Semi-Blind Estimation of FDD Massive MIMO Channels using Correlative Coded Analog Feedback

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***Abstract*—Knowledge of accurate channel state information (CSI) is crucial for Massive MIMO systems in order to unfold the full potential of spatial diversity. In TDD systems, uplink (UL) and downlink (DL) channels are estimated by transmitting pilots in the uplink and exploiting channel reciprocity. FDD systems are more complicated because they require a dedicated downlink training and corresponding CSI feedback in the UL. For the latter task, linear analog modulation has been proposed which avoids digitizing and coding the CSI (see e.g., Echo-MIMO). We investigate the possibility to utilize these feedback signals for the (blind) estimation of UL channels, making a separate UL training obsolete. The method requires the correlative coding of the feedback signals prior to transmission. As we will show, the blind channel estimation benefits from the large amount of DL CSI feedback that arises in Massive MIMO systems. Consequently, for sufficiently many base station antennas M (i.e., $M > 32$ with $K = 4$ terminals) and low SNR, our method exhibits larger coherent processing gains than the Echo-MIMO scheme which requires K additional uplink training symbols. On the other hand, employing Echo-MIMO with $M/2$ uplink training symbols is always superior to SBCE method.**

***Index Terms*—Massive MIMO systems, frequency-division duplex, closed-loop training, channel estimation, analog feeback, blind identification**

I. Introduction

Massive MIMO is a multi-user MIMO technology that employs a very large number (e.g., tens or hundreds) of antennas at the base station (BS), which utilizes these to communicate simultaneously with K single-antenna terminals. Such systems have recently drawn considerable interest because they allow to alleviate inter-user interference with a simple linear precoder and receive combiner [1]. However, the coherent signal processing at the BS requires accurate knowledge of the channel state information (CSI). While time-division duplex systems acquire the high-dimensional CSI by means of uplink (UL) training (whose overhead depends only on K) and then exploit the channel reciprocity, the CSI acquisition in frequency-division duplex (FDD) systems becomes very costly due to the separate downlink (DL) training and CSI feedback in the UL. The fact that the amount of transmission resources required by both tasks grows with M, motivated several research groups to propose DL CSI acquisition methods which reduce this overhead. Generally speaking, these methods assume that there is either some kind of channel sparsity that can be utilized, or some a-priori channel knowledge such as long-term channel statistics. The reduction of the DL training has been considered in e.g., [2], where the spatial and temporal channel correlations are exploited for the careful design of the training pilots, and in [3] which exploits some (hidden) joint sparsity structure in the user channel matrices by means of a distributed, compressive sensing (CS) technique. Methods that focus on the reduction of the DL CSI feedback (i.e., feedback compression) can be found in e.g., [4] (projection based), [5] (CS based), [6] (pattern based). The application of these techniques to analog feedback schemes[1] is proposed in [12] for feedback reduction, and in [13], [14] to improve the CSI estimation using CS techniques.

Although the mentioned sparsity assumptions may hold at millimeter wave frequencies, measurements (see e.g., Figure 4 in [15]) indicate that spatial sparsity assumptions are questionable at lower frequencies. This gives rise to the following question: *How to reduce the overhead of the (closed-loop) CSI acquisition in absence of any channel structure?*
We tackle this question by investigating a method that utilizes the analog DL CSI feedback for the estimation of the UL channels; that is, no dedicated UL training is required. Though the overhead reduction may be quite limited, we can show performance gains with respect to the Echo-MIMO scheme for specific SNR regimes.

One should note that the uplink channels and feedback signals are unknown. Thus, we have to employ a blind estimation technique, which typically relies on distinct characteristics of the (feedback) signals in the UL, e.g., cyclostationarity [16], higher order cumulants [17] or second-order statistics [18]. We adopt the latter approach which has led to the AMUSE method (i.e., Algorithm for Multiple Unknown Signals Extraction) in [19]. Analog to [20], we artificially shape the spectra of our feedback signals by utilizing correlative filters at the terminals. Note that the inherent indeterminacy (i.e., unknown phase rotations of the UL channels and feedback signals) of the AMUSE algorithm does not impede our MIMO system because the data signals usually comprise demodulation reference signals.

[1]The linear analog modulation of the DL CSI feedback, which avoids digitizing and coding, has been analyzed in [7], and utilized in the Echo-MIMO scheme [8]. In [9], [10] it is shown for the i.i.d. Rayleigh fading case, that the analog feedback is optimal the sense of mean square error of the downlink CSI if the number of feedback symbols equals the number of feedback channel uses. The practicability of this feedback scheme is demonstrated in [11].

The paper's body is organized into four major sections, addressing the system model, the blind identification (AMUSE) method, the proposed semi-blind channel estimation method, and numerical experiments in order to assess the benefits and weak points of our method.

