Tensor-Based Approach for Time-Delay Estimation

Bilal Hammoud[1], Felix Antreich[1], Josef A. Nossek[2], João Paulo C. L. da Costa[3], and André L. F. de Almeida[4]

[1]German Aerospace Center (DLR), Institute for Communications and Navigation, Wessling, Germany
[2]Institute for Circuit Theory and Signal Processing, Technical University Munich, Munich, Germany
[3]Department of Electrical Engineering, University of Brasília (UnB), Brasília, Distrito Federal, Brazil
[4]Department of Teleinformatics Engineering, Federal University of Ceará (UFC), Fortaleza, Brazil

***Abstract*—Multipath (ML) signals reflected from the surrounding objects of a Global Navigation Satellite Systems (GNSS) receiver lead to a bias in the time-delay estimation of the line-of-sight (LOS) signal. In several cases, this bias is reflected as errors in the pseudo-range estimation of the receiver.**

In this paper, we derive a tensor-based filtering approach using an antenna array and a compression method based on canonical components (CC) with a bank of signal matched correlators in order to mitigate multipath and to estimate the time-delay of the LOS signal of a GNSS satellite. First, we resort to multi-dimensional filtering based on the principal singular vectors of the multi-dimensional data. In order to separate highly correlated signal components in the multi-dimensional signal subspace methods like forward-backward averaging (FBA), spatial smoothing (SPS), and the recently developed expanded spatial smoothing (SPS-EXP) are applied. Afterwards, time-delay estimation of the LOS signal is performed with a simple interpolation based on the multi-dimensional filtered cross-correlation values of the bank of correlators. An advantage of such an approach is that no multi-dimensional nonlinear problems need to be solved and also no model order estimation is required.

I. Introduction

The quality of the ranging data provided by a Global Navigation Satellite Systems (GNSS) receiver largely depends on the synchronization error, that is, on the accuracy of the propagation time-delay estimation of the line-of-sight (LOS) with respect to each satellite. In case the LOS signal is corrupted by several superimposed delayed replicas (reflective, diffractive, or refractive multipath), the estimation of the propagation time-delay and thus the position can be severely degraded using state-of-the-art GNSS receivers [1], [2], [3]. Especially, for high precision and safety-critical applications, *e.g.* aviation, maritime, rail, precision farming, surveying or automotive applications, multipath mitigation is very important in order to enable robust and reliable positioning.

Several techniques have been proposed in the literature for solving the multipath problem in GNSS using one antenna, see *e.g.* [4], [5], [6]. When using antenna arrays high resolution parameter estimation algorithms provide high accurate results [7], [8], [9], but they entail rather high complexity in the parameter estimation as multi-dimensional nonlinear problems have to be solved. Furthermore, they also require the use of accurate model order estimation algorithms [7]. A-J van der Veen et al in [10] uses a two-dimensional (2-D) ESPRIT-like shift-invariance technique to separate and estimate the phase shifts due to delay and direction-of-incidence. Tensor algebra have been used in blind multi-user detection and multipath estimation in [11], but resolution of highly correlated multipath signals was not considered with respect to time-delay estimation of the LOS signal. The considered performance metric in [11] is bit error rate. Furthermore, tensor formulations and low-rank modeling have been applied in studies using multiple input multiple output (MIMO) systems in order to resolve different propagation paths [12], [13], but time-delay estimation of a LOS signal in case of highly correlated multipath was not considered.

In this work, we present an approach for which no multi-dimensional nonlinear problem needs to be solved and also no model order estimation is required. We derive a tensor-based filtering approach using an antenna array and a compression method based on canonical components (CC) with a bank of signal-matched correlators [9] in order to mitigate multipath and to estimate the time-delay of the LOS signal. First, we resort to multi-dimensional filtering based on the principal singular vectors of the received data tensor. In order to separate highly correlated signal components in the multi-dimensional signal subspace methods like forward-backward averaging (FBA) [14], spatial smoothing (SPS) [15], and the recently developed expanded spatial smoothing (SPS-EXP) [16] are applied. Afterwards, time-delay estimation of the LOS signal is performed with a simple interpolation based on the multi-dimensional filtered cross-correlation values of the bank of correlators.

The proposed pre-processing schemes require that the antenna array response is left centro-hermitian. In case the array response is not left centro-hermitian, signal adaptive array interpolation methods can be applied to transform the array response to a centro-hermitian array response [17].

The proposed approach is capable of separating highly correlated and even coherent signals and is approaching the respective Cramer-Rao lower bound (CRLB) for time-delay estimation in the compressed time domain.

The rest of this paper is organized as follows: Section II defines the pre- and post-correlation signal model. Section III presents the tensor-based approach for time delay estimation. The computational complexity of each proposed algorithm is studied in Section IV. Simulation parameters and results are shown in Section V, and Section VI concludes the paper.

II. Signal Model

In the following, we define the pre- and post-correlation signal model for a multi-antenna GNSS receiver and we introduce a compression method based on a bank of signal matched correlators.

A. Pre-correlation Signal Model

The complex baseband signal of one GNSS satellite with bandwidth B that is received by an antenna array with M sensor elements can be given as

$$\mathbf{x}(t) = \mathbf{s}(t) + \mathbf{n}(t) = \sum_{\ell=1}^{L} \mathbf{s}_\ell(t) + \mathbf{n}(t) \tag{1}$$

where $\mathbf{s}(t) \in \mathbb{C}^{M\times 1}$ denotes the superimposed signal replicas

$$\mathbf{s}_\ell(t) = \mathbf{a}\left(\phi_\ell\right)\ \gamma_\ell\ c(t-\tau_\ell). \tag{2}$$

$\mathbf{a}\left(\phi_\ell\right) \in\ \mathbb{C}^{M\times 1}$ defines the steering vector of an antenna array with azimuth angle ϕ_ℓ, $c(t-\tau_\ell)$ denotes a periodically repeated pseudo random (PR) sequence $c(t)$ with time-delay τ_ℓ, chip duration T_c, and period $T = N_c T_c$ with $N_c \in \mathbb{N}$. γ_ℓ is the complex amplitude. Additionally, we assume temporally and spatially white complex Gaussian noise $\mathbf{n}(t) \in \mathbb{C}^{M\times 1}$. In the following the parameters of the line-of-sight (LOS) signal are indicated with $\ell = 1$ and the parameters of the non-LOS (NLOS) signals (multipath) with $\ell = 2, \ldots, L$. We define the signal parameter vectors

