \\ &+ \big(1-\rho_1^2\big)\,(\boldsymbol{I}_Q \otimes \boldsymbol{b}_1)\,\boldsymbol{R}_{\boldsymbol{u}}\,(\boldsymbol{I}_Q \otimes \boldsymbol{b}_1)^{\mathrm{H}}\end{aligned} \tag{19}$$

and therefore the mulitpath interference temporal covariance matrix is

$$\boldsymbol{R}_{\boldsymbol{u}} = \frac{\boldsymbol{R}_{\boldsymbol{q}_1} - \rho_1^2 \boldsymbol{R}_{\boldsymbol{q}_0}}{1-\rho_1^2}. \tag{20}$$

Eq. (20) can be simplified by noting that for highly temporally correlated LOS and multipath signals, we have

$$\boldsymbol{R}_{\boldsymbol{q}_1} \approx \boldsymbol{R}_{\boldsymbol{q}_0}. \tag{21}$$

Inserting (21) into (20) yields the approximation

$$\boldsymbol{R}_{\boldsymbol{u}} \approx \boldsymbol{R}_{\boldsymbol{q}_0}. \tag{22}$$

Using (20) or (22) improves the performance of the LOS time-delay estimation when the CP model is applied. Even though the CP model is based on the assumption of $L = 1$ multipath signals, simulation results show that it also performs well in the case of more than one multipath signal, if the signals are highly temporally or spatially correlated [10]. In this case ρ_1 reflects the overall correlation between LOS and multipath, $\boldsymbol{R_u}$ is the overall multipath interference temporal covariance matrix and $\boldsymbol{b}_1$ is the overall multipath spatial signature. To emphasize these properties we denote the correlation between the LOS and multipath with ρ while $\boldsymbol{b}_{\text{CP}}$ denotes the overall multipath spatial signature. The CP model is finally given by

$$\boldsymbol{Z} = (\boldsymbol{b}_0 + \rho \boldsymbol{b}_{\text{CP}})\, \boldsymbol{q}(\tau_0)^{\text{T}} + \sqrt{1-\rho^2}\boldsymbol{b}_{\text{CP}}\boldsymbol{u}^{\text{T}} + \boldsymbol{E_R} \tag{23}$$

with parametrization

$$\boldsymbol{\xi}_{\text{CP}} = \left[\tau_0, \boldsymbol{b}_0^{\text{T}}, \boldsymbol{b}_{\text{CP}}^{\text{T}}, \rho, \sigma_{\boldsymbol{\eta}}^2\right] \in \mathbb{C}^{2M+3}. \tag{24}$$

IV. Parameter Estimation

In the following we show how to estimate the parameters of the CP model $\boldsymbol{\xi}_{\text{CP}}$. Assuming Gaussian noise, the probability density function of $\boldsymbol{Z}$ given $\boldsymbol{\xi}_{\text{CP}}$ is

$$\begin{aligned}\text{p}(\boldsymbol{Z}|\boldsymbol{\xi_{CP}}) =& \frac{1}{\pi^{MN}\det(\boldsymbol{R}(\boldsymbol{\xi}_{\text{CP}}))}\\ &\cdot \exp\left(-\text{vec}(\boldsymbol{Z}-\boldsymbol{M}(\boldsymbol{\xi}_{\text{CP}}))^{\text{H}}\boldsymbol{R}(\boldsymbol{\xi}_{\text{CP}})^{-1}\right.\\ &\left.\cdot\text{vec}(\boldsymbol{Z}-\boldsymbol{M}(\boldsymbol{\xi}_{\text{CP}}))\right)\end{aligned} \tag{25}$$

with mean and covariance matrix

$$\boldsymbol{M}(\boldsymbol{\xi}_{\text{CP}}) = (\boldsymbol{b}_0 + \rho\boldsymbol{b}_{\text{CP}})\,\boldsymbol{q}(\tau_0)^{\text{T}} \tag{26}$$