Notation: Vectors and matrices are given in lowercase and uppercase boldface letters, respectively. $(\cdot)^*$,$(\cdot)^T$, and $(\cdot)^H$ denote the complex conjugate, the transposed, and the Hermitian transpose. The symbol $\mathrm{E}[\cdot]$ denotes the expectation operator. $\boldsymbol{I}_M$ is the $M \times M$ identity matrix. The superscript $\#$ denotes the Moore-Penrose pseudo-inverse. $\delta(\tau)$ is the Kronecker delta.

II. System Model

We consider a time-invariant, frequency-flat, multi-user MIMO channel[2], where a base station, equipped with M antennas, serves K single-antenna terminals. For a coherent signal processing, the base station needs to learn both the up- and downlink channels via training, which is described in the following two sections.

A. Downlink Training

The downlink channel is learned through known pilots that are transmitted by the base station array. It is well-known (see e.g., [21]) that orthogonal training signals with equal power allocation per transmit antenna are optimal in i.i.d. Rayleigh fading channels. Let $\boldsymbol{\Phi}$ be a $\phi \times M$ unitary complex matrix (i.e., $\boldsymbol{\Phi}^H\boldsymbol{\Phi} = \boldsymbol{I}_M$), whose columns represent the training signals of the individual transmit antennas. Then, the CSI-bearing signal, received at the k-th terminal, is given by the vector

$$\boldsymbol{r}_k^T = [r_k(0)\dots r_k(\phi-1)]^T = \sqrt{\phi/M}\boldsymbol{b}_k^T\boldsymbol{\Phi}^T + \boldsymbol{m}_k^T \tag{1}$$

where $\boldsymbol{b}_k \in \mathbb{C}^M$ is downlink propagation vector from the array to the k-th terminal, and $\boldsymbol{m}_k^T = [m_k(0)\dots m_k(\phi-1)]^T$ contains both (downlink) receiver noise and interference. Note that we keep the instantaneous transmit power of the array constant; that is, we reduce the transmit power per antenna with increasing M.

B. Correlative Coding and Uplink Training

The uplink channel is learned through the analog feedback of the received downlink signals. In order to enable the separation of the superimposed signals at the base station, the k-th terminal passes its feedback signal (prior to transmission) through a distinct (see Section III,(A2)) correlative filter of order L_k with impulse response $c_k(t) = \sum_{i=0}^{L_k} c_{k,i}\delta(t-i)$. The symbols of the transmit signal $\boldsymbol{s}_k^T = [s_k(0),\dots,s_k(\phi-1)]^T$ have the form

$$s_k(t) = \sqrt{\beta_k}\sum_{i=0}^{L_k} c_{k,i}\cdot r_k(t-i), \tag{2}$$

where $\sqrt{\beta_k}$ denotes the power control coefficient, which ensures the power constraint $\mathrm{E}[s_k(t)s_k(t)^*] = 1$. The CSI-bearing signal $\boldsymbol{X} \in \mathbb{C}^{M\times\phi}$, received at the base station, is

$$\boldsymbol{X} = \boldsymbol{A}\left[\boldsymbol{s}_1,\dots,\boldsymbol{s}_K\right]^T + \boldsymbol{N} \tag{3}$$

where $\boldsymbol{A} = [\boldsymbol{a}_1,\dots,\boldsymbol{a}_K]$ is the $M \times K$ uplink propagation matrix and $\boldsymbol{N} = [\boldsymbol{n}(0),\dots,\boldsymbol{n}(\phi-1)]$ is a $M \times \phi$ matrix comprising both (uplink) receiver noise and interference.

III. Review: Blind Identification Algorithm

In this Section, we recapitulate a variant of the AMUSE algorithm [19] for the blind identification problem, which arises from the uplink equation (3). Therefore, we consider the *instantaneous* (noisy) mixture $\boldsymbol{x}(t)$ of the source symbols $\boldsymbol{s}(t) = [s_1(t),\dots,s_K(t)]^T$, given by

$$\boldsymbol{x}(t) = \boldsymbol{y}(t) + \boldsymbol{n}(t) = \boldsymbol{A}\boldsymbol{s}(t) + \boldsymbol{n}(t). \tag{4}$$

The blind identification problem is to identify both, the unknown mixing matrix $\boldsymbol{A} \in \mathbb{C}^{M\times K}$ and the unkown source symbols $\boldsymbol{s}(t)$, given the measurements $\boldsymbol{x}(t)$.

We start by listing a set of assumptions under which the blind identification algorithm is derived.