$$\begin{aligned} \boldsymbol{\eta} &= [\mathrm{Re}\{\boldsymbol{\gamma}\}^{\mathrm{T}}, \mathrm{Im}\{\boldsymbol{\gamma}\}^{\mathrm{T}}, \boldsymbol{\phi}^{\mathrm{T}}, \boldsymbol{\tau}^{\mathrm{T}}]^{\mathrm{T}} & (3)\\ \boldsymbol{\eta}_\ell &= [\mathrm{Re}\{\gamma_\ell\}, \mathrm{Im}\{\gamma_\ell\}, \phi_\ell, \tau_\ell]^{\mathrm{T}} & (4)\end{aligned}$$

with $\boldsymbol{\gamma} = [\gamma_1, \ldots, \gamma_L]^{\mathrm{T}}$, $\boldsymbol{\phi} = [\phi_1, \ldots, \phi_L]^{\mathrm{T}}$ and $\boldsymbol{\tau} = [\tau_1, \ldots, \tau_L]^{\mathrm{T}}$. The spatial observations are collected in K periods of the PR sequence of N time instances, thus $\mathbf{x}[(k-1)N + n] = \mathbf{x}(((k-1)N + n)\,T_s)$ with $n = 1, \ldots, N$, $k = 1, \ldots, K$, and the sampling frequency $\frac{1}{T_s} = 2B$. The channel parameters are assumed constant at least during the k-th period of the observation interval. Collecting the samples of the k-th period of the observation interval leads to

$$\mathbf{X}[k] = \left[\mathbf{x}[(k-1)N+1], \ldots, \mathbf{x}[(k-1)N+N]\right]\ \in\ \mathbb{C}^{M\times N} \tag{5}$$

$$\mathbf{N}[k] = \left[\mathbf{n}[(k-1)N+1], \ldots, \mathbf{n}[(k-1)N+N]\right]\ \in\ \mathbb{C}^{M\times N} \tag{6}$$

$$\mathbf{S}[k;\boldsymbol{\eta}] = \left[\mathbf{s}[(k-1)N+1], \ldots, \mathbf{s}[(k-1)N+N]\right]\ \in\ \mathbb{C}^{M\times N} \tag{7}$$

$$\mathbf{S}_\ell[k;\boldsymbol{\eta}_\ell] = \left[\mathbf{s}_\ell[(k-1)N+1], \ldots, \mathbf{s}_\ell[(k-1)N+N]\right]\ \in\ \mathbb{C}^{M\times N}. \tag{8}$$

Thus, the signal can be written in matrix notation as

$$\begin{aligned} \mathbf{X}[k] = \mathbf{S}[k;\boldsymbol{\eta}] + \mathbf{N}[k] &= \sum_{\ell=1}^{L} \mathbf{S}_\ell[k;\boldsymbol{\eta}_\ell] + \mathbf{N}[k] \\ &= \mathbf{A}[k]\ \boldsymbol{\Gamma}[k]\ \mathbf{C}[k] + \mathbf{N}[k] \qquad (9)\end{aligned}$$

where

$$\mathbf{A}[k] = [\mathbf{a}(\phi_1), \ldots, \mathbf{a}(\phi_\ell), \ldots, \mathbf{a}(\phi_L)] \in \mathbb{C}^{M\times L} \tag{10}$$

denotes the steering matrix for the k-th period of the observation interval, while

$$\boldsymbol{\Gamma}[k] = \mathrm{diag}\{\boldsymbol{\gamma}\}\ \in \mathbb{C}^{L\times L} \tag{11}$$

is a diagonal matrix whose entries are the complex amplitudes of the signal replicas $\boldsymbol{\gamma} = [\gamma_1, \ldots, \gamma_L]^{\mathrm{T}}$ during the k-th period of the observation interval. Furthermore,

$$\mathbf{C}[k] =\ [\mathbf{c}[k;\tau_1] \cdots \mathbf{c}[k;\tau_\ell] \cdots \mathbf{c}[k;\tau_L]]^{\mathrm{T}} \in\ \mathbb{R}^{L\times N} \tag{12}$$

contains the sampled and shifted $c(t)$ for each impinging wavefront

$$\begin{aligned} \mathbf{c}[k;\tau_\ell] = [&c(((k-1)N+1)T_s - \tau_\ell), \ldots, & (13)\\ &\ldots, c(((k-1)N+N)T_s - \tau_\ell)]^{\mathrm{T}}. & (14)\end{aligned}$$

In general $||\mathbf{c}[k;\tau_\ell]||_2^2 \neq N$ for all τ_ℓ, however in many cases[1] we can assume that $||\mathbf{c}[k;\tau_\ell]||_2^2 \approx N, \forall \tau_\ell \quad \forall k$ and if additionally $N \geq N_c$ and $N/N_c \in \mathbb{N}$ we get $\mathbf{c}[k;\tau_\ell] = \mathbf{c}(\tau_\ell), \quad \forall k$. In the following we assume that the array response $\mathbf{A}[k]$ is left centro-hermitian with

$$\mathbf{A}[k] = \boldsymbol{\Pi}_M \mathbf{A}^*[k] \tag{15}$$

where

$$\boldsymbol{\Pi}_M = \begin{bmatrix} & & 1 \\ & \iddots & \\ 1 & & \end{bmatrix}\ \in\ \mathbb{R}^{M\times M}. \tag{16}$$

B. Post-correlation Signal Model

A Fisher Information preserving compression applying a bank of Q correlators at the output of each antenna is used. We follow a canonical component (CC) method where the information about the signal parameters are extracted from the received signal by correlating with several delayed replicas of the signal with relative delays associated to a regular grid [9]. Thus, the signal at the output of the q-th correlator of the bank of correlators at the output of each antenna element with $q = 1, \ldots, Q$ can be written

$$\mathbf{y}_q[k] = \mathbf{X}[k](\mathbf{c}[k;\kappa_q])^* \in \mathbb{C}^{M\times 1} \tag{17}$$

where κ_q denotes the time-delay for the correlator tap q. We can define the output signal of the bank of correlators by

$$\mathbf{Y}[k] = [\mathbf{y}_1[k], \ldots, \mathbf{y}_q[k], \ldots, \mathbf{y}_Q[k]] = \mathbf{X}[k]\mathbf{Q}[k] \in \mathbb{C}^{M\times Q} \tag{18}$$

and the compression matrix

$$\mathbf{Q}[k] = [\mathbf{c}[k;\kappa_1], \ldots, \mathbf{c}[k;\kappa_q], \ldots, \mathbf{c}[k;\kappa_Q]] \in \mathbb{R}^{N\times Q}. \tag{19}$$

Thus, we can write

$$\mathbf{Y}[k] = \mathbf{A}[k]\boldsymbol{\Gamma}[k]\mathbf{C}[k]\mathbf{Q}[k] + \mathbf{N}[k]\mathbf{Q}[k]. \tag{20}$$

The so-called thin singular value decomposition (SVD) or also called economy size SVD of $\mathbf{Q}[k]$ in case $Q \ll N$ is given by $\mathbf{Q}[k] = \mathbf{U}\boldsymbol{\Sigma}\mathbf{V}^{\mathrm{H}}$, where the columns of $\mathbf{U} \in \mathbb{C}^{N\times Q}$ and $\mathbf{V} \in \mathbb{C}^{Q\times Q}$ only refer to the non-zero singular values and thus all diagonal elements of $\boldsymbol{\Sigma} \in \mathbb{C}^{Q\times Q}$ are larger than zero. We

[1] e.g. in case of GPS C/A PR sequences with bandwidth $B \geq 1.023$ MHz.

define the compression matrix $\mathbf{Q}_\omega[k]$, with $\mathbf{Q}_\omega^{\mathrm{H}}[k]\mathbf{Q}_\omega[k]=\mathbf{I}_Q$, that preserves the input noise properties at the output of the bank of correlators using the thin SVD as follows:

$$\mathbf{Q}_\omega[k]=\mathbf{Q}[k](\boldsymbol{\Sigma}\mathbf{V}^{\mathrm{H}})^{-1}=\mathbf{U}\ \in\mathbb{C}^{N\times Q} \tag{21}$$

Here, $\mathbf{I}_Q$ denotes a $Q\times Q$ identity matrix.

In the following, we assume that the time-delays τ_ℓ , the azimuth angles ϕ_ℓ and thus also the compression matrix $\mathbf{Q}_\omega[k]$ are constant with respect to K periods. This is a reasonable assumption for GNSS *e.g.* for moving users in an urban city center where the average life span of echoes is approximately 1 m [18]. Life span refers to the motion distance across which a multipath signal is observable, i.e. active. In the latter case, for an observation time of 30 ms ($K=30$ for a GPS C/A signal with $N=2046$ and $B=1.023$ MHz), a maximum velocity of 100 km/h, and a spatial resolution of $c/B\approx 293$ m, the multipath time-delays can be assumed constant, where c denotes the speed of light. The time-delay of the LOS signal τ_1 in general can be assumed constant for an even longer observation time. Also the azimuth angles of LOS and NLOS signals ϕ_ℓ can be assumed constant for such an observation time.

Thus, we can write

$$\begin{aligned}\bar{\mathbf{Y}}[k] &= \mathbf{A}\boldsymbol{\Gamma}[k]\mathbf{C}\mathbf{Q}_\omega+\mathbf{N}[k]\mathbf{Q}_\omega && (22)\\ &= \mathbf{A}\boldsymbol{\Gamma}[k]\mathbf{C}\mathbf{Q}_\omega+\mathbf{N}_\omega[k] \quad \in\mathbb{C}^{M\times Q}. && (23)\end{aligned}$$

Applying the vec-operator on matrix $\bar{\mathbf{Y}}[k]$, we get

$$\begin{aligned}\tilde{\mathbf{y}}[k] &= \mathrm{vec}\{\bar{\mathbf{Y}}[k]\}=\underbrace{\mathrm{vec}\{\mathbf{A}\boldsymbol{\Gamma}[k]\mathbf{C}\mathbf{Q}_\omega\}}_{=\tilde{\mathbf{s}}[k]}+\underbrace{\mathrm{vec}\{\mathbf{N}_\omega[k]\}}_{=\tilde{\mathbf{n}}[k]}\\ &= ((\mathbf{C}\mathbf{Q}_\omega)^{\mathrm{T}}\diamond\mathbf{A})\boldsymbol{\gamma}[k]+\tilde{\mathbf{n}}[k] \quad \in\mathbb{C}^{MQ\times 1} && (24)\end{aligned}$$

where $\diamond$ denotes the Khatri-Rao product. Collecting the data samples during K periods, we obtain

$$\tilde{\mathbf{Y}}=\underbrace{((\mathbf{C}\mathbf{Q}_\omega)^{\mathrm{T}}\diamond\mathbf{A})\tilde{\boldsymbol{\Gamma}}}_{=\tilde{\mathbf{S}}}+\tilde{\mathbf{N}} \quad \in\mathbb{C}^{MQ\times K} \tag{25}$$

with

$$\begin{aligned}\tilde{\mathbf{Y}} &= [\tilde{\mathbf{y}}[1],\dots,\tilde{\mathbf{y}}[k],\dots,\tilde{\mathbf{y}}[K]] && (26)\\ &= \begin{bmatrix}\mathbf{y}_1[1],\dots,\mathbf{y}_1[k],\dots,\mathbf{y}_1[K]\\ \vdots\\ \mathbf{y}_q[1],\dots,\mathbf{y}_q[k],\dots,\mathbf{y}_q[K]\\ \vdots\\ \mathbf{y}_Q[1],\dots,\mathbf{y}_Q[k],\dots,\mathbf{y}_Q[K]\end{bmatrix} && (27)\\ \tilde{\mathbf{S}} &= [\tilde{\mathbf{s}}[1],\dots,\tilde{\mathbf{s}}[k],\dots,\tilde{\mathbf{s}}[K]]\in\mathbb{C}^{MQ\times K} && (28)\\ \tilde{\mathbf{N}} &= [\tilde{\mathbf{n}}[1],\dots,\tilde{\mathbf{n}}[k],\dots,\tilde{\mathbf{n}}[K]]\in\mathbb{C}^{MQ\times K} && (29)\\ \tilde{\boldsymbol{\Gamma}} &= [\boldsymbol{\gamma}[1],\dots,\boldsymbol{\gamma}[k],\dots,\boldsymbol{\gamma}[K]]\in\mathbb{C}^{L\times K}. && (30)\end{aligned}$$

We define the tensor $\boldsymbol{\mathcal{S}}\in\mathbb{C}^{K\times Q\times M}$ collecting the signal data and a tensor $\boldsymbol{\mathcal{N}}\in\mathbb{C}^{K\times Q\times M}$ collecting the white noise data, respectively. The three different matrix unfoldings of the tensor $\boldsymbol{\mathcal{S}}$ can be expressed as [19]

$$\begin{aligned}[\boldsymbol{\mathcal{S}}]_{(1)} &= \tilde{\boldsymbol{\Gamma}}^{\mathrm{T}}((\mathbf{C}\mathbf{Q}_\omega)^{\mathrm{T}}\diamond\mathbf{A})^{\mathrm{T}}\in\mathbb{C}^{K\times QM} && (31)\\ [\boldsymbol{\mathcal{S}}]_{(2)} &= (\mathbf{C}\mathbf{Q}_\omega)^{\mathrm{T}}(\mathbf{A}\diamond\tilde{\boldsymbol{\Gamma}}^{\mathrm{T}})^{\mathrm{T}}\in\mathbb{C}^{Q\times MK} && (32)\\ [\boldsymbol{\mathcal{S}}]_{(3)} &= \mathbf{A}(\tilde{\boldsymbol{\Gamma}}^{\mathrm{T}}\diamond(\mathbf{C}\mathbf{Q}_\omega)^{\mathrm{T}})^{\mathrm{T}}\in\mathbb{C}^{M\times KQ}. && (33)\end{aligned}$$

Finally, we can write the tensor signal model

$$\boldsymbol{\mathcal{Y}}=\boldsymbol{\mathcal{S}}+\boldsymbol{\mathcal{N}}\ \in\mathbb{C}^{K\times Q\times M}. \tag{34}$$

It is instructive to mention that the signal tensor $\boldsymbol{\mathcal{S}}$ follows a third-order Parallel Factors (PARAFAC) decomposition [19], [20] with matrix factors $\tilde{\boldsymbol{\Gamma}}^{\mathrm{T}}$, $(\mathbf{C}\mathbf{Q}_\omega)^{\mathrm{T}}$, and $\mathbf{A}$.

III. Proposed Tensor-Based Approach for Time-Delay Estimation

In this section, we present different algorithms that use multi-linear algebra in order to estimate the time-delay of the LOS signal while mitigating the effect of the NLOS signals (multipath). In the following, we assume that the receive power of the LOS signal is larger than those of the NLOS signals.

A. Multi-dimensional Filtering using High Order Singular Value Decomposition (HOSVD)

Applying HOSVD on our signal model, we can write [19]

$$\boldsymbol{\mathcal{Y}}=\boldsymbol{\mathcal{R}}\times_1\mathbf{U}^{(1)}\times_2\mathbf{U}^{(2)}\times_3\mathbf{U}^{(3)} \tag{35}$$

with the data tensor $\boldsymbol{\mathcal{Y}}$ and the core tensor $\boldsymbol{\mathcal{R}}\in\mathbb{C}^{K\times Q\times M}$, and the unitary matrices $\mathbf{U}^{(1)}\in\mathbb{C}^{K\times K}$, $\mathbf{U}^{(2)}\in\mathbb{C}^{Q\times Q}$, and $\mathbf{U}^{(3)}\in\mathbb{C}^{M\times M}$. Here, the operator $\times_n$ denotes the so-called n-mode product of a tensor by a matrix [19]. Based on the core tensor $\boldsymbol{\mathcal{R}}$ ordering properties, we find that the n-mode singular vectors $\mathbf{u}_i^{(n)}$ are ordered in the unitary matrices $\mathbf{U}^{(n)}$ in a decreasing order of the magnitude of its corresponding singular values. Therefore, we can now define the vector $\mathbf{q}$ as

$$\mathbf{q}=\left(\left(\boldsymbol{\mathcal{Y}}\times_1\left(\mathbf{u}_1^{(1)}\right)^{\mathrm{H}}\times_3\left(\mathbf{u}_1^{(3)}\right)^{\mathrm{H}}\right)\boldsymbol{\Sigma}\mathbf{V}^{\mathrm{H}}\right)^{\mathrm{T}} \quad \in\mathbb{C}^{Q\times 1} \tag{36}$$

where $\mathbf{q}$ represents the multi-dimensionally filtered cross-correlation values at each tap of the correlator bank.

Based on $\mathbf{q}$ and a cubic spline interpolation using the absolute value of its entries, we can derive the cost function $F(\tau)$ and then estimate the time-delay of the LOS signal by solving the problem

$$\hat{\tau}_1=\arg\max_{\tau}\{F(\tau)\}. \tag{37}$$

B. Multi-dimensional Filtering using HOSVD with Forward Backward Averaging (FBA)

In general, LOS and NLOS signals are highly correlated in case of GNSS. If a left centro-hermitian sensor array is assumed, the separation of LOS and NLOS signals by an adaptive multi-dimensional filtering as given in (36) can be improved using FBA [14].

The *extended* 3-mode unfolding of $\boldsymbol{\mathcal{Y}}$ can be given as

$$\mathbf{Z}=\begin{bmatrix}[\boldsymbol{\mathcal{Y}}]_{(3)} & \boldsymbol{\Pi}_M[\boldsymbol{\mathcal{Y}}]_{(3)}^{*}\boldsymbol{\Pi}_{KQ}\end{bmatrix}\in\mathbb{C}^{M\times 2KQ}. \tag{38}$$

We define the new tensor $\boldsymbol{\mathcal{Y}}_{\mathrm{FBA}} \in \mathbb{C}^{2K \times Q \times M}$ such that

$$[\boldsymbol{\mathcal{Y}}_{\mathrm{FBA}}]_{(3)} = \mathbf{Z} \tag{39}$$

By applying HOSVD on $\boldsymbol{\mathcal{Y}}_{\mathrm{FBA}}$,

$$\boldsymbol{\mathcal{Y}}_{\mathrm{FBA}} = \boldsymbol{\mathcal{R}}_{\mathrm{FBA}} \times_1 \mathbf{U}_{\mathrm{FBA}}^{(1)} \times_2 \mathbf{U}_{\mathrm{FBA}}^{(2)} \times_3 \mathbf{U}_{\mathrm{FBA}}^{(3)} \tag{40}$$

we can select the first columns of $\mathbf{U}_{\mathrm{FBA}}^{(3)}$ and $\mathbf{U}_{\mathrm{FBA}}^{(1)}$, $\mathbf{u}_{1,\mathrm{FBA}}^{(3)} \in \mathbb{C}^{M \times 1}$ and $\mathbf{u}_{1,\mathrm{FBA}}^{(1)} \in \mathbb{C}^{2K \times 1}$ respectively. Consequently, we can use $\mathbf{u}_{1,\mathrm{FBA}}^{(3)}$ instead of $\mathbf{u}_1^{(3)}$ and $\mathbf{u}_{1,\mathrm{FBA}}^{(1)}$ instead of $\mathbf{u}_1^{(1)}$ in (36) in order to derive an improved space-time filtered vector of cross-correlations denoted as $\mathbf{q}_{\mathrm{FBA}}$.

C. Multi-dimensional Filtering using HOSVD with Spatial Smoothing (SPS)

SPS [15] is another pre-processing scheme that can be used to *de-correlate* the impinging wavefronts in case of a left centro-hermitian or Vandermonde sensor array. To this end, a uniform linear array (ULA) with M sensors can be divided into L_s subarrays, each containing $M_s = M - L_s + 1$ sensor elements. The selection matrix corresponding to the ℓ_s-th subarray with $\ell_s = 1, \dots, L_s$ can be defined as

$$\mathbf{J}_{\ell_s}^{(M)} = \begin{bmatrix} \mathbf{0}_{M_s \times \ell_s - 1} & \mathbf{I}_{M_s} & \mathbf{0}_{M_s \times L_s - \ell_s} \end{bmatrix} \in \mathbb{R}^{M_s \times M}. \tag{41}$$

Therefore, the spatially smoothed *extended* 3-mode unfolding of $\boldsymbol{\mathcal{Y}}$ is given by

$$\mathbf{W} = \begin{bmatrix} \mathbf{J}_1^{(M)} [\boldsymbol{\mathcal{Y}}]_{(3)} \cdots \mathbf{J}_{\ell_s}^{(M)} [\boldsymbol{\mathcal{Y}}]_{(3)} \cdots \mathbf{J}_{L_s}^{(M)} [\boldsymbol{\mathcal{Y}}]_{(3)} \end{bmatrix} \in \mathbb{C}^{M_s \times KQL_s}. \tag{42}$$

We define the new tensor $\boldsymbol{\mathcal{Y}}_{\mathrm{SPS}} \in \mathbb{C}^{L_s K \times Q \times M_s}$ such that

$$[\boldsymbol{\mathcal{Y}}_{\mathrm{SPS}}]_{(3)} = \mathbf{W}. \tag{43}$$

By applying HOSVD on $\boldsymbol{\mathcal{Y}}_{\mathrm{SPS}}$,

$$\boldsymbol{\mathcal{Y}}_{\mathrm{SPS}} = \boldsymbol{\mathcal{R}}_{\mathrm{SPS}} \times_1 \mathbf{U}_{\mathrm{SPS}}^{(1)} \times_2 \mathbf{U}_{\mathrm{SPS}}^{(2)} \times_3 \mathbf{U}_{\mathrm{SPS}}^{(3)} \tag{44}$$

we can get $\mathbf{u}_{1,\mathrm{SPS}}^{(3)} \in \mathbb{C}^{M_s \times 1}$ and $\mathbf{u}_{1,\mathrm{SPS}}^{(1)} \in \mathbb{C}^{L_s K \times 1}$.
Consequently, we can calculate the new correlation vector $\mathbf{q}_{\mathrm{SPS}}$ using

$$\mathbf{q}_{\mathrm{SPS}} = \left(\left(\boldsymbol{\mathcal{Y}}_{\mathrm{SPS}} \times_1 \left(\mathbf{u}_{1,\mathrm{SPS}}^{(1)} \right)^{\mathrm{H}} \times_3 \left(\mathbf{u}_{1,\mathrm{SPS}}^{(3)} \right)^{\mathrm{H}} \right) \boldsymbol{\Sigma} \mathbf{V}^{\mathrm{H}} \right)^{\mathrm{T}}. \tag{45}$$

After cubic spline interpolation based on the absolute value of the entries of $\mathbf{q}_{\mathrm{SPS}}$, the estimation of the time-delay of the LOS signal is obtained by solving problem (37).

D. Multi-dimensional Filtering using HOSVD with FBA and SPS

For this algorithm, the two pre-processing algorithms discussed earlier, FBA and SPS, are applied. Therefore, the forward-backward averaged and spatially smoothed *extended* 3-mode unfolding of $\boldsymbol{\mathcal{Y}}$ is given by

$$\mathbf{E} = \begin{bmatrix} \mathbf{J}_1 \mathbf{Z} \cdots \mathbf{J}_l \mathbf{Z} \cdots \mathbf{J}_L \mathbf{Z} \end{bmatrix} \in \mathbb{C}^{M_s \times 2L_s KQ} \tag{46}$$

where $\mathbf{Z}$ is defined in (38).

We define the new tensor $\boldsymbol{\mathcal{Y}}_{\mathrm{FBA+SPS}} \in \mathbb{C}^{2L_s K \times Q \times M_s}$. The estimate for τ_1 is derived following the same procedure following (44) and (45).

E. Multi-dimensional Filtering using HOSVD with Expanded Spatial Smoothing (SPS-EXP)

The idea of expanded spatial smoothing (SPS-EXP) recently proposed in [16] is to use a fourth dimension for the subarrays instead of accumulating the spatially smoothed *extended* data in the time dimension.

We define the 4-th order tensor $\boldsymbol{\mathcal{Y}}_{\mathrm{SPS-EXP}} \in \mathbb{C}^{K \times Q \times M_s \times L_s}$ using

$$[\boldsymbol{\mathcal{Y}}_{\mathrm{SPS-EXP}}]_{(3)} = \begin{bmatrix} \mathbf{J}_1^{(M)} [\boldsymbol{\mathcal{Y}}]_{(3)} \cdots \mathbf{J}_{\ell_s}^{(M)} [\boldsymbol{\mathcal{Y}}]_{(3)} \cdots \mathbf{J}_{L_s}^{(M)} [\boldsymbol{\mathcal{Y}}]_{(3)} \end{bmatrix}. \tag{47}$$

By applying HOSVD on $\boldsymbol{\mathcal{Y}}_{\mathrm{SPS-EXP}}$, we can get the singular vectors corresponding to the strongest singular values of the SVDs of the unfoldings in K, M_s and L_s dimensions. Thus, we can calculate an extended spatially smoothed multi-dimensionally filtered cross-correlation vector as

$$\begin{aligned} &\mathbf{q}_{\mathrm{SPS-EXP}} = \\ &\left(\left(\boldsymbol{\mathcal{Y}}_{\mathrm{SPS-EXP}} \times_1 \left(\mathbf{u}_{1,\mathrm{SPS-EXP}}^{(1)} \right)^{\mathrm{H}} \times_3 \right.\right. \\ &\left.\left. \left(\mathbf{u}_{1,\mathrm{SPS-EXP}}^{(3)} \right)^{\mathrm{H}} \times_4 \left(\mathbf{u}_{1,\mathrm{SPS-EXP}}^{(4)} \right)^{\mathrm{H}} \right) \boldsymbol{\Sigma} \mathbf{V}^{\mathrm{H}} \right)^{\mathrm{T}}. \end{aligned} \tag{48}$$

After spline interpolation based on the absolute value of the entries of $\mathbf{q}_{\mathrm{SPS-EXP}}$ estimation of the time-delay of the LOS signal can be performed as given in (37).

F. Two dimensional Filtering using Simple Matrix Approach (MA) with FBA and SPS

In order to estimate the time-delay of the LOS signal without using the tensor approach we can refer to a simple matrix approach described below.

Having our signal model in (34), we can smooth the collected data in (22) and we can write

$$\mathbf{Y}_{\mathrm{MA}} = \frac{1}{K} \sum_{k=1}^{K} \bar{\mathbf{Y}}[k]. \tag{49}$$

$\mathbf{Y}_{\mathrm{MA}}$ represents the average of the data taken over the snapshot dimension K. In a next step one can extend the data using the two pre-processing algorithms FBA and SPS to obtain the forward-backward averaged and spatially smoothed data. Afterwards, we apply the thin SVD on $\mathbf{Y}_{\mathrm{MA+FBA+SPS}} \in \mathbb{C}^{M_s \times 2QL_s}$ to get the principal singular vector, $\mathbf{u}_{\mathrm{MA+FBA+SPS}} \in \mathbb{C}^{Ms \times 1}$, corresponding to the strongest singular value. The principal singular vector is used to filter the data in the spatial domain to finally obtain the vector $\mathbf{q}_{\mathrm{MA+FBA+SPS}} \in \mathbb{C}^{Q \times 1}$ following

$$\mathbf{q}_{\mathrm{MA+FBA+SPS}} = \left(\mathbf{u}_{\mathrm{MA+FBA+SPS}}^{\mathrm{H}} \left(\mathbf{J}_1 \bar{\mathbf{Y}}_{\mathrm{MA}} \right) \left(\boldsymbol{\Sigma} \mathbf{V}^{\mathrm{H}} \right) \right)^{\mathrm{T}}. \tag{50}$$

After cubic spline interpolation based on the absolute value of the entries of $\mathbf{q}_{\mathrm{MA+FBA+SPS}}$, the estimation of the time-delay of the LOS signal is obtained by solving problem (37).

IV. Complexity Evaluation

The computational complexity (in terms of FLoating point OPeration (flop) counts) of the multiplication of two complex matrices, $\mathbf{A} \in \mathbb{C}^{M \times N}$ and $\mathbf{B} \in \mathbb{C}^{N \times L}$, is assumed to be $\mathcal{O}(MNL)$ [21].

Since we need only the principal singular vector corresponding to the strongest singular value, we can apply simplified methods instead of a full SVD. According to [22][23], the numerical complexity obtaining the d strongest singular vectors (in our case d=1) from applying thin SVD on $\mathbf{A} \in \mathbb{C}^{M \times N}$, is given by $\mathcal{O}(MN)$.

We will assume that the unfolding (going from tensor representation to matrix representation) and the inverse-unfolding (building a tensor from its matrix unfolding) are not considered in the computational complexity since both functions are only about data representation rather than operating on the data. Therefore, in this assessment we will ignore any complexity related to unfolding and inverse-unfolding. In addition, this assessment will focus only on the mathematical steps that are different from one algorithm to another. So it is more about the relative complexity difference between the different algorithms rather than the absolute complexity of each algorithm. Thus, we will ignore the de-whitening stage where we need to multiply the solution vector by $\mathbf{\Sigma V}^{\mathrm{H}}$. Following the above reasoning the complexity for each algorithm is listed in Table I.

Table I
Numerical Complexity of the algorithms

Algorithm	Complexity
MA+FBA+SPS	$\mathcal{O}\Big((M)KQ\Big)$
HOSVD	$\mathcal{O}\Big(3(M)KQ\Big)$
HOSVD+FBA	$\mathcal{O}\Big(3(2M)KQ\Big)$
HOSVD+SPS	$\mathcal{O}\Big(3(M_sL_s)KQ\Big)$
HOSVD+SPS-EXP	$\mathcal{O}\Big(4(M_sL_s)KQ\Big)$
HOSVD+FBA+SPS	$\mathcal{O}\Big(3(2M_sL_s)KQ\Big)$

We can observe that the numerical complexity of the MA+FBA+SPS with respect to HOSVD is $\mathcal{O}(\frac{1}{3})$. HOSVD+FBA has double the complexity than the simple HOSVD. Based on our model, when dividing the ULA into different subarrays, some specific sensor elements from the original array will be used many times in the different subarrays, therefore the product M_sL_s representing the total number of sensor elements used in all subarrays is bigger than twice the number of ULA sensor elements M. This makes HOSVD+SPS computationally slightly more expensive than the HOSVD+FBA with a ratio of $\mathcal{O}(\frac{M_sL_s}{2M} \approx 1.25)$. The computational complexity of HOSVD+SPS-EXP is even higher than the complexity of HOSVD+SPS. The computational complexity of HOSVD+FBA+SPS is twice the computational complexity of HOSVD+SPS. The complexity ratio between HOSVD with and without SPS is given by $\mathcal{O}(M_sL_s)$.

V. Simulations

We assume a left centro-hermitian ULA with $M = 8$ isotropic sensor elements with half-wavelength spacing ($\Delta = \lambda/2$). The received signal is a GPS C/A signal with bandwidth $B = 1.023$ MHz and carrier frequency $f_c = 1575.42$ MHz. We consider a two-path scenario with a LOS and one NLOS signal ($L = 2$). For the SPS and SPS-EXP, we assume that the ULA is divided into $L_s = 5$ subarrays, each containing $M_s = 4$ sensor elements. The number of samples taken within one observation period k is $N = 2046$. The number of observation periods $K = 30$ and we assume that all the channel parameters are constant over K. The azimuth angle difference between LOS and NLOS signal is $\Delta\phi = 60°$. The signal phase for LOS and NLOS signals, denoted by $\arg\{\gamma_1\}$ and $\arg\{\gamma_2\}$, are assumed independent and identically distributed (i.i.d.) for each Monte Carlo simulation and drawn from a uniform distribution $[0, 2\pi[$. We performed 2000 Monte Carlo simulations to derive the root mean square error of the time-delay of the LOS signal $\mathrm{RMSE}(\tau_1)$. The number of correlators in the bank is $Q = 11$. The carrier to noise density ratio is $C/N_0 = 48$ dB-Hz. Thus, the pre-correlation SNR approximately is -15 dB, and the post-correlation SNR approximately is 15 dB. The signal to multipath ratio SMR $= 5$ dB. The square root of the expectation of the Cramer-Rao Lower Bound (CRLB) of the time-delay of the LOS signal with respect to the random signal phases $\arg\{\gamma_1\}$ and $\arg\{\gamma_2\}$ denoted by $\sqrt{\mathrm{E}[\mathrm{CRLB}(\tau_1)]}$ is derived to be used as a lower bound for comparison of the performance of the proposed multi-dimensional filters and subsequent time-delay estimation. The time-delay difference between LOS and NLOS signal is normalized by T_c and is denoted by $\Delta\tau/T_c$. The $\mathrm{RMSE}(\tau_1)$ for the different methods presented above, HOSVD, HOSVD+FBA, HOSVD+SPS, HOSVD+FBA+SPS, and HOSVD+SPS-EXP as well as MA, the simple matrix approach with FBA and SPS denoted by MA+FBA+SPS and $\sqrt{\mathrm{E}[\mathrm{CRLB}(\tau_1)]}$ are presented in Figure 1. For all tensor approaches, the error is maximum when $\Delta\tau/T_c \approx [0.2 - 0.5]$. In fact, this is due to the superposition of the the LOS and NLOS signal and to the properties of their respective autocorrelation functions. The maximal $\sqrt{\mathrm{E}[\mathrm{CRLB}(\tau_1)]}$, approximately reaching 18 m in ranging error, is obtained when using HOSVD algorithm without any pre-processing algorithm. When $\Delta\tau/T_c > 1$, the multipath is one chip duration (around 300 m) later than the LOS, and thus its effect is considered to be small. The introduction of the FBA decreases the error from 18 m to around 12 m. The HOSVD+SPS, HOSVD+SPS-EXP, and HOSVD+FBA+SPS all have similar performance and show substantially better time-delay estimation of the LOS signal, with a maximum error of around 1.8 m.

When the LOS and the multipath signal are strongly correlated, i.e. $\Delta\tau/T_c \approx [0 - 0.2]$, the decomposition of the ULA into smaller subarrays (in all algorithms using SPS extension) helps to decrease the error to a maximum of 1.8 m by spatial de-correlation of the signals. However, when the LOS and the

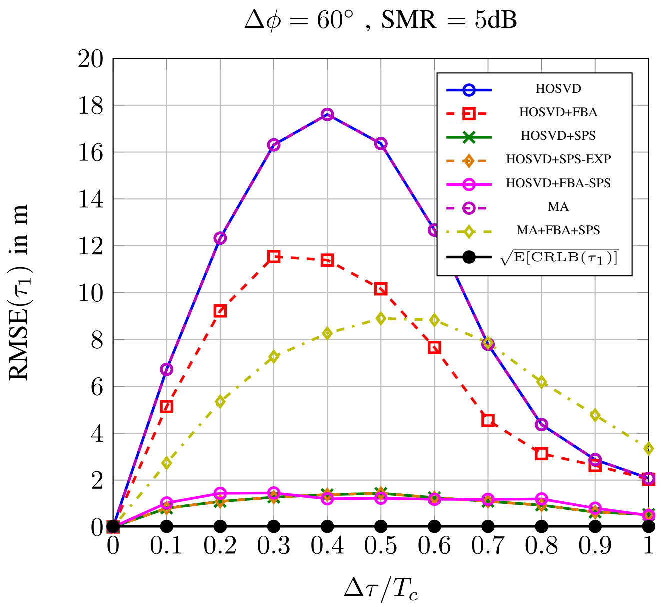

Figure 1. RMSE of the LOS time delay estimation obtained using different algorithms in a constant channel environment

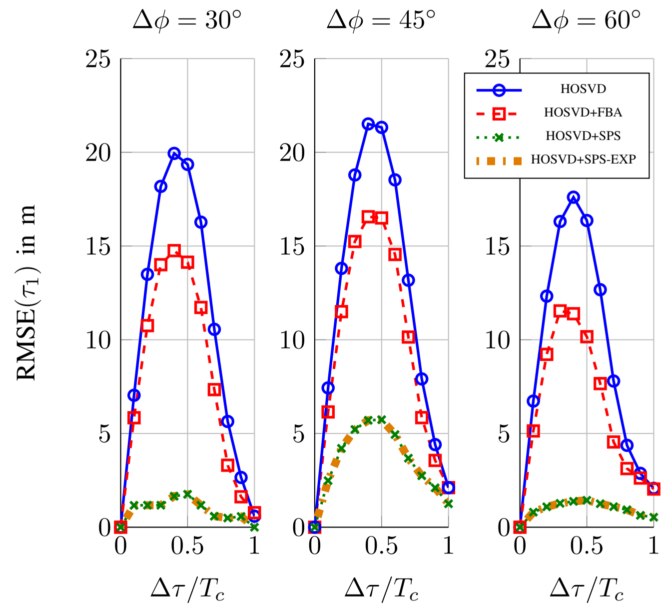

Figure 2. RMSE of the LOS time delay estimation obtained using different algorithms in a constant channel environment with SMR = 5 dB and different $\Delta\phi$

multipath signal are weakly correlated i.e. $\Delta\tau/T_c \approx [0.8-1]$, SPS performance is worse than HOSVD or HOSVD+FBA.

The similar performance behavior that is shown by all algorithms using SPS is due to the fact that SPS de-correlates the impinging wavefronts in the spatial domain, and also extends the time dimension K using a decomposition into subarrays. For the HOSVD+SPS and HOSVD+SPS-EXP, the subarrays dimension L_s is appended to the snapshot dimension K and to another fourth dimension respectively. Since in both algorithms we apply the smoothing over the different dimensions containing all the data, the performance between these two algorithms is very similar. HOSVD+FBA+SPS is not providing much benefit compared to HOSVD+SPS due to the fact that we are using only one multipath component besides the LOS signal. All the subarrays are used to de-correlate one signal from the LOS signal. The introduction of more than one multipath may show different behavior for this algorithm, where HOSVD+FBA may introduce more gain in the estimation.

The $\mathrm{RMSE}(\tau_1)$ obtained by the simple MA is converging to the same results obtained from the 3D approach using only the simple HOSVD. MA+FBA+SPS takes benefit of SPS to de-correlate the LOS and the multipath components when they are highly correlated by using the different sensors in the introduced subarrays. However, it hen loses the benefit of having $M = 8$ sensors when it uses only $M_s = 4$ sensors in each subarray. The multi-dimensionally filtered cross-correlation values in $\mathbf{q}_{\mathrm{MA+FBA+SPS}}$ are obtained from the operation on half of the data collected, $\left(\mathbf{J}_1\bar{\mathbf{Y}}_{\mathrm{MA}}\right)$, and not all the data $\bar{\mathbf{Y}}_{\mathrm{MA}}$.

The difference in azimuth angle between LOS and multipath signal has an effect on the LOS time-delay estimation. Figure 2 shows results for four different algorithms with different $\Delta\phi$. The worst maximal error values for each algorithm are obtained when $\Delta\phi = 45°$. The smallest error for SPS is obtained when $\Delta\phi = 30°$ (the case in which the array steering vectors for the LOS and multipath signal are spatially orthogonal). For the HOSVD, HOSVD+FBA and HOSVD+SPS-EXP, the lowest maximal error is obtained when $\Delta\phi = 60°$.

VI. Conclusion

In this paper, we derived a tensor-based filtering approach using an antenna array and a compression method based on CC with a bank of signal matched correlators in order to mitigate multipath and to estimate the time-delay of the LOS signal of a GNSS satellite. First, we resort to multi-dimensional filtering based on the principal singular vectors of the multi-dimensional data. In order to separate highly correlated signal components in the multi-dimensional signal subspace, methods like FBA, SPS, and the recently developed SPS-EXP are applied. Afterwards, time-delay estimation of the LOS signal is performed with a simple interpolation based on the multi-dimensional filtered cross-correlation values of the bank of correlators. In addition, the performance of the algorithms was assessed. A simulation based comparison between the tensor approach and a simple proposed 2-D matrix approach was included in order to show the benefits of using a three-dimensional approach. Finally, the computational complexity of each proposed algorithm was studied in details.

One advantage of the presented tensor-based approach is that no multi-dimensional nonlinear problems need to be solved. In addition, no model order estimation is required. Based on our work, the multi-linear model is a very good tool to perform parameter estimation without the need to have knowledge about all other parameters in a given model. The proposed approach is flexible in terms of its extensibility to add more dimensions which can be of interest (for example different GNSS frequency bands). In terms of performance, the presented approaches based on the HOSVD showed very good results in terms of accuracy with the cost of higher complexity with respect to other simpler lower dimensional approaches assessed in this work.

VII. Acknowledgement

The research leading to these results has received funding from Coordenação de Aperfeiçoamento de Pessoal de Nível Superior (CAPES) under the PVE grant number 88881.030392/2013-01.

References

[1] M. S. Braasch and A. J. van Dierendonck, "GPS Receiver Architecture and Measurements," *Proceedings of the IEEE*, vol. 87, January 1999.

[2] R. D. J. van Nee, "Spread-Spectrum Code and Carrier Synchronization Errors Caused by Multipath and Interference," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 29, October 1993.

[3] B. W. Parkinson and J. J. Spilker, eds., *Global Positioning System: Theory and Applications*, vol. 1. Progress in Astronautics and Aeronautics, 1996.

[4] R. D. J. van Nee and P. C. Fenton, "The Multipath Estimating Delay Lock Loop Approaching Theoretical Accuracy Limits ," in *Proceedings of IEEE Position Location and Navigation Symposium*, (Las Vegas, NV, USA), April 1994.

[5] P. Closas, C. Fernandes-Prades, and J. A. Fernandes-Rubio, "A Particle Filtering Tracking Algorithm for GNSS Synchronization Using Laplacet's Method," in *Proceedings of the 33rd IEEE International Conference on Acoustics, Speech and Signal Processing ICASSP*, (Las Vegas, U.S.A.), April 2008.

[6] B. Krach, M. Lentmaier, and P. Robertson, "Joint Baysian Positioning and multipath Mitigation in GNSS," in *Proceedings of the 33rd IEEE International Conference on Acoustics, Speech and Signal Processing ICASSP*, (Las Vegas, U.S.A.), April 2008.

[7] F. Antreich, J. A. Nossek, G. Seco-Granados, and L. A. Swindlehurst, "The Extended Invariance Principle for Signal Parameter Estimation in an Unknown Spatial Field," *IEEE Transactions on Signal Processing*, vol. 59, July 2011.

[8] G. Seco-Granados, J. A. Fernández-Rubio, and C. Fernández-Prades, "ML Estimator and Hybrid Beamformer for Multipath and Interference Mitigation in GNSS Receivers," *IEEE Transactions on Signal Processing*, vol. 53, March 2005.

[9] J. Selva-Vera, *Efficient Multipath Mitigation in Navigation Systems*. PhD thesis, Department of Signal Signal Theory and Communications, Universitat Politècnica de Catalunya, Spain, 2004.

[10] A.-J. Van der Veen, M. C. Vanderveen, and A. J. Paulraj, "Joint angle and delay estimation using shift-invariance properties," *Signal Processing Letters, IEEE*, vol. 4, no. 5, pp. 142–145, 1997.

[11] N. Sidiropoulos, G. Giannakis, and R. Bro, "Blind PARAFAC Receivers for DS-CDMA Systems," *Signal Processing, IEEE Transactions on*, vol. 48, no. 3, pp. 810–823, 2000.

[12] D. Nion and N. D. Sidiropoulos, "Adaptive Algorithms to Track the PARAFAC Decomposition of a Third-order Tensor," *Signal Processing, IEEE Transactions on*, vol. 57, no. 6, pp. 2299–2310, 2009.

[13] D. Nion and N. D. Sidiropoulos, "Tensor Algebra and Multidimensional Harmonic Retrieval in Signal Processing for MIMO Radar," *Signal Processing, IEEE Transactions on*, vol. 58, no. 11, pp. 5693–5705, 2010.

[14] S. Pillai and B. H. Kwon, "Forward/backward Spatial Smoothing Techniques for Coherent Signal Identification ," *IEEE Transactions on Acoustics, Speech and Signal Processing*, vol. 37, pp. 8–9, January 1989.

[15] T. J. Shan, M. Wax, and T. Kailath, "Spatial Smoothing Approach for Location Estimation of Coherent Sources ," *IEEE Transactions on Acoustics, Speech and Signal Processing*, vol. 33, pp. 806–811, August 1985.

[16] P. R. B. Gomes, A. L. F. de Almeida, and J. P. C. L. da Costa, "Tensor-based methods for blind spatial signature estimation under arbitrary and unknown source covariance structure," *submitted*, 2015.

[17] M. A. M. Marinho, F. Antreich, J. P. C. L. da Costa, and J. A. Nossek, "A Signal Adaptive Array Interpolation Approach with Reduced Transformation Bias for DOA estimation of Highly Correlated Signals," in *Proceedings of the IEEE International Conference on Acoustics, Speech, and Signal Processing (ICASSP) 2014*, (Florence, Italy), May 2014.

[18] A. Lehner, *Multipath Channel Modelling for Satellite Navigation Systems*. PhD thesis, Universität Erlangen-Nürnberg, 2007.

[19] L. de Lathauwer, B. de Moor, and J. Vandewalle, "A multilinear singular value decomposition," *SIAM J. Matrix Anal. Appl.*, vol. 21, no. 4, 2000.

[20] R. A. Harshman, "Foundations of the parafac procedure: Models and conditions for an "explanatory" multi-modal factor analysis," *UCLA Working Papers in Phonetics*, vol. 16, 1970.

[21] A. J. Stothers, "On the complexity of matrix multiplication," 2010.

[22] N. Halko, P.-G. Martinsson, and J. A. Tropp, "Finding structure with randomness: Probabilistic algorithms for constructing approximate matrix decompositions," *SIAM review*, vol. 53, no. 2, pp. 217–288, 2011.

[23] M. Brand, "Incremental singular value decomposition of uncertain data with missing values," in *Computer Vision?ECCV 2002*, pp. 707–720, Springer, 2002.