$$\begin{aligned}\boldsymbol{R}(\boldsymbol{\xi}_{\text{CP}}) &= \text{Cov}\left[\sqrt{1-\rho^2}\boldsymbol{b}_{\text{CP}}\boldsymbol{u}^{\text{T}} + \boldsymbol{E}_Q\right]\\ &= \boldsymbol{R_u} \otimes \left(1-\rho^2\right)\boldsymbol{b}_{\text{CP}}\boldsymbol{b}_{\text{CP}}^{\text{T}} + \sigma_{\boldsymbol{\eta}}^2\boldsymbol{I}_Q \otimes \boldsymbol{I}_{2M}.\end{aligned} \tag{27}$$

The optimum estimator for $\boldsymbol{\xi}_{\text{CP}}$ in (23) is given by the ML estimate

$$\hat{\boldsymbol{\xi}}_{\text{CP}} = \arg\max_{\boldsymbol{\xi}_{\text{CP}}} \text{p}(\boldsymbol{Z}|\boldsymbol{\xi_{CP}}) \tag{28}$$

$$= \arg\min_{\boldsymbol{\xi}_{\text{CP}}} l(\boldsymbol{Z}|\boldsymbol{\xi_{CP}}), \tag{29}$$

where the log-likelihood function is

$$\begin{aligned}l(\boldsymbol{Z}|\boldsymbol{\xi_{CP}}) =& \ln\det(\boldsymbol{R}(\boldsymbol{\xi}_{\text{CP}})) +\\ &\text{vec}(\boldsymbol{Z}-\boldsymbol{M}(\boldsymbol{\xi}_{\text{CP}}))^{\text{H}}\boldsymbol{R}(\boldsymbol{\xi}_{\text{CP}})^{-1}\text{vec}(\boldsymbol{Z}-\boldsymbol{M}(\boldsymbol{\xi}_{\text{CP}})).\end{aligned} \tag{30}$$

To solve for the estimate $\hat{\boldsymbol{b}}_0$ we take the derivative of (30) with respect to $\boldsymbol{b}_0$

$$\begin{aligned}&\frac{\partial l(\boldsymbol{Z}|\boldsymbol{\xi_{CP}})}{\partial \boldsymbol{b}_0} =\\ &2\left(\boldsymbol{q}(\tau_0)\otimes\boldsymbol{I}_{2M}\right)^{\text{H}}\boldsymbol{R}(\boldsymbol{\xi}_{\text{CP}})^{-1}\text{vec}(\boldsymbol{Z}-\boldsymbol{M}(\boldsymbol{\xi}_{\text{CP}}))\end{aligned} \tag{31}$$

and equating (31) to $\boldsymbol{0}$ to find

$$\hat{\boldsymbol{b}}_0(\tau_0) = \frac{\boldsymbol{Z}\boldsymbol{q}(\tau_0)}{\|\boldsymbol{q}(\tau_0)\|} - \rho\boldsymbol{b}_{\text{CP}}. \tag{32}$$

Inserting (32) into (30) the optimization problem reduces to

$$\begin{aligned}\left[\tau_0, \boldsymbol{b}_{\text{CP}}^{\text{T}}, \rho, \sigma_{\boldsymbol{\eta}}^2\right] =& \arg\min \text{vec}\Big(\boldsymbol{Z} - \hat{\boldsymbol{b}}_0(\tau_0)\,\boldsymbol{q}(\tau_0)^{\text{T}}\Big)^{\text{H}}\boldsymbol{R}(\boldsymbol{\xi}_{\text{CP}})^{-1}\\ &\cdot\text{vec}\Big(\boldsymbol{Z} - \hat{\boldsymbol{b}}_0(\tau_0)\,\boldsymbol{q}(\tau_0)^{\text{T}}\Big).\end{aligned} \tag{33}$$

Due to the structure of $\boldsymbol{R}(\boldsymbol{\xi}_{\text{CP}})$, a closed-form expression for the parameters of the multipath interference and noise $\hat{\boldsymbol{b}}_{\text{CP}}$, $\hat{\sigma}_{\boldsymbol{\eta}}^2$, and $\hat{\rho}$ based on (33) is not straightforward to derive. Therefore, we estimate these parameters with another approach and then solve for the LOS delay τ_0 using a line-search under the assumption that $\boldsymbol{R}(\boldsymbol{\xi}_{\text{CP}})$ is known. In the following we show how $\hat{\boldsymbol{b}}_{\text{CP}}$, $\hat{\sigma}_{\boldsymbol{\eta}}^2$, and $\hat{\rho}$ can be estimated.

A. Estimation of $\boldsymbol{R}(\boldsymbol{\xi}_{CP})$

The multipath interference plus noise spatio-temporal covariance matrix $\boldsymbol{R}(\boldsymbol{\xi}_{\text{CP}})$ is

$$\hat{\boldsymbol{R}}(\boldsymbol{\xi}_{\text{CP}}) = \text{E}\left[\text{vec}\Big(\boldsymbol{Z} - \hat{\boldsymbol{b}}_0(\tau_0)\,\boldsymbol{q}(\tau_0)^{\text{T}}\Big)\,\text{vec}\Big(\boldsymbol{Z} - \hat{\boldsymbol{b}}_0(\tau_0)\,\boldsymbol{q}(\tau_0)^{\text{T}}\Big)^{\text{H}}\right]. \tag{34}$$

However, for only one sample of $\boldsymbol{Z}$ this covariance matrix cannot be estimated with sufficient accuracy. Therefore, we exploit the structure of $\boldsymbol{R}(\boldsymbol{\xi}_{\text{CP}})$. Under the assumption that the multipath interference plus noise temporal covariance is still approximately white, i.e.

$$\text{E}\left[\Big(\boldsymbol{Z} - \hat{\boldsymbol{b}}_0(\tau_0)\,\boldsymbol{q}(\tau_0)\Big)^{\text{H}}\Big(\boldsymbol{Z} - \hat{\boldsymbol{b}}_0(\tau_0)\,\boldsymbol{q}(\tau_0)\Big)\right] \approx \boldsymbol{I}_Q \tag{35}$$

we can approximate

$$\begin{aligned}&\left(1-\rho^2\right)\boldsymbol{b}_{\text{CP}}\boldsymbol{b}_{\text{CP}}^{\text{T}} + \sigma_{\boldsymbol{\eta}}^2\boldsymbol{I}_{2M}\\ &\approx \Big(\boldsymbol{Z} - \hat{\boldsymbol{b}}_0(\tau_0)\,\boldsymbol{q}(\tau_0)^{\text{T}}\Big)\Big(\boldsymbol{Z} - \hat{\boldsymbol{b}}_0(\tau_0)\,\boldsymbol{q}(\tau_0)^{\text{T}}\Big)^{\text{H}} \qquad (36)\\ &= \boldsymbol{B}(\tau_0). \qquad (37)\end{aligned}$$

The white noise assumption is not exact, as $\boldsymbol{R_u}$ will usually have one dominant eigenvalue. However, simulations show that this approach still leads to good results. Eq. (37) can be rearranged to

$$\boldsymbol{b}_{\text{CP}}\boldsymbol{b}_{\text{CP}}^{\text{H}} = \frac{\boldsymbol{B}(\tau_0) - \sigma_{\boldsymbol{\eta}}^2\boldsymbol{I}_{2M}}{1-\rho^2}. \tag{38}$$

This allows us to use the singular value decomposition approach presented in [10] to calculate $\hat{\boldsymbol{b}}_{\text{CP}}$, $\hat{\sigma}_{\boldsymbol{\eta}}^2$ and $\hat{\rho}$. A reasonable estimate $\hat{\boldsymbol{b}}_{\text{CP}}$ is given by the eigenvector corresponding to the largest eigenvalue of (38). For the estimation of $\sigma_{\boldsymbol{\eta}}^2$ and ρ, we notice that (38) is of the form $\boldsymbol{G} = \boldsymbol{H} - \alpha\boldsymbol{I}_M$, where

$$\boldsymbol{H} = \frac{\boldsymbol{B}(\tau_0)}{(1-\rho^2)} \tag{39}$$

$$\alpha = \frac{\sigma_{\boldsymbol{\eta}}^2}{1-\rho^2}. \tag{40}$$

The eigenvalues of $\boldsymbol{G}$ are in general given by [11, p. 31]

$$\lambda_{\boldsymbol{G},i} = \lambda_{\boldsymbol{H},i} - \alpha. \tag{41}$$

Therefore, the following system of equations holds

$$\begin{aligned}\lambda_{\boldsymbol{\eta}}^{-1} &= \frac{\lambda_{\boldsymbol{B},1}}{(1-\rho^2)} - \frac{\sigma_{\boldsymbol{\eta}}^2}{1-\rho^2}\\ 0 &= \frac{\lambda_{\boldsymbol{B},i}}{(1-\rho^2)} - \frac{\sigma_{\boldsymbol{\eta}}^2}{1-\rho^2} \quad \text{for } i = 2\dots 2M,\end{aligned} \tag{42}$$

where $\lambda_{\boldsymbol{B},1} \leq \lambda_{\boldsymbol{B},2} \leq \ldots \leq \lambda_{\boldsymbol{B},2M}$ are the sorted eigenvalues of $\boldsymbol{B}(\tau_0)$. Eq. (42) can be solved by

$$\begin{bmatrix}\hat{\sigma}_{\boldsymbol{\eta}}^2\\ 1-\hat{\rho}^2\end{bmatrix} = \begin{bmatrix}1 & \lambda_{\boldsymbol{\eta}}^{-1}\\ 1 & 0\\ \vdots & \vdots\\ 1 & 0\end{bmatrix}^{+}\begin{bmatrix}\lambda_{\boldsymbol{B},1}\\ \lambda_{\boldsymbol{B},2}\\ \vdots\\ \lambda_{\boldsymbol{B},2M}\end{bmatrix} \tag{43}$$

in the minimum mean-squared error (MMSE) sense, where $\bullet^+$ denotes the Moore-Penrose pseudo inverse [11]. Inserting $\hat{\boldsymbol{b}}_{\mathrm{CP}}$, $\hat{\rho}$ and $\hat{\sigma}^2_{\boldsymbol{\eta}}$ into the optimization problem (33) allows us then to solve for the LOS estimate $\hat{\tau}_0$. $\boldsymbol{R}_{\boldsymbol{u}}$ is calculated together with τ_0 in the line search algorithm if (22) is used or calculated with an additional search over τ_1 if (20) is used.

V. Simulation Results

We assume a GPS C/A code with chip duration $T_{\mathrm{c}} = 997.52\,\mathrm{ns}$, bandwidth $B = 1.023\,\mathrm{MHz}$ and $N_{\mathrm{d}} = 1023$ chips per code period as transmit signal $c(t)$. The receive array is a 2 antenna dual-polarization uniform linear array (ULA) with 10 dB cross-polar isolation between RHCP and LHCP channels. The RHCP channel signal-to-noise ratio (SNR) is

$$\mathrm{SNR} = \mathrm{C/N_0} - 10\log_{10}(2B) + 10\log_{10}(N_{\mathrm{c}}), \qquad (44)$$

with carrier-to-noise density $\mathrm{C/N_0} = 40.3\,\mathrm{dB\text{-}Hz}$ and number of observed code periods $N_{\mathrm{c}} = 1$. During the observation interval, the channel parameters are assumed to be constant. The LOS azimuth angle-of-arrival is $\phi_0 = 70°$ and the signal to multipath ratio (SMR) is 6 dB. The spatial signatures $\boldsymbol{b}_l$ are calculated with the structured dual-polarization multipath model introduced in [10]

$$\boldsymbol{b}_l = \begin{cases} \gamma_0 \begin{bmatrix} \boldsymbol{s}_{\mathrm{R,c}}(\phi_0) \\ \boldsymbol{s}_{\mathrm{L,x}}(\phi_0) \end{bmatrix} & \text{for } l = 0 \\ \gamma_l \begin{bmatrix} \alpha_{\mathrm{R},l}\boldsymbol{s}_{\mathrm{R,c}}(\phi_l) + \alpha_{\mathrm{L},l}\boldsymbol{s}_{\mathrm{R,x}}(\phi_l) \\ \alpha_{\mathrm{L},l}\boldsymbol{s}_{\mathrm{L,c}}(\phi_l,\theta_l) + \alpha_{\mathrm{R},l}\boldsymbol{s}_{\mathrm{L,x}}(\phi_l,\theta_l) \end{bmatrix} & \text{for } l = 1 \dots L. \end{cases} \qquad (45)$$

$\alpha_{\mathrm{R},l}$ and $\alpha_{\mathrm{L},l}$ determine the polarization of the signal

$$\alpha_{\mathrm{R},l} = \begin{cases} 1 & \text{for } l = 0 \\ 1/\sqrt{2} & \text{for } l = 1 \dots L \end{cases} \qquad (46)$$

$$\alpha_{\mathrm{L},l} = \begin{cases} 0 & \text{for } l = 0 \\ 1/\sqrt{2} & \text{for } l = 1 \dots L. \end{cases} \qquad (47)$$

$\boldsymbol{s}_{\mathrm{R,c}}$, $\boldsymbol{s}_{\mathrm{R,x}}$, $\boldsymbol{s}_{\mathrm{L,c}}$, and $\boldsymbol{s}_{\mathrm{L,x}}$ denote the steering vectors of a ULA with a small random phase distortion to make sure the vectors are linearly independent. For comparison the Cramer Rao Lower Bound (CRLB) for the estimation of the structured model and the root mean squared error (RMSE) of a single path estimator [8] are applied. The single path estimator is equivalent to $\rho = 0$ and models the multipath signal as uncorrelated with the LOS signal.

A. One Multipath Signal

First, we consider the case $L = 1$. Figure 1 shows the CRLB and RMSE of the estimate $\hat{\tau}_0$ for different choices of $\boldsymbol{Q}_{\boldsymbol{u}}$ over the delay difference $\Delta\tau = \tau_1 - \tau_0$. The multipath azimuth angle of arrival is $\phi_1 = 150°$. All CP model methods have a better performance than estimating with a single path assumption. In comparison to the white multipath interference assumption the approximate temporal multipath interference covariance matrix leads to a better performance for the LOS time-delay estimation. Using the exact multipath approximation (20) instead of (22) does not yield a better estimation performance for τ_0 even though the computational effort is significantly higher, due to the second line search which has to be performed. Figure 2 shows the CRLB and RMSE of the estimate $\hat{\tau}_0$ for different choices of $\boldsymbol{Q}_{\boldsymbol{u}}$ over the multipath azimuth angle of arrival ϕ_1. The delay difference is $\Delta\tau = 0.2T_c$. Again all CP model algorithms perform better than the single path algorithm.

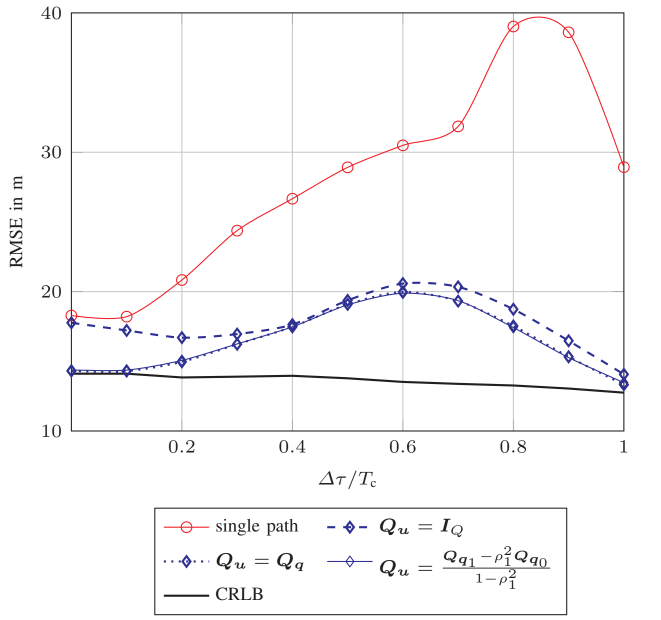

Figure 1. Estimation Performance of the CP Model for Different Choices of the Multipath Interference Temporal Covariance Matrix $\boldsymbol{Q}_{\boldsymbol{u}}$ over $\Delta\tau/T_{\mathrm{c}}$ $(L = 1)$

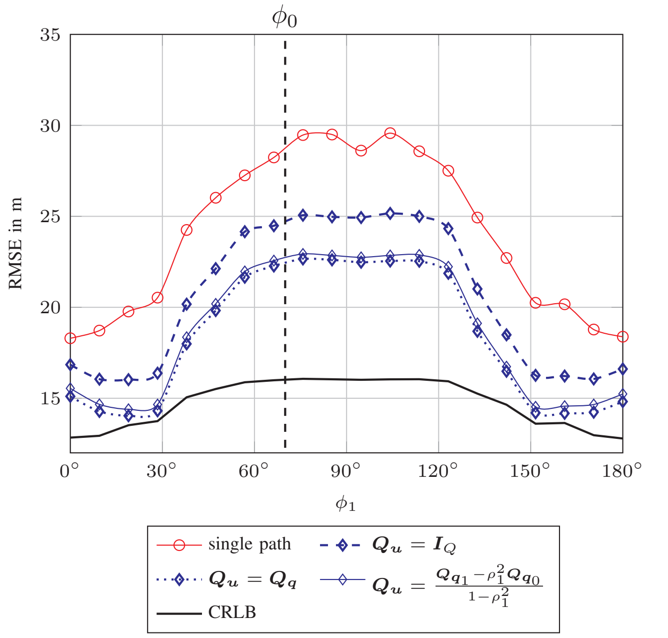

Figure 2. Estimation Performance of the CP Model for Different Choices of the Multipath Interference Temporal Covariance Matrix $\boldsymbol{Q}_{\boldsymbol{u}}$ over ϕ_1 $(L = 1)$

B. $L = 6$ Multipath Signals

The CP path algorithm is designed under the assumption of $L = 1$ multipath signals. However, in [10] it has been shown that an even better performance can be obtained in the case of multiple multipath signals if these are highly temporally and spatially correlated. Figure 3 shows the RMSE of the estimate $\hat{\tau}_0$ for different choices of $\boldsymbol{Q}_{\boldsymbol{u}}$ over the mean delay difference $\Delta\bar{\tau} = \frac{1}{L}\sum_{l=1}^{L}\tau_l - \tau_0$. The τ_l are evenly spread

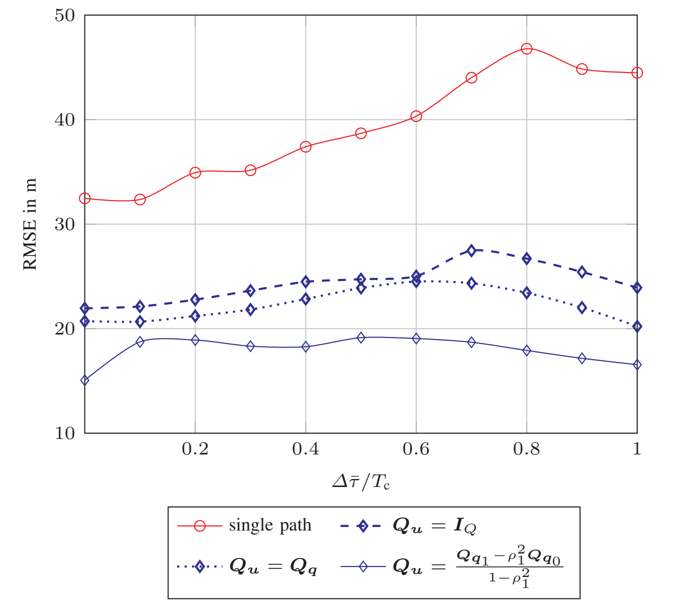

Figure 3. Estimation Performance of the CP Model for Different Choices of the Multipath Interference Temporal Covariance Matrix $\boldsymbol{Q_u}$ over $\Delta\bar{\tau}/T_c$ $(L = 6)$

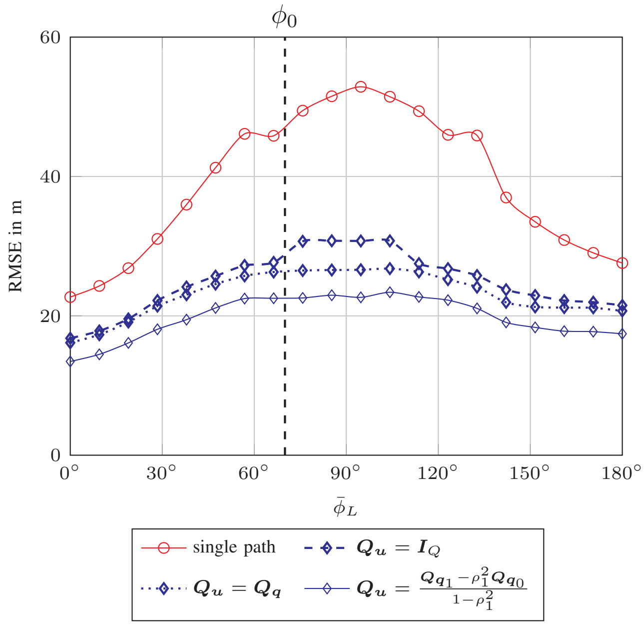

Figure 4. Estimation Performance of the CP Model for Different Choices of the Multipath Interference Temporal Covariance Matrix $\boldsymbol{Q_u}$ over $\bar{\phi}_L$ $(L = 6)$

within an interval of $0.4T_c$. The multipath azimuth angles of arrival are spread within $155°$ and $185°$. The CRLB cannot be calculated for the problem where all parameters are estimated, as the multipath signals are highly correlated and therefore the multipath parameters are not identifiable [13]. All CP-based methods yield a better performance than estimating with the single path assumption. Using the temporal interference covariance matrices (20) or (22) yields a better LOS time-delay estimation performance than assuming white multipath interference. Applying the exact multipath approximation (20) instead of (22) yields an even better estimation performance for τ_0 in this case. Figure 4 shows the RMSE of the estimate $\hat{\tau}_0$ for different choices of $\boldsymbol{Q_u}$ over the mean azimuth angle of arrival $\bar{\phi}_L = \frac{1}{L}\sum_{l=1}^{L}\phi_l$. The mean delay difference is $\Delta\bar{\tau} = 0.2T_c$. The ϕ_l are evenly spread within an interval of $30°$. This example shows that especially in the case of highly spatially correlated LOS and multipath signals the CP model can yield a large performance gain in comparison with the single path model. Again the exact multipath interference covariance matrix (20) shows the best performance.

VI. Conclusion

In this paper we have derived the CP model for temporally correlated multipath interference. Under the assumption that the multipath plus noise covariance matrix is still approximately white, the parameters of the CP model can be estimated in the same way as in the case of white multipath interference. For the estimation of the LOS time delay however, the temporal covariance of the multipath interference has to be considered. In comparison to the CP model with assumption, the approach presented in this paper achieves a better performance while still offering the benefits of the CP model, i.e no model order estimation, a limited number of parameters to estimate and no separation of the LOS delay estimate from the multipath delay estimates. Simulation results show that the CP model with temporally correlated multipath interference leads to significantly better estimation results for the LOS time-delay than a single path estimator or the CP model with the white noise assumption. This is not only the case for a single multipath signal, but especially for multiple highly temporally and spatially correlated multipath signals.

VII. Acknowledgement

The research leading to these results has been carried out under the framework of the project ”R&D for the maritime safety and security and corresponding real time services”. The project started in January 2013 and is led by the Program Coordination Defence and Security Research within the German Aerospace Center (DLR). This work was supported by the Technische Universität München - Institute for Advanced Study, funded by the German Excellence Initiative and the European Union Seventh Framework Programme under grant agreement No. 291763, by the European Union under the Marie Curie COFUND Program and has received funding from Coordenação de Aperfeiçoamento de Pessoal de Nível Superior (CAPES) under the PVE grant number 88881.030392/2013-01.

References

[1] M. Braasch and A. van Dierendonck, “GPS receiver architectures and measurements,” *Proceedings of the IEEE*, vol. 87, no. 1, pp. 48–64, Jan 1999.

[2] S. M. Kay, *Fundamentals of Statistical Signal Processing, Volume 1: Estimation Theory*. Prentice Hall PTR, 1993.

[3] E. Lohan, R. Hamila, A. Lakhzouri, and M. Renfors, “Highly efficient techniques for mitigating the effects of multipath propagation in DS-CDMA delay estimation,” *Wireless Communications, IEEE Transactions on*, vol. 4, no. 1, pp. 149–162, Jan 2005.

[4] N. Blanco-Delgado and F. D. Nunes, “Multipath Estimation in Multicorrelator GNSS Receivers using the Maximum Likelihood Principle NURIA,” *IEEE Transactions on Aerospace and Electronic Systems*, vol. 48, no. 4, p. 2012, Oktober 2012.

[5] J. A. Fessler and A. Hero, "Space-Alternating Generalized Expectation-Maximization Algorithm," *IEEE Transactions on Signal Processing*, vol. 42, no. 10, pp. 2664–2677, October 1994.

[6] F. Antreich, J. Nossek, G. Seco-Granados, and A. Swindlehurst, "The Extended Invariance Principle for Signal Parameter Estimation in an Unknown Spatial Field," *IEEE Transactions on Signal Processing*, vol. 59, no. 7, pp. 3213–3225, July 2011.

[7] S. Rougerie, G. Carri´e, F. Vincent, L. Ries, and M. Monnerat, "A new Multipath Mitigation Method for GNSS Receivers Based on an Antenna Array," *International Journal of Navigation and Observation*, vol. 2012, 2012.

[8] G. Seco-Granados, J. Fernandez-Rubio, and C. Fernandez-Prades, "ML estimator and hybrid beamformer for multipath and interference mitigation in GNSS receivers," *Signal Processing, IEEE Transactions on*, vol. 53, no. 3, pp. 1194–1208, March 2005.

[9] O. Bialer, D. Raphaeli, and A. Weiss, "Efficient Time of Arrival Estimation Algorithm Achieving Maximum Likelihood Performance in Dense Multipath," *IEEE Transactions on Signal Processing*, vol. 60, no. 3, pp. 1241–1252, March 2012.

[10] F. Wendler, F. Antreich, J. A. Nossek, and A. L. Swindlehurst, "Dual-Polarization Time Delay Estimation for Multipath Mitigation," in *WSA 2015; 19th International ITG Workshop on Smart Antennas; Proceedings of.* VDE, 2015, pp. 1–6.

[11] K. B. Petersen and M. S. Pedersen. (2012, November) The Matrix Cookbook. Electronic Book. Version 20121115. [Online]. Available: http://www2.imm.dtu.dk/pubdb/p.php?3274

[12] J. S. Vera, "Efficient Multipath Mitigation in Navigation Systems," Ph.D. dissertation, Universitat Politécnica de Catalunya, Spain, 2004, spain.

[13] P. Stoica and T. Marzetta, "Parameter estimation problems with singular information matrices," *IEEE Transactions on Signal Processing*, vol. 49, no. 1, 2001.