(A1) The source symbol vector $\boldsymbol{s}(t)$ is assumed to be a stationary multivariate random process with known covariance matrices

$$\begin{aligned}\boldsymbol{R}_{\boldsymbol{s}}(\tau) &= \mathrm{E}\left[\boldsymbol{s}(t+\tau)\boldsymbol{s}(t)^H\right]\\ &= \mathrm{diag}\left(\rho_1(\tau),\dots,\rho_K(\tau)\right).\end{aligned}$$

The component processes $\boldsymbol{s}_k(t)$, $1 \le k \le K$ are mutually uncorrelated, and each possesses the auto-covariance $\rho_k(\tau) = \mathrm{E}\left[\boldsymbol{s}_k(t+\tau)\boldsymbol{s}_k(t)^*\right]$.

(A2) We assume that the source signals have unit variance $\rho_k(0) = 1, \forall k$, and that for some time lag $\tau \neq 0$ the auto-correlation coefficients $\rho_k(\tau), \forall k$ are mutually distinct (i.e., $\rho_k(\tau) \neq \rho_l(\tau)$ for $k \neq l$), and known to the base station[3].

(A3) The additive noise $\boldsymbol{n}(t)$ is modeled as a stationary, spatially[4] and temporally white, zero-mean complex random process independent of the source signals,

$$\boldsymbol{R}_{\boldsymbol{n}}(\tau) = \mathrm{E}\left[\boldsymbol{n}(t+\tau)\boldsymbol{n}(t)^H\right] = \delta(\tau)\sigma_{\mathrm{U}}^2\boldsymbol{I}_M.$$

Under the above assumptions, the covariance matrices of the array output exhibit the following structure:

$$\begin{aligned}\boldsymbol{R}_{\boldsymbol{x}}(\tau) &= \mathrm{E}\left[\boldsymbol{x}(t+\tau)\boldsymbol{x}(t)^H\right]\\ &= \boldsymbol{A}\boldsymbol{R}_{\boldsymbol{s}}(\tau)\boldsymbol{A}^H + \delta(\tau)\sigma_{\mathrm{U}}^2\boldsymbol{I}_M\end{aligned} \tag{5}$$

The first step of the blind identification algorithm consists of whitening the signal part $\boldsymbol{y}(t)$ of the observation (4). The $K \times M$ whitening matrix $\boldsymbol{W}$ is chosen such that

$$\mathrm{E}\left[\boldsymbol{W}\boldsymbol{y}(t)\boldsymbol{y}(t)^H\boldsymbol{W}^H\right] = \boldsymbol{W}\boldsymbol{A}\boldsymbol{A}^H\boldsymbol{W}^H = \boldsymbol{I}_K. \tag{6}$$

[2]The case of multi-antenna terminals is a straightforward extension of the proposed method, by treating every terminal antenna as a separate terminal. In addition, the discussed concepts can be extended to frequency-selective channels by means of subband-wise processing.

[3]As we will see in Section IV, this assumption implies that the base station has the knowledge of the (static) filter impulse responses $c_k(t)$,$1 \le k \le K$, and the power control coefficients β_k, $1 \le k \le K$.

[4]The algorithm may be extended to the case of spatially colored noise, see [18, Section III.A].

The matrix $\boldsymbol{W}$ can be derived from the eigendecomposition of $\boldsymbol{A}\boldsymbol{A}^H$, which in turn can be calculated using the array output covariance $\boldsymbol{R}_{\boldsymbol{x}}(0)$; that is,

$$\boldsymbol{A}\boldsymbol{A}^H = \boldsymbol{R}_{\boldsymbol{x}}(0) - \sigma_{\mathrm{U}}^2 \boldsymbol{I}_M. \tag{7}$$

Note that for any whitening matrix $\boldsymbol{W}$, there exists a $K \times K$ unitary matrix $\boldsymbol{U}$ such that $\boldsymbol{W}\boldsymbol{A} = \boldsymbol{U}$. As a consequence, the matrix $\boldsymbol{A}$ can be factored as $\boldsymbol{A} = \boldsymbol{W}^\sharp \boldsymbol{U}$.

In the second step, the matrix $\boldsymbol{U}$ is identified up to an arbitrary phase rotation of its columns. The whitened observations obey the linear model

$$\boldsymbol{z}(t) = \boldsymbol{W}\boldsymbol{x}(t) = \boldsymbol{U}\boldsymbol{s}(t) + \boldsymbol{W}\boldsymbol{n}(t)$$

where the signal part is now a unitary mixture of the source symbols. The covariance matrix $\boldsymbol{R}_{\boldsymbol{z}}(\tau)$ of the process $\boldsymbol{z}(t)$ is given for some $\tau \neq 0$ by

$$\begin{aligned}\boldsymbol{R}_{\boldsymbol{z}}(\tau) &= \boldsymbol{W}\boldsymbol{R}_{\boldsymbol{x}}(\tau)\boldsymbol{W}^H \\ &= \boldsymbol{U}\boldsymbol{R}_{\boldsymbol{s}}(\tau)\boldsymbol{U}^H.\end{aligned}$$

Since $\boldsymbol{U}$ is unitary and $\boldsymbol{R}_{\boldsymbol{s}}(\tau)$ is diagonal, the matrix $\boldsymbol{R}_{\boldsymbol{z}}(\tau)$ is a normal matrix and thus exhibits the same eigenvalues and algebraic multiplicities as $\boldsymbol{R}_{\boldsymbol{s}}(\tau)$. Since $\boldsymbol{R}_{\boldsymbol{s}}(\tau)$ is assumed to have distinct eigenvalues (cf. (A2)), the eigendecomposition of $\boldsymbol{R}_{\boldsymbol{z}}(\tau)$ identifies the matrix $\boldsymbol{U}$ up to a permutation of the columns and a column-wise phase rotation. The first indeterminacy is resolved by matching the order of the eigenvalues of $\boldsymbol{R}_{\boldsymbol{z}}(\tau)$ with the order of the diagonal entries of $\boldsymbol{R}_{\boldsymbol{s}}(\tau)$ (compare [20, IV.A]).

Given $\boldsymbol{W}$ and $\boldsymbol{U}$, the mixing matrix $\boldsymbol{A}$ can be identified by $\hat{\boldsymbol{A}} = \boldsymbol{W}^\sharp \boldsymbol{U}$, and the source signals are recovered by $\hat{s}(t) = \boldsymbol{U}^H \boldsymbol{W}\boldsymbol{x}(t)$, both up to a terminal-specific phase rotation.

IV. Joint UL/DL Channel Estimation

A. System Model Requirements

Employing the AMUSE method for the up- and downlink channel estimation requires specific system model assumptions. The following requirements assure assumption (A1):

- (A1.a) For the downlink, we have mutually uncorrelated i.i.d. Rayleigh fading channels.
- (A1.b) The additive noise processes $m_k(t)$, $1 \le k \le K$ are modeled as stationary, temporally white, mutually uncorrelated, zero-mean complex random processes; that is, $\mathrm{E}\left[m_k(t+\tau)m_l(t)^*\right] = \delta(\tau)\delta(k-l)\sigma_{\mathrm{D}}^2$. (For simplicity, it is assumed that the downlink noise variances are equal for all terminals.)
- (A1.c) The pilot matrix $\boldsymbol{\Phi}$ is square; i.e., $\phi = M$.

According to (A1.a)-(A1.c), we have $\mathrm{E}\left[r_k(t+\tau)r_k(t)^*\right] = \delta(\tau)\mu_k$ for some $\mu_k > 0$. Consequently, the auto-covariance of the k-th feedback signal $s_k(t)$ is given by

$$\rho_k(\tau) = \mathrm{E}\left[s_k(t+\tau)s_k(t)^*\right] = \beta_k \mu_k \sum_{i=\tau}^{L_k} c_{k,i}^* c_{k,i-\tau}.$$

Obviously, Assumption (A2) can be satisfied by an appropriated choice of the filter coefficients $c_{k,i}, \forall k, i$, as illustrated in Section V.

B. Semi-Blind Channel Estimation (SBCE) Algorithm

The SBCE method comprises the following steps:

1) Compute the sample covariance matrix
$$\hat{\boldsymbol{R}}_{\boldsymbol{x}}(0) = \phi^{-1}\boldsymbol{X}\boldsymbol{X}^H. \tag{8}$$
Denote by $\lambda_1, \dots, \lambda_K$ the K largest eigenvalues and $\boldsymbol{v}_1, \dots, \boldsymbol{v}_K$ the corresponding eigenvectors of $\hat{\boldsymbol{R}}_{\boldsymbol{x}}(0)$.
2) Estimate the uplink noise variance σ_{U}^2 by averaging the $M-K$ smallest eigenvalues of $\hat{\boldsymbol{R}}_{\boldsymbol{x}}(0)$. Then, perform a whitening of the received signal $\boldsymbol{X}$ by left-multiplying it with the matrix
$$\hat{\boldsymbol{W}} = \left[(\lambda_1 - \hat{\sigma}_{\mathrm{U}}^2)^{-\frac{1}{2}}\boldsymbol{v}_1, \dots, (\lambda_K - \hat{\sigma}_{\mathrm{U}}^2)^{-\frac{1}{2}}\boldsymbol{v}_K\right]^H,$$
yielding the $K \times \phi$ matrix
$$\boldsymbol{Z} = [\boldsymbol{z}(0), \dots, \boldsymbol{z}(\phi-1)] = \hat{\boldsymbