Peak-to-Average-Power Reduction for FBMC-based Systems

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***Abstract*—In this study, different peak-to-average-power-ratio (PAPR) reduction schemes proposed for OFDM are analyzed for Filter-Bank Multicarrier/Offset Quadrature Amplitude Modulation (FBMC/OQAM) systems, namely clipping, tone reservation (TR), active constellation extension (ACE), joint tone reservation and active constellation extension (TRACE) and selected mapping (SLM). A modification of selected mapping (SLM) is introduced, where the overlapping structure of FBMC/OQAM signals is exploited. Further, a novel set of schemes for PAPR reduction that can be applied to any multi-carrier modulation technique called *two-stage PAPR reduction* is proposed. This set of schemes is found to offer similar PAPR reduction performance of clipping, however with an improved SER performance that varies depending on the complexity and the type of the used first stage. It is observed, that the PAPR reduction can benefit for FBMC by taking the specific structure of the time-domain signal into account.**

***Index Terms*—OFDM, FBMC, PAPR reduction.**

I. Introduction

Filter-Bank Multicarrier/Offset Quadrature Amplitude Modulation (FBMC/OQAM) [1] has recently attracted a lot of attention as a potential competitor to the widely adopted cyclic-prefix Orthogonal Frequency-Division Multiplexing (CP-OFDM) [2]. Despite of the superior frequency localization that makes FBMC/OQAM a potential enabler in the context of opportunistic networks, it still suffers from high peak-to-average power ratio (PAPR) [3].

Generally, a high PAPR requires highly linear power amplifiers (expensive) or a large input back-off (energy inefficient) in order to protect the signal from severe degradation due to the non-linearities. Hence, the idea of PAPR reduction schemes was proposed in the literature in order to combat such problem without the loss in power efficiency. An overview of the different schemes for OFDM is given in [4], [5]. The same idea was applied to FBMC/OQAM [6] using some of the proposed schemes. In this paper, two new PAPR reduction schemes are introduced. First, a set of *two-stage PAPR reduction* schemes are introduced which is applicable to any multi-carrier modulation technique. The second scheme is a modification of a scheme from the literature of OFDM, called *modified selected mapping (mSLM)*, which takes the special structure of the FBMC/OQAM into consideration. This paper is organized as follows, Section II starts by motivating the reason behind the increased interest in FBMC/OQAM and then introduces the single-input/single-output (SISO) FBMC/OQAM transceiver structure based on the efficient implementation proposed in the European Union funded project PHYDYAS [3]. It is then concluded by the introduction of the PAPR metric used in defining the problem and the evaluation of the different PAPR reduction schemes' performances. Section III introduces the various schemes from the literature of OFDM that are used as the basis for the proposed schemes. These schemes are then used to introduce the algorithm for the newly proposed *two-stage PAPR reduction* set of schemes. Additionally, the SLM scheme from the literature of OFDM is discussed and a modification that benefits from the structure of the FBMC/OQAM time signal is introduced. Section IV presents simulation results for the different schemes presented for FBMC/OQAM in comparison to CP-OFDM. Finally, Section V gives a wrap up of all the results presented.

II. System Model

Filter-Bank Multicarrier (FBMC) is a generalization of the MC modulation concept, where a well-designed prototype filter shapes the modulated signal on each sub-carrier. This prototype filter differs from the traditional rectangular pulse-shape filter used in CP-OFDM with its sinc-shaped spectrum. The filter suppresses the high out of band leakage observed in the sub-carrier frequency response of OFDM. Fig. 1 gives the FBMC/OQAM transceiver block diagram based on the implementation proposed in PHYDYAS [3].

The fundamental parts of this block diagram are the OQAM pre-processing and the synthesis filter bank (SFB) at the transmitter side as well as the analysis filter bank (AFB) and the OQAM post-processing at the receiver side. Using the derivations from [7], the discrete-time baseband FBMC/OQAM signal can be expressed as

$$\begin{aligned} s\left[k\right] &= \sum_{n=-\infty}^{\infty} \sum_{m=0}^{M-1} d_{m,n} \theta_{m,n} \beta_{m,n} p\left[k - n\frac{M}{2}\right] \mathrm{e}^{\mathrm{j}\frac{2\pi}{M}mk} \\ &= \sum_{n=-\infty}^{\infty} s_n[k] \end{aligned} \tag{1}$$

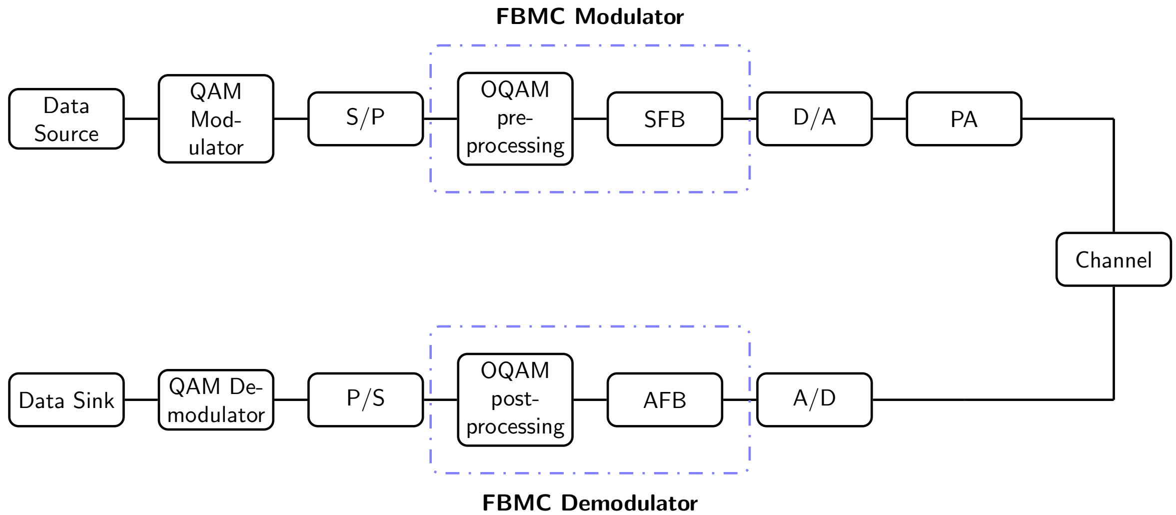

Fig. 1. FBMC/OQAM single-input single-output (SISO) transceiver structure based chain based on the PHYDIAS implementation [3] using synthesis filter bank (SFB) and analysis filter bank (AFB).

where k is the high-data-rate (M/T) sample index at the output of the SFB, m is the sub-carrier index for the M sub-carriers FBMC/OQAM system, n is the time index at the OQAM symbol rate $(2/T)$, $d_{m,n}$ is the real data sequence before phase mapping. The symbol $\theta_{m,n}$ is defined as $\theta_{m,n} = \mathrm{j}^{m+n}$ controlling the phase of the real data sequence at the input of the SFB and $\beta_{m,n}$ is defined as $\beta_{m,n} = (-1)^{mn} \cdot \mathrm{e}^{-\mathrm{j}\frac{2\pi m}{M}\left(\frac{L_p-1}{2}\right)}$, where L_p is the prototype filter length. As impulse response of the prototype filter $p\,[k]$, the one proposed in [8] is used. Finally, it should be mentioned that unlike CP-OFDM, FBMC/OQAM has overlapping time symbols as shown in Fig. 2. Finally, as previously mentioned that a common drawback shared among all MC modulation techniques is the large dynamic range of the transmitted signal due to the statistical independence of the sub-carriers, which make them highly sensitive to the non-linear characteristics of the PA. This non-linearity would then induce in-band and out-of-band spurious products, which, as a result, might degrade the system's performance. A simple metric that can be used to describe the dynamic behaviour of the transmitted signal $s\,(t)$ is the PAPR which is defined as

$$\gamma = \frac{\max\left\{|s\,(t)|^2\right\}}{\mathrm{E}\left\{|s\,(t)|^2\right\}}, \tag{2}$$

where $|s\,(t)|$ is the amplitude of the transmitted signal and $\mathrm{E}\{\cdot\}$ is the expected value. This PAPR can be expressed in dB as

$$\mathrm{PAPR_{dB}} = 10\log_{10}(\gamma)\,. \tag{3}$$

To tackle such problem, several PAPR reduction schemes were developed for OFDM, where each has its own advantages and disadvantages. The properties of these PAPR reduction techniques are the focus of this paper and thus will be further discussed in the following.

III. PAPR Reduction Schemes

In this paper, five different schemes from the literature of OFDM will be compared: clipping [5], tone reservation (TR) [9], active constellation extension (ACE) [10], joint tone reservation and active constellation extension (TRACE) [10], and selected mapping (SLM) [11]. The first four schemes are going to be included in our presented set of schemes called two-stage PAPR reduction. Finally, SLM will be modified in order to benefit from the special structure of the FBMC/OQAM's time signal, which is called mSLM.

A. Clipping

Clipping is the simplest and most effective method to reduce the peaks to a desired level, and thus reducing the PAPR of the signal. The output clipped signal $s^c\,[k]$ of a soft limiter can be written as

$$s^c\,[k] = \begin{cases} s\,[k]\,, & |s\,[k]| < A_{\max} \\ A_{\max}\mathrm{e}^{\mathrm{j}\phi(s[k])}, & |s\,[k]| \geq A_{\max}, \end{cases} \tag{4}$$

where $|s\,[k]|$ is the magnitude of $s\,[k]$, $A_{\max}$ is the clipping level, and $\phi\,(s\,[k])$ represents the phase of $s\,[k]$. Although this scheme ensures peak reduction, the distortion caused by amplitude clipping can be viewed as another source of noise which is either in-band or out-of-band noise. In-band distortion means that it generates self-interference by distorting the amplitude of the signal which degrades the system performance as a result, while out-of-band noise means that it gives rise to the regrowth of the high frequency components reducing the spectral efficiency as a result.

B. Iterative Clipping Schemes

Fig. 3 gives the general block diagram for the different iterative clipping based schemes. These schemes differ in the method used for symbols modification, which are discussed in the following. Note that at the PAPR measurement block the signal follows path (a) if the PAPR is less than or equal to the

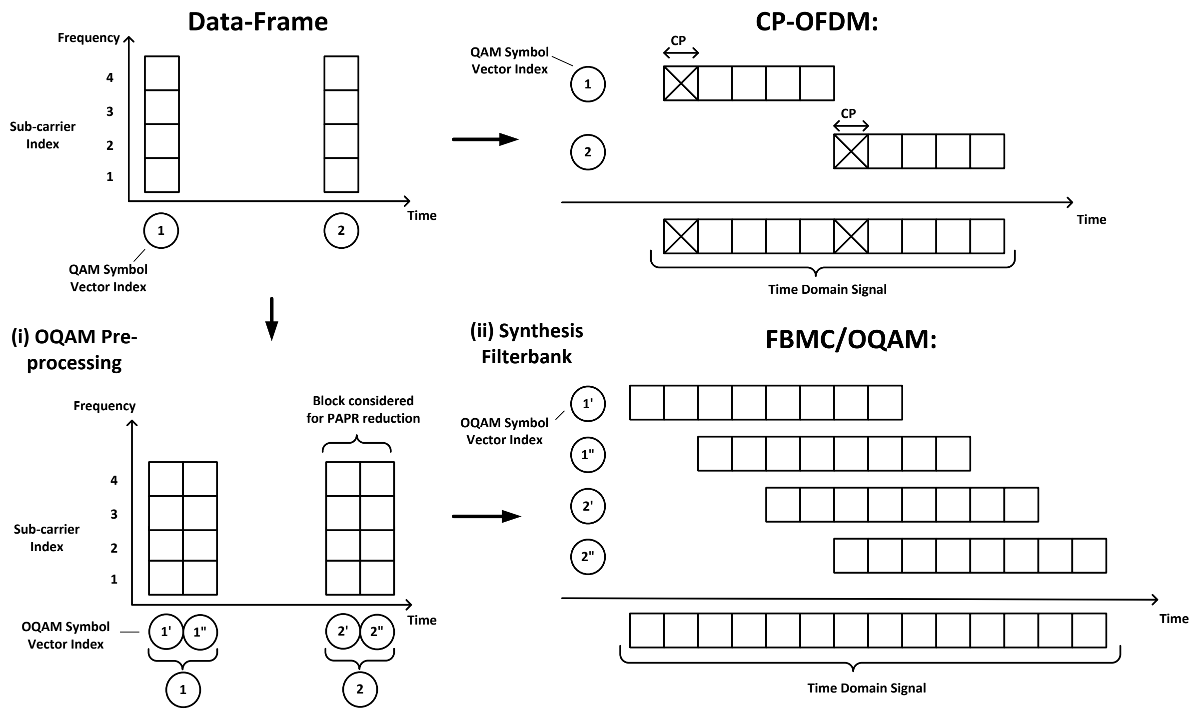

Fig. 2. Overlapping FBMC time symbols: for this example, the number of sub-carriers is $M = 4$, the overlapping factor is $K = 2$, the prototype filter length is $L_{\text{length}} = K \cdot M = 8$ and number of QAM symbol vectors is $n_{\text{symbol}} = 2$.

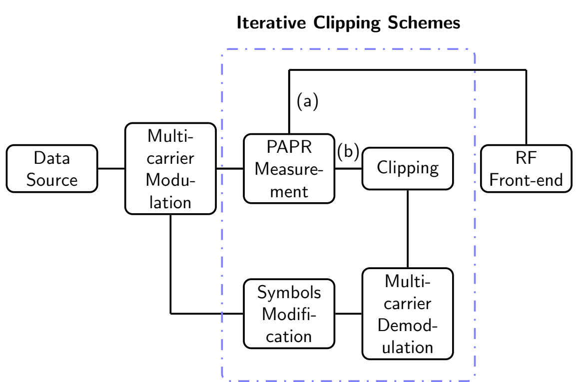

Fig. 3. Iterative clipping block diagram: the signal follows path (a) if the PAPR is less than or equal to the predefined threshold or if the maximum number of iterations set for the scheme is reached. Otherwise it will follow path (b).

predefined threshold or if the maximum number of iterations set for the scheme is reached. Otherwise it will follow path (b).

1) Tone Reservation: The Tone Reservation (TR) scheme reduces the PAPR of the transmitted time domain OFDM signal by reserving a small subset of sub-carriers, called peak reduction tones (PRT), with no data in them. These reserved sub-carriers are then optimized in order to reduce the PAPR. This technique was developed for the digital subscriber line (DSL) system, hence the naming of tones instead of sub-carriers. In [9] an efficient optimization of the non-data carrying sub-carriers is proposed based on the fairly efficient projection-onto-convex-sets (POCS) algorithm. In this scheme all the symbols in the data carrying sub-carriers are changed back to their original values before clipping, while the symbols of the PRT are kept unchanged. Despite the relative simplicity of this technique it suffers from several drawbacks, the first is the data rate loss in reserving a subset of sub-carriers with no data in them. Additionally, the choice of the PRT position sets significantly affect performance and it is known to be a non-deterministic polynomial-time hard (NP-hard) problem as it has to be optimized over all possible sets. Unlike in a DSL system, where typically a number of tones with low SNR could be used as PRT, wireless systems do not have such luxury and so in reserving a random subset of sub-carriers would lead to loss in bandwidth.

2) Active Constellation Extension: This scheme reduces the PAPR by changing the constellation of the modulated signal. It maps the outer constellation points into arbitrary positions in order to reduce the PAPR while taking into consideration not to decrease the minimum distance of the modulation scheme in order not to affect the SER performance. In [10], similar to the TR scheme, an efficient optimization of the constellation points based on the projection-onto-convex-sets (POCS) algorithm was proposed. In this scheme, all symbols that penetrate areas in the constellation that would decrease the minimum distance of this modulation scheme are changed back to their original values, while the symbols of the outer constellation points that are extended to regions that won't affect the minimum distance of the modulation scheme are left unchanged. Fig. 4 gives an example of the allowed regions for symbols extension in a 4-QAM constellation diagram.

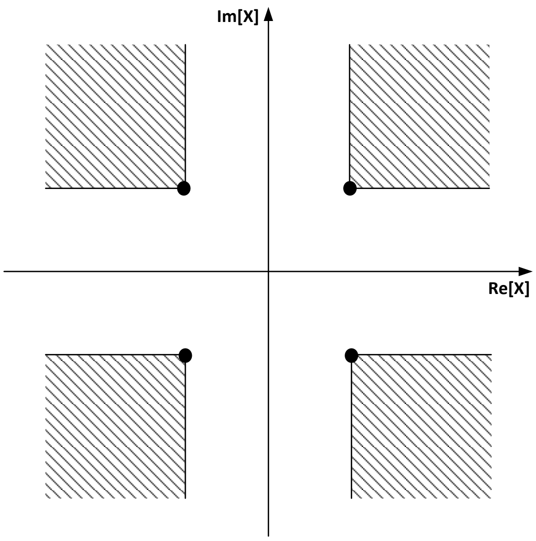

Fig. 4. Allowed regions for symbol extensions in a 4-QAM constellation diagram for active constellation extension (ACE).

This scheme maintains the SER performance with no loss in data rate as in TR. However, this comes at the expense of increased transmit power. Additionally, such scheme distorts the signal constellation and thus soft-decision detection can not be applied directly. Finally, high transmitter complexity is one of the major drawbacks.

3) Joint Tone Reservation and Active Constellation Extension: This scheme is simply a combination of TR and ACE, where a small subset of PRT are reserved with no data in them and are optimized for PAPR reduction. Additionally, the outer constellation points of the data carrying sub-carriers are mapped into arbitrary positions in order to reduce the PAPR while taking into consideration not to decrease the minimum distance of the modulation scheme in order not to affect the SER performance. This idea was first introduced in [10], where it was expected to offer a significant improvement in PAPR reduction performance compared to applying either scheme independently. Moreover, it was presented in [6] for PAPR reduction of FBMC/OQAM signals. This scheme has a faster convergence of PAPR reduction compared to the case of applying either scheme, TR or ACE, independently. However, this comes at the expense of suffering from the disadvantages of both schemes.

C. Two-Stage PAPR Reduction

This is the proposed set of PAPR reduction schemes, where the reduction is done on two separate stages. The first stage is one of the iterative clipping based schemes (i.e., TR, ACE, TRACE) and then clipping is applied at the second and final stage. This means that it is only a version of the iterative schemes, where after the predefined maximum number of iterations is reached, a consecutive clipping is applied giving the final output. Fig. 5 gives a simplified block diagram for the two-stage PAPR reduction. These schemes are expected

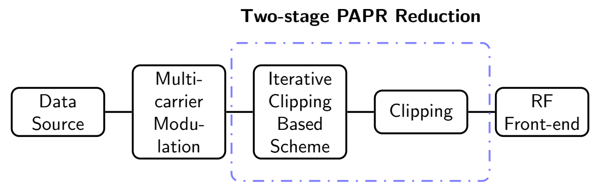

Fig. 5. Two-stage PAPR reduction scheme block diagram. In the first stage, one of the iterative clipping based schemes (i.e., TR, ACE, TRACE) is applied followed by a consecutive clipping at the second stage.

to achieve the PAPR reduction performance of clipping, while at the same time achieving a better symbol error rate (SER) performance. The attractive aspect of this set of schemes would be its potential in finding a compromise between SER performance degradation and the invested computational complexity, while achieving the desired PAPR reduction performance.

D. Modified Selected Mapping

The SLM scheme was first presented in [11], where the transmitter generates several, say U, different candidate symbol vectors $\mathbf{X}^u$ based on the same input symbol vector $\mathbf{X}$ and choose the one which gives the minimum PAPR for transmission. SLM has the advantage of being compatible to work with any modulation scheme and with arbitrary number of sub-carriers. Moreover, it completely avoids signal distortion and has an increased reduction of PAPR with an increasing number of candidates U. However, it must be noted that as U increases, there is an increase in the data rate loss due to additional overhead, i.e., the index information $\tilde{u}$ transmitted to the receiver in order to be able to recover the original input symbol vector. It also has to be noted that such disadvantage is avoidable as a modified algorithm for SLM was presented where there is no need for transmission of side information [12]. Additionally, as U increases the computational complexity increases. In this work the SLM is modified in order to benefit from the special structure of the FBMC/OQAM's time signal, Fig. 2. The mSLM scheme is similar to the scheme proposed in [13] for wavelet OFDM, where it considers U^K hypotheses instead of just U as in the regular SLM. Offering a potential for improved PAPR reduction performance as a result, however at the expense of a rather great increase in computational complexity. The algorithm of mSLM scheme involves the following steps.

1) Several different candidate QAM symbol vectors $\mathbf{X}^u = \left[X_0^u, X_1^u, \ldots, X_{M-1}^u\right]^T$ based on the same input QAM symbol vector $\mathbf{X} = \left[X_0, X_1, \ldots, X_{M-1}\right]^T$ are generated through component wise multiplication of $\mathbf{X}$ with the phase sequences $\mathbf{P}^u = \left[P_0^u, P_1^u, \ldots, P_{M-1}^u\right]^T, u = 1, \ldots, U$.

2) The components $P^u_m, m = 0, \ldots, M-1$, of the phase sequence $\mathbf{P}^u$ are unit magnitude complex numbers, where $P^u_m = \exp(\mathrm{j}\varphi^u_m), \varphi^u_m \in [0, 2\pi)$. Generally $P^u_m \in \{\pm 1\}$ or $P^u_m \in \{\pm 1, \pm \mathrm{j}\}$ are often used.
3) The U different candidate QAM symbol vectors $\mathbf{X}^u$ are then modulated using the respective FBMC/OQAM modulator, giving U different SLM signal vectors $\mathbf{y}^u$.
4) Until step (3) this is exactly as for the regular SLM. However, now for FBMC/OQAM the overlap between the symbols are to be taken into consideration by considering K overlapping QAM symbol vectors instead of considering each individually. This means that we now have U^K hypotheses

$$\hat{s}^{(\mathbf{u})}[k] = \sum_{n'=-\infty}^{n-1} s_{n'}[k + (n-n')M] + \sum_{\nu=0}^{K-1} s^{(u_\nu)}_{n+\nu}[k - \nu M] \quad (5)$$

Where $\mathbf{u} \triangleq [u_\nu] = [u_0, u_1, \ldots, u_{K-1}]$, with the first part of the equation denoting the signal that has been generated until step n, and the second part of the equation considers the next K FBMC symbols with all U possible variations of each symbol.
5) Now a PAPR value is calculated for every $\hat{s}^{(\mathbf{u})}[k]$, from which the optimum $\tilde{u}$ that gives the minimum PAPR is identified. Then finally $\hat{s}^{(\tilde{u}_0)}[k]$ is selected for transmission at step n.
6) In the next step $n+1$, all the hypotheses with $u_0 = \tilde{u}_0$ can be reused, and the U candidates of the next FBMC/OQAM symbol $n+K$ are considered.

IV. Results

In this section, the results obtained from simulating the various PAPR reduction schemes that were previously explained for both OFDM and FBMC/OQAM based systems are presented. Additionally, an evaluation in terms of SER performance is given for schemes that has an effect on it. To ensure a fair comparison, both OFDM and FBMC/OQAM have the same parameters, where the total number of sub-carriers used are $M = 64$ with the DC sub-carrier forced to zero. An oversampling of $L = 4$ is used, in order for the obtained results to give a good prediction of the PAPR characteristics in the analog domain [3]. Moreover, 4-QAM modulation is used. For the FBMC/OQAM specific parameters, the filter used is the PHYDYAS filter with an overlapping factor of $K = 4$ and a prototype filter length of $L_p = KM - 1$. While, for the OFDM specific parameters the cyclic prefix (CP) length is chosen such that an overhead of 7.8% is achieved as in the LTE normal mode operation. Note that the complementary cumulative distribution function (CCDF) is used for the evaluation of the PAPR of all the reduction schemes. The clipping ratio (CR) is set to 3 dB for all schemes, where it is defined as $\mathrm{CR_{dB}} = 20\log\left(\frac{A_{\max}}{\sqrt{P_s}}\right)$, P_s is the average power of the time signal $s(t)$ before PAPR reduction and $A_{\max}$ is the maximum allowed amplitude. Finally, the maximum number of iterations for the iterative clipping schemes is fixed to 2 iterations, so by doing this there are 2 modulation and 2 demodulation procedures which are either OFDM or FBMC/OQAM. To further ensure having a fixed complexity for all schemes, the number of phase sequences U of the SLM scheme is set to 4; thus, having a total of 4 modulation procedures, where both modulation and demodulation procedures have similar complexities. Fig. 6 gives the CCDF of the PAPR distribution for 10^6 blocks for the proposed two-stage (TR /TRACE /ACE /+ Clipping) and selective mapping (SLM) PAPR reduction for OFDM and FBMC. It can be seen that for the defined CR of 3 dB and 2 iterations for the iterative clipping based schemes, a PAPR similar reduction of about 7 dB for both FBMC/OQAM and OFDM is achieved at a probability of 10^{-4}. Additionally, it can be observed that the SLM achieves a PAPR reduction performance of approximately 2.5 dB at the same probability of 10^{-4}. Fig. 7 gives the CCDF of the PAPR distribution for

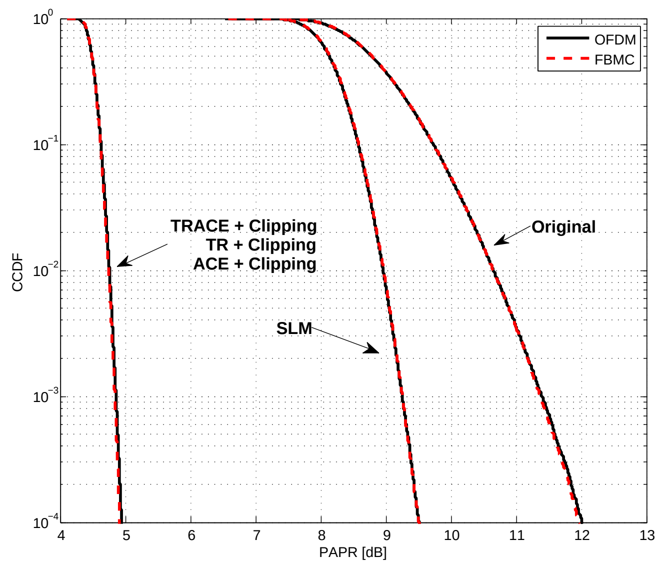

Fig. 6. Comparison of Two-stage (TR /TRACE /ACE /+ Clipping) and selective mapping (SLM) based PAPR reduction for OFDM and FBMC: for this figure, the number of sub-carriers is $M = 64$, oversampling factor of $L = 4$, the overlapping factor is $K = 4$, the prototype filter length is $L_{\text{length}} = K \cdot M - 1 = 255$, a constellation size of 4, and a clipping level of $\mathrm{CR_{dB}} = 3$ dB

mSLM versus SLM for different number of phase sequences $U = 2, 4, 6$. It can be observed, that the mSLM scheme achieves a PAPR reduction of 1.8 dB/3 dB/3.5 dB in contrast to 1 dB/1.5 dB/2 dB for regular SLM at a probability of 10^{-2}, for different number of phase sequences $U = 2, 4, 6$. Note that mSLM only can be applied to FBMC due to the overlapping time symbols. Fig. 8 gives the symbol error rate (SER) performance before and after the application of the various PAPR reduction schemes under investigation. It can be observed in this figure that there is no deterioration in the SER performance when using the SLM scheme, which is the same for mSLM. Moreover, a deterioration in the SER performance for all previously presented combinations of the two-stage

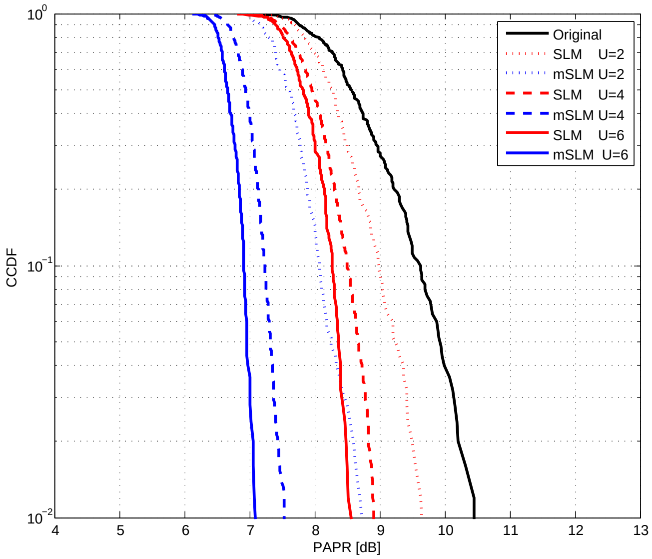

Fig. 7. Comparison of mSLM and SLM for different number of phase sequences $U = 2, 4, 6$ for FBMC: for this figure, the number of sub-carriers is $M = 64$, oversampling factor of $L = 4$, the overlapping factor is $K = 4$, the prototype filter length is $L_{\text{length}} = K \cdot M - 1 = 255$, a constellation size of 4, and a clipping level of $\text{CR}_{\text{dB}} = 3\ \text{dB}$

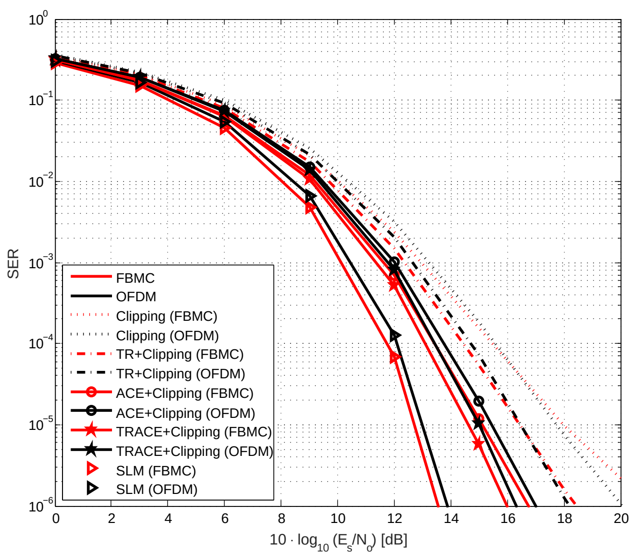

Fig. 8. Symbol error rate (SER) performance of OFDM and FBMC before and after the application of PAPR reduction schemes under investigation: for this figure, the number of sub-carriers is $M = 64$, oversampling factor of $L = 4$, the overlapping factor is $K = 4$, the prototype filter length is $L_{\text{length}} = K \cdot M - 1 = 255$, a constellation size of 4, and a clipping level of $\text{CR}_{\text{dB}} = 3\ \text{dB}$

