Stable Matching with Externalities for Beamforming and User Assignment in Multi-cell MISO Systems

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***Abstract*—We consider the problem of distributed joint user association and beamforming in multi-cell multiple-input single-output systems. Assuming perfect local channel state information, each base station applies a distributed beamforming scheme called WSLNR-MAX [1] which depends on the user association in the network. We determine the user association by a proposed stable matching with externalities algorithm which also takes the beamforming vectors at the base stations into account. The merit in the stable matching model is the distributed implementation aspects. Each user asks to be matched with a base station according to his preferences, and each base station decides independently which users to accept. Simulation results reveal efficient distributed operation of the system compared to matching without externalities.**

I. Introduction

Efficient assignment of users to base stations in a multi-cell network is decisive for achieving spectral efficiency. Assuming perfect channel state information (CSI) at the base stations which are equipped with multiple antennas, the user assignment problem for maximizing the systems' sum rate is coupled with the beamforming design at the base stations [2], [3]. In [2], this problem is addressed in multi-cell multiple-input multiple-output (MIMO) networks and an alternating optimization algorithm is proposed which reaches a local optimum of the original nonconvex problem. In [3], a two-stage user association and beamforming algorithm has been proposed motivated by the fact that user association takes place at a larger time scale than the update of the beamforming vectors.

In this work, we are interested in distributed algorithms for the joint user assignment and beamforming in multi-cell MISO systems. We use a beamforming scheme at the base stations according to weighted signal-to-leakage-and-noise ratio maximizing beamforming (WSLNR-MAX) [1, Section 4.2.2] which is defined for a given user association to the base stations. This distributed beamforming scheme, which is shown to give high sum rate efficiency, can be applied at the base stations requiring local CSI only, i.e., each base stations needs only know the downlink channels from itself to the users. We use the beamforming scheme within a stable matching framework which we propose in this paper in order to determine the joint user assignment and beamforming in a distributed way. Specifically, the users propose (i.e., ask to be matched to) to the base stations in an order according to the channel norms. The base stations decide on which users to accept based on the achieved power gains with the WSLNR-MAX beamforming scheme.

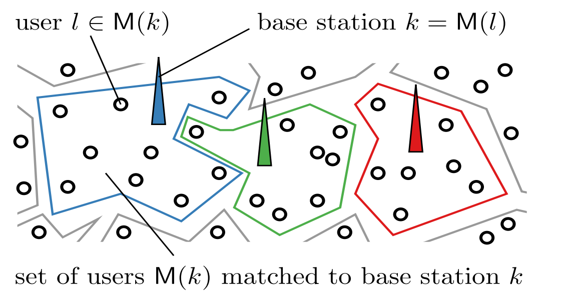

Fig. 1. System Model

Since a base station's user choice depends on the users matched to the other base stations, the proposed framework relates to matching with externalities [4], [5]. Applications of stable matching with externalities for user association in single-antenna interference networks can be found in [6], [7]. In a multi-cell MIMO setting, a stable matching with externalities algorithm has been developed in [8] incorporating different efficient precoding schemes.

Notations: Column vectors and matrices are given in lowercase and uppercase boldface letters, respectively. $\|\mathbf{a}\|$ is the Euclidean norm of $\mathbf{a} \in \mathbb{C}^N$. $|b|$ and $|\mathcal{S}|$ denote the absolute value of $b \in \mathbb{C}$, and the cardinality of a set $\mathcal{S}$, respectively. $(\cdot)^\dagger$ denotes the Hermitian transpose. The power set of $\mathcal{A}$ is $2^{\mathcal{A}}$.

II. System Model and Problem Formulation

We consider a set of base stations $\mathcal{K} = \{1, \ldots, K\}$ and a set of users $\mathcal{U} = \{1, \ldots, U\}$. Each base station k uses N_k antennas. The channel vector from base station k to user l is $\mathbf{h}_{k,l} \in \mathbb{C}^{N_k}$. We assume that each base station has local channel state information (CSI). That is, each base station knows the channel vectors from itself to the users.

We assume that each user can be assigned to at most one base station and each base station can serve multiple users. The user association will be determined by a *matching* defined as follows [9]:

Definition 1: A matching M is a mapping from $\mathcal{U} \cup \mathcal{K}$ to $2^{\mathcal{U} \cup \mathcal{K}}$ which satisfies

i. $\mathsf{M}(k) \in 2^{\mathcal{U}}$ and $|\mathsf{M}(k)| \leq U$ if $k \in \mathcal{K}$,
ii. $\mathsf{M}(l) \in 2^{\mathcal{K}}$ and $|\mathsf{M}(l)| \leq 1$ if $l \in \mathcal{U}$,
iii. $l \in \mathsf{M}(k)$ if and only if $k = \mathsf{M}(l)$.

$$r_l(\mathsf{M},\{\mathbf{w}\}) = \log_2\left(1+\frac{\left\|\mathbf{h}_{k,l}^{\dagger}\mathbf{w}_{k,l}\right\|^2}{\underbrace{\sum_{j\in\mathsf{M}(k)\setminus\{l\}}\left\|\mathbf{h}_{k,l}^{\dagger}\mathbf{w}_{k,j}\right\|^2}_{\text{intra-cell inteference}}+\underbrace{\sum_{n\in\mathcal{K}\setminus\{k\}}\sum_{m\in\mathsf{M}(n)}\left\|\mathbf{h}_{n,l}^{\dagger}\mathbf{w}_{n,m}\right\|^2}_{\text{inter-cell interference}}+\sigma^2}\right) \text{ for } k = \mathsf{M}(l) \quad (1)$$

For $k \in \mathcal{K}$, $\mathsf{M}(k)$ is the set of users assigned to base station k. Similarly, $\mathsf{M}(l)$ is the base station assigned to user l (see Fig. 1). In Definition 1, (i) restricts that each BS can serve a set of users from the set $\mathcal{U}$, (ii) restricts that each user is served by at most one BS, and (iii) ensures symmetry in the matching. Note that if $\mathsf{M}(k)=\emptyset$, then base station k is unmatched and thus switches its transmission off (the same applies for an unmatched user).

Given a matching M, the signal received at a user l is

$$y_l = \sum_{k\in\mathcal{K}}\sum_{j\in\mathsf{M}(k)}\mathbf{h}_{k,l}^{\dagger}\mathbf{w}_{k,j}x_j + z_l \quad (2)$$

where $\mathbf{w}_{k,j}\in\mathbb{C}^{N_k}$ is the transmit beamforming vector associated with user j at base station k, $x_j \sim \mathcal{CN}(0,1)$ is the signal intended for user j, and $z_l \sim (0,\sigma^2)$ is additive white Gaussian noise at receiver l. The achievable rate of user l, matched to base station $k=\mathsf{M}(l)$, is given in (1).

We are interested in the problem of maximizing the weighted sum rate in the network through joint beamforming design and user association:

$$\underset{\mathsf{M}\in\mathcal{M},\{\mathbf{w}\}}{\text{maximize}} \quad \sum_{l\in\mathcal{U}}\omega_l r_l(\mathsf{M},\{\mathbf{w}\}) \quad (3a)$$

$$s.t. \quad \sum_{j\in\mathsf{M}(k)}\|\mathbf{w}_{k,j}\|^2 \le P_k, \text{ for all } k\in\mathcal{K}, \quad (3b)$$

$$|\mathsf{M}(k)| \le q_k, \text{ for all } k\in\mathcal{K}. \quad (3c)$$

In Problem (3), $\mathcal{M}$ is the set of all feasible matchings, and we assume a total power constraint P_k at a base station k. The constraint in (3c) restricts the maximum number of users that can be assigned to a base station k to $q_k\in\mathbb{N}$, which can be used for balancing the load between the base stations. Problem (3) is NP-hard [2] even for a fixed matching M [10].

Our approach in this paper emphasizes on a distributed implementation for joint beamforming and user association. The matching M will be determined by a stable matching algorithm (proposed in the next section) and will depend on the beamforming scheme at the base stations. For a given matching M, we fix the beamforming scheme to WSLNR-MAX beamforming [1, Section 4.2.2] at a base station k serving user l as:

$$\mathbf{v}_{k,l}^{\text{WSLNR}}(\mathsf{M}) = \frac{\left(\sum\limits_{j\in\mathcal{A}\setminus\{l\}}\frac{\lambda_j}{\sigma^2}\mathbf{h}_{k,j}\mathbf{h}_{k,j}^{\dagger}+\frac{\mu_k}{P_k}\mathbf{I}\right)^{-1}\mathbf{h}_{k,l}}{\left\|\left(\sum\limits_{j\in\mathcal{A}\setminus\{l\}}\frac{\lambda_j}{\sigma^2}\mathbf{h}_{k,j}\mathbf{h}_{k,j}^{\dagger}+\frac{\mu_k}{P_k}\mathbf{I}\right)^{-1}\mathbf{h}_{k,l}\right\|} \quad (4)$$

where

- the set $\mathcal{A} := \bigcup_{n\in\mathcal{K}}\mathsf{M}(n)$ contains all users assigned to the base stations under matching M.
- the parameters $\mu_k := |\mathsf{M}(k)|/\sum_{n\in\mathcal{K}}|\mathsf{M}(n)|$ are selected heuristically as in [1, Equation 4.37].
- the parameters $\lambda_j := \omega_j/\sum_{m\in\mathcal{A}}\omega_m$ are selected as in [1, Equation 4.36] which is a heuristic choice suitable for the weighted sum rate optimization.

Including power control, the beamforming vector at base station k for user l is written as

$$\mathbf{w}_{k,l}^{\text{WSLNR}}(\mathsf{M}) = \sqrt{p_{k,l}(\mathsf{M}(k))}\,\mathbf{v}_{k,l}^{\text{WSLNR}}(\mathsf{M}), \quad (5)$$

where the power allocation $p_{k,j}(\mathsf{M}(k))$ at base station k depends only on which users are in its cell $\mathsf{M}(k)$ and is determined using [1, Theorem 3.16].

III. Stable Matching

In a stable matching problem, there exists two sets of agents. Each agent in one set wants to be matched with one or more agents in the other set. In our case, the two sets correspond to the set of users $\mathcal{U}$ and the set of base stations $\mathcal{K}$. A matching between the two sets is defined in Definition 1.

Let a set of users $\mathcal{L}$ want to be matched with a base station k. The *choice function* of a base station k selects the users out of $\mathcal{L}$ which it prefers most. One way to define the choice function is by using the channel norm as a measure to select the users as

$$\mathsf{C}_k^{\text{no-ext}}(\mathcal{L}) = \underset{\mathcal{B}\subseteq\mathcal{L}}{\arg\max}\sum_{l\in\mathcal{B}}\|\mathbf{h}_{k,l}\| \quad s.t.\ |\mathcal{B}|\le q_k. \quad (6)$$

In (6), the preference of a base station does not depend on the matching M and hence there is no effects of externalities in the choice function. Another way to model the choice function of a base station is by exploiting the used beamforming scheme in (5) as follows:

$$\mathsf{C}_k^{\text{ext}}(\mathsf{M},\mathcal{L}) = \underset{\mathcal{B}\subseteq\mathcal{L}}{\arg\max}\sum_{l\in\mathcal{B}}\left|\mathbf{h}_{k,l}^{\dagger}\mathbf{w}_{k,l}^{\text{WSLNR}}(\mathsf{M}_{(k,\mathcal{L})})\right| \quad (7a)$$

$$s.t.\ \max_{l\in\mathcal{B}}\left\{\left|\mathbf{h}_{k,l}^{\dagger}\mathbf{w}_{k,l}^{\text{WSLNR}}(\mathsf{M}_{(k,\mathcal{L})})\right|\right\} \ge \alpha\max_{j\in\mathcal{L}}\left\{\left|\mathbf{h}_{k,j}^{\dagger}\mathbf{w}_{k,j}^{\text{WSLNR}}(\mathsf{M}_{(k,\mathcal{L})})\right|\right\}, \quad (7b)$$

$$|\mathcal{B}|\le q_k, \quad (7c)$$

where $\alpha\in[0,1]$ is a design parameter[1] and $\mathsf{M}_{(k,\mathcal{L})}$ is the matching induced from M according to the following [4].

[1]For large α, the choice function is more restrictive than for smaller α which will affect the size of $\mathsf{C}_k^{\text{ext}}(\mathsf{M},\mathcal{L})$.

Definition 2: Given a matching M and a pair $(k, \mathcal{L})$ with $k \in \mathcal{K}$ and $\mathcal{L} \subseteq \mathcal{U}$, define the matching $\mathsf{M}_{(k,\mathcal{L})}$ as

i. if $l \in \mathcal{L}$, then $\mathsf{M}_{(k,\mathcal{L})}(l) = \{k\}$
ii. if $l \in \mathsf{M}(k) \setminus \mathcal{L}$, then $\mathsf{M}_{(k,\mathcal{L})}(l) = \emptyset$
iii. if $l \notin \mathsf{M}(k) \cup \mathcal{L}$, then $\mathsf{M}_{(k,\mathcal{L})}(l) = \mathsf{M}(l)$

In Definition 2, the matching $\mathsf{M}_{(k,\mathcal{L})}$ is induced from M by "unmatching" the users in $\mathsf{M}(k) \cup \mathcal{L}$ from their current matchings in M and matching $\mathcal{L}$ with base station k.

In order to calculate the choice function, base station k needs to calculate the beamforming vectors according to WSLNR-MAX beamforming in (5), which requires local CSI only. It is additionally required that each base station knows which users are matched to the other base stations in order to determine the unintended receivers. This information should then be exchanged between the base stations.

Due to the existence of externalities in the choice functions of the base stations, i.e., the decision at a base station depends on which users are matched to the other base stations, then we need to design a user proposal method which takes the externalities into account. For this purpose, we define for each user $l \in \mathcal{U}$ a *proposal budget* $b_{l,k} \in \mathbb{N}$ which limits the total number of times this user asks to be matched with a base station $k \in \mathcal{K}$. The motivation for a proposal budget, besides dealing with the issue of externalities, is to limit the signaling overhead between the users and base stations. Using the proposal budget, we model the utility of a user l for base station k to depend on the channel norm, which is available information at the users, as:

$$u_{l,k}(\boldsymbol{\gamma}_l, \tilde{u}_l) = \begin{cases} \|\mathbf{h}_{k,l}\| & \gamma_{l,k} \leq b_{l,k} \text{ and } \gamma_{l,k} = \min_{j \in \mathcal{C}}\{\gamma_{l,j}\} \\ & \text{with } \mathcal{C} := \{j \in \mathcal{K} \mid \|\mathbf{h}_{j,l}\| > \tilde{u}_l\} \\ 0 & \text{otherwise} \end{cases} \tag{8}$$

In (8), $\gamma_{l,k}$ is the number of times a user $l \in \mathcal{U}$ has already proposed to base station $k \in \mathcal{K}$. The condition $\gamma_{l,k} = \min_{j \in \mathcal{C}}\{\gamma_{l,j}\}$ with $\mathcal{C} := \{j \in \mathcal{K} \mid \|\mathbf{h}_{j,l}\| > \tilde{u}_l\}$ indicates that user l is interested only in the base stations in $\mathcal{C}$ (associated with higher channel norms than a given value $\tilde{u}_l$), and more specifically those base stations in $\mathcal{C}$ to which it has proposed the least number of times. The value of $\tilde{u}_l$ will be selected as follows:

$$\tilde{u}_l(\mathsf{M}(l)) = \begin{cases} \|\mathbf{h}_{\mathsf{M}(l),l}\| & \text{if } \mathsf{M}(l) \neq \emptyset \\ \beta \max_{k \in \mathcal{K}}\{\|\mathbf{h}_{k,l}\|\} & \text{otherwise} \end{cases} \tag{9}$$

which is the channel norm corresponding to the base station $\mathsf{M}(l)$ matched to user l, or a minimum utility preference for a user l, given by $\beta \max_{k \in \mathcal{K}}\{\|\mathbf{h}_{k,l}\|\}$, where $\beta \in [0, 1]$ is a design parameter. Similar to α in (7), β can be used to affect the number of base stations which a user can potentially be matched to. The minimum utility preference is modeled as the utility of a user l when unmatched, i.e. $\mathsf{M}(l) = \emptyset$, since in a stable matching (requirements described below) a user would not accept to be matched with a base station which gives lower utility than the utility corresponding to being unmatched.

We describe the stability requirements in a stable matching in the following.

Algorithm 1 Stable matching with proposal budget.

Initialize: matching M such that $\mathsf{M}(l) = \emptyset$ for all $l \in \mathcal{U}$

repeat
    for all $l \in \mathcal{U}$ **do**
        user l proposes to it best base station

$$k_l^* = \{k \in \mathcal{K} \mid u_{l,k}(\boldsymbol{\gamma}_l, \tilde{u}_l(\mathsf{M}(l))) > u_{l,j}(\boldsymbol{\gamma}_l, \tilde{u}_l(\mathsf{M}(l))) \text{ for all } j \in \mathcal{K}\} \tag{10}$$

        update proposal count $\gamma_{l,k_l^*} = \gamma_{l,k_l^*} + 1$
    for all $k \in \mathcal{K}$ **do**
        set of users proposing to base station k

$$\mathcal{P}_k := \{\mathcal{A} \subseteq \mathcal{U} \mid l \in \mathcal{A} \text{ if } k_l^* = k\} \cup \mathsf{M}(k). \tag{11}$$

        accept $\mathsf{C}_k(\mathsf{M}, \mathcal{P}_k)$
        reject $\mathcal{P}_k \setminus \mathsf{C}_k(\mathsf{M}, \mathcal{P}_k)$
        update $\mathsf{M} = \mathsf{M}_{(k, \mathsf{C}_k(\mathsf{M}, \mathcal{P}_k))}$
until no proposal from any user is made

Definition 3: Matching M is individually rational if

i. for all $l \in \mathcal{U}$, $\|\mathbf{h}_{\mathsf{M}(l),l}\| > \beta \max_{k \in \mathcal{K}}\{\|\mathbf{h}_{k,l}\|\}$, and
ii. for all base stations $k \in \mathcal{K}$, $\mathsf{C}_k(\mathsf{M}(k), \mathsf{M}) = \mathsf{M}(k)$.

Individually rationality ensures that each user prefers being in its current matching rather than being unmatched, and that each base station should be matched to the users determined by its choice function.

Definition 4: Matching M is pairwise stable if there does not exist a pair $(l, k) \in \mathcal{U} \times \mathcal{K}$ such that

i. $u_{l,k}(\boldsymbol{\gamma}_l, \tilde{u}_l(\mathsf{M}(l))) > u_{l,\mathsf{M}(l)}(\boldsymbol{\gamma}_l, \tilde{u}_l(\mathsf{M}(l)))$, and
ii. $l \in \mathsf{C}_k(\mathsf{M}(k) \cup \{l\})$.

Pairwise stability requires that there exist no base station k and no user l which are not matched to each other but prefer a matching between themselves.

Definition 5: A matching M is *stable* if it is individually rational and pairwise stable.

Algorithm 1 has similarities with the deferred acceptance algorithm [9] which reaches a stable matching in settings without externalities. First, each user proposes to its best base station according to the utility model in (8). Given the proposals from the users, each base station selects its best users according to its choice function. Here, we use the choice function in (6) for the case without externalities and the choice function in (7) for the case with externalities. The algorithm terminates when no user proposes to any base station.

The reached matching satisfies the individual rationality condition and pairwise stability since the algorithm iterates over all possible opportunities for pairing any user and base station which prefer each other. Note that algorithm 1 converges to a stable matching with a worst case total number of proposals $\sum_{l \in \mathcal{U}} \sum_{k \in \mathcal{K}} b_{l,k}$ having the proposal budget in the utility model in (8).

Observe that in the case without externalities, Algorithm 1 corresponds to the deferred acceptance algorithm [9] in which a user proposes at most once to a base station. This is unlike

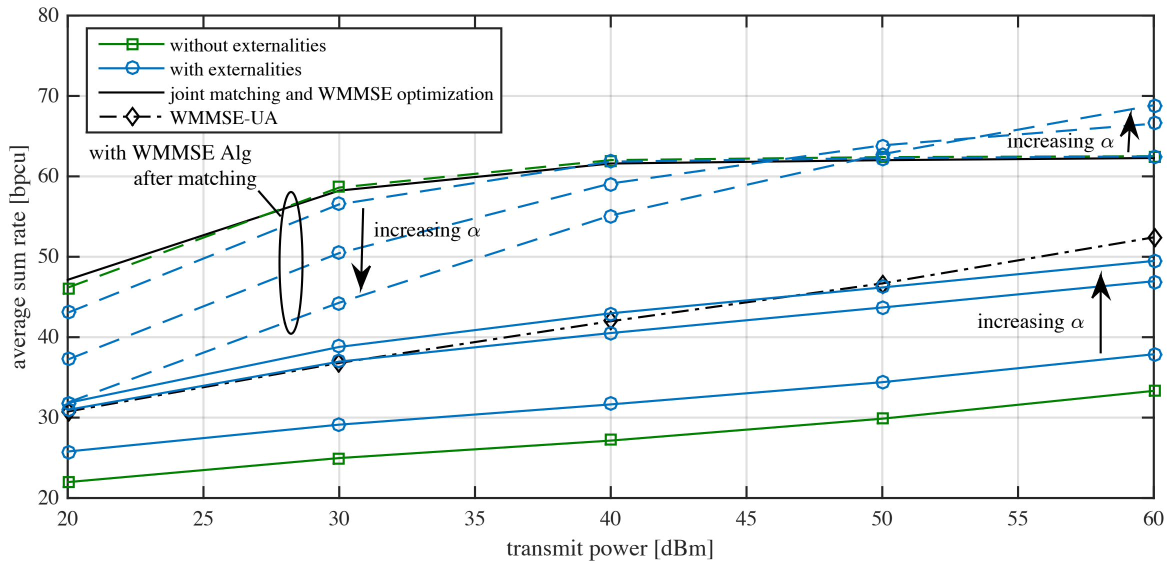

Fig. 2. Average sum rate for a setting with 20 users. The parameter α for the stable matching with externalities is chosen from $\{0.1, 0.5, 0.9\}$.

in the case with externalities in which a user may propose more than one time to the same base station.

IV. Simulation Results

In the simulations, we consider a multi-cell system where the base stations and the users are dropped uniformly at random in a square. We following the simulation setup in [11], and choose the dimension of the square region to ensure that the average cell size is the same as a setup in which the cells are hexagonal with an apothem of 250 m [12]. We also adopt from [11] the simulation parameters, which rely on 3GPP Case 2 [12], as is shown in Table I.

TABLE I
Simulation Parameters

Bandwidth	10 MHz
Carrier frequency	2 GHz
Pathloss	$15.3 + 3.76 \log_{10}(\text{distance[m]})$
Shadow fading	Log-normal i.i.d. with standard deviation 8 dB
Noise PSD	-174 dBm/Hz
Receiver noise figure	9 dB

In the following, we consider five base stations where each base station is equipped with five antennas. We generate 200 random instances for the base station and user deployments and the channel realizations to calculate the average performance. The base station quota constraints, which are used in the choice functions in (7) and (6), are chosen as $q_k = 5$, equal to the number of antennas at each base station. We set the users' proposal budget in (8) as $b_{l,k} = 2$, and the parameter β used in (9) is set to 0.25.

In Fig. 2, we evaluate the average sum rate achieved with the stable matching algorithms for different power levels at the base stations. Stable matching with externalities is shown to outperform the matching scheme without externalities. By increasing α (which affects the choice function in (7))

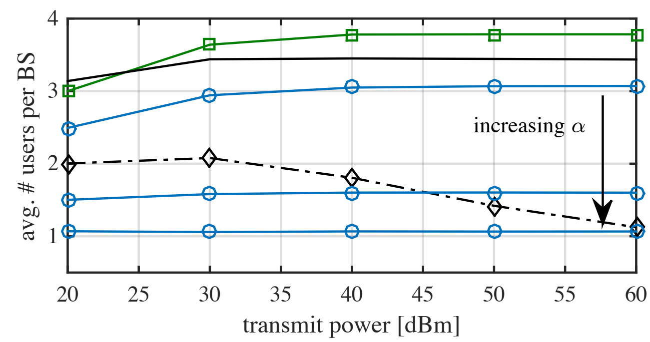

Fig. 3. Average number of users assigned per base station corresponding to the curves in Fig. 2.

the gains increase. Note that, we apply the fully distributed beamforming scheme in (5). Applying the WMMSE algorithm [13] after the user assignment process, we see a significant performance increase according to the curves with dashed lines. However, the gains in applying the WMMSE algorithm come at a cost of a necessary feedback overhead which is required for implementing the algorithm. Interestingly, the performance of stable matching without externalities including WMMSE algorithm is higher than that of stable matching with externalities. The reason for this is, stable matching without externalities assigns more users per base station than stable matching without externalities as is seen in Fig. 3. Consequently, the WMMSE algorithm [13] has larger degrees of freedom which is exploited in the sum rate optimization to give higher performance.

The joint matching and WMMSE optimization algorithm in [8] applies stable matching with externalities using the achievable rates found by WMMSE optimization as utility functions for the users. This scheme has higher complexity than our proposed algorithm due to the higher feedback

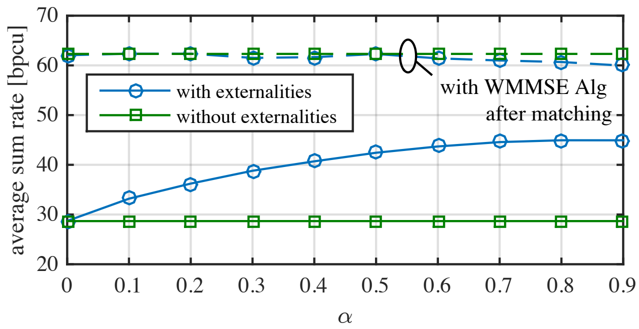

Fig. 4. Effect of the choice of α on the average sum rate for a setting with 20 users. The transmission power is set to 46 dBm.

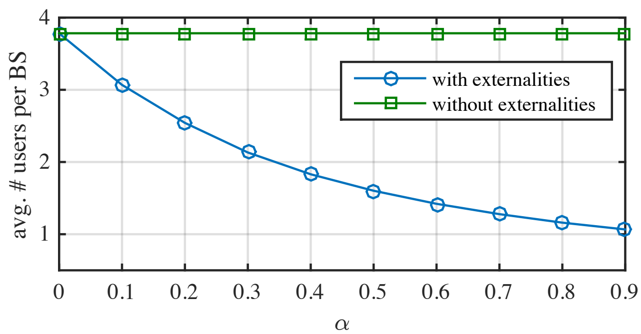

Fig. 5. Average number of users assigned per base station corresponding to the curves in Fig. 4.

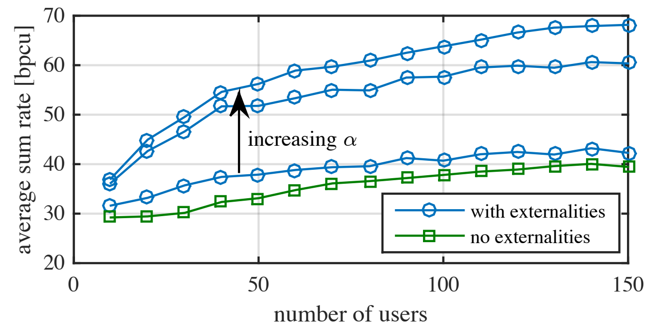

Fig. 6. Average sum rate for increasing number of users. The transmission power is set to 46 dBm.

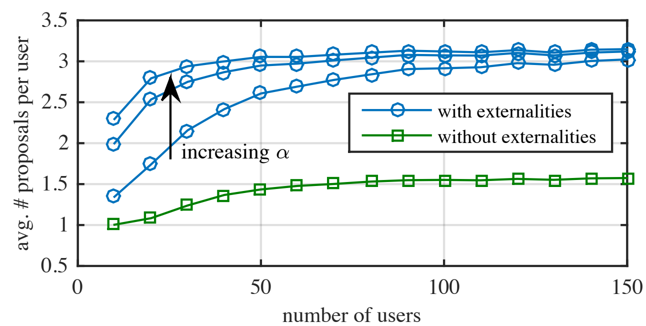

Fig. 7. Average number of proposals per user corresponding to the curves in Fig. 6.

overhead for WMMSE optimization in each iteration of the stable matching algorithm. For the joint user assignment and WMMSE algorithm (WMMSE-UA) from [2], we restrict the number of iterations of the algorithm to 20 and choose a gradient projection step-size of one for the user assignment optimization step. Unexpectedly, this algorithm gave poor performance in our simulations.

In Fig. 4, we show the average sum rate achieved with the stable matching algorithms for different choices of α. The corresponding average number of users assigned per base station is plotted in Fig. 5. With the distributed beamforming scheme in (5), stable matching with externalities achieves higher performance for large α. These gains are due to the selection of fewer number of users for each base station which are associated with highest desired power gains with the WSLNR-MAX beamforming scheme. The performance of the stable matching schemes with and without externalities become comparable when the WMMSE algorithm is applied after the matching phase.

For increasing number of users, we plot the average sum rate achieved with stable matching in Fig. 6. The corresponding average number of proposals per user is shown in Fig. 7. Comparing different choices of α in Fig. 6, we also observe that a large choice of α brings higher efficiency. In addition, the gap between the curves increases with the number of users. Regarding complexity of the stable matching algorithms, it can be observed in Fig. 7 that the average number of proposals per user is very low, only up to three in a network with 150 users.

V. Conclusion

We have proposed a distributed joint beamforming and user association scheme based on stable matching with externalities in a multi-cell MISO network. The scheme relies on the users to propose to their mostly preferred base stations (using channel norm information), and each base station exploits its WSLNR-MAX beamforming scheme for selecting the users from set of users who propose it. Simulation results reveal high efficiency improvement compared to user selection based on channel norm at the base stations. In addition, the scheme has low complexity which makes it suitable for applications in large or dense networks.

Acknowledgment

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