schemes in comparison with the case when no PAPR reduction and when clipping are used is seen. It can be observed that their is a loss of approximately $5\,\text{dB}$ at a probability of 10^{-5} in SER performance when comparing clipping with the case when no PAPR reduction is used. The two-stage TR+Clipping scheme shows a gain of about $1.5\,\text{dB}$ in SER performance over clipping at the same probability. This gain is further improved for ACE+Clipping amounting to about $2.5\,\text{dB}$. The final combination of TRACE+Clipping offers the best gain of approximately $2.7\,\text{dB}$.

V. Conclusion

In this paper, a study of different PAPR reduction schemes with focus on applicability to FBMC/OQAM systems was conducted. First, a set of two-stage PAPR reduction schemes was proposed that showed superior PAPR reduction performance similar to that of clipping, however with an improved SER performance. This schemes show a potential compromise between the deterioration of the signal and the computational complexity, while offering the desired PAPR reduction performance. Further, a modification of the SLM scheme proposed for OFDM is presented. It is shown, that exploiting the time domain symbol structure from FBMC improves the PAPR reduction compared to SLM and does not deteriorate the signal. However, the PAPR reduction performance of mSLM increases a U increases to the cost of increased computational complexity, since U^K hypotheses have to be considered instead of just U for regular SLM.

References

[1] M. Bellanger, D. Le Ruyet, D. Roviras, M. Terré, J. Nossek, L. Baltar, Q. Bai, D. Waldhauser, M. Renfors, T. Ihalainen *et al.*, "FBMC physical layer: a primer," *PHYDYAS, January*, 2010.

[2] J. A. Bingham, "Multicarrier modulation for data transmission: An idea whose time has come," *IEEE Communications Magazine*, vol. 28, no. 5, pp. 5–14, 1990.

[3] A. Viholainen, M. Bellanger, and M. Huchard, "Prototype filter and structure optimization," *website: www.ict-phydyas.org: Document D5.1*, 2009.

[4] S. H. Han and J. H. Lee, "An overview of peak-to-average power ratio reduction techniques for multicarrier transmission," *IEEE Wireless Communications*, vol. 12, no. 2, pp. 56–65, 2005.

[5] D.-W. Lim, S.-J. Heo, and J.-S. No, "An overview of peak-to-average power ratio reduction schemes for OFDM signals," *Journal of Communications and Networks*, vol. 11, no. 3, pp. 229–239, 2009.

[6] Z. Kollár, L. Varga, and K. Czimer, "Clipping-based iterative papr-reduction techniques for FBMC," in *Proceedings of 17th International OFDM Workshop 2012 (InOWo'12)*, 2012, pp. 1–7.

[7] M. Tanda, T. Fusco, M. Renfors, J. Louveaux, and M. Bellanger, "Data-aided synchronization and initialization (single antenna)," *website: www.ict-phydyas.org: Document D2.1*, 2008.

[8] K. W. Martin, "Small side-lobe filter design for multitone data-communication applications," *IEEE Transactions on Circuits and Systems II: Analog and Digital Signal Processing*, vol. 45, no. 8, pp. 1155–1161, 1998.

[9] A. Gatherer and M. Polley, "Controlling clipping probability in DMT transmission," in *IEEE Conference Record of the Thirty-First Asilomar Conference on Signals, Systems, and Computers*, vol. 1, 1997, pp. 578–584.

[10] D. L. Jones, "Peak power reduction in OFDM and DMT via active channel modification," in *IEEE Conference Record of the Thirty-Third Asilomar Conference on Signals, Systems, and Computers*, vol. 2, 1999, pp. 1076–1079.

[11] R. W. Bäuml, R. F. H. Fischer, and J. B. Huber, "Reducing the peak-to-average power ratio of multicarrier modulation by selected mapping," *Electronics Letters*, vol. 32, no. 22, pp. 2056–2057, 1996.

[12] M. Breiling, S. H. Müller-Weinfurtner, and J. B. Huber, "SLM peak-power reduction without explicit side information," *IEEE Communications Letters*, vol. 5, no. 6, pp. 239–241, 2001.

[13] M. Hoch, S. Heinrichs, and J. B. Huber, "Peak-to-average power ratio and its reduction in wavelet-OFDM," in *International OFDM Workshop*, 2011, pp. 56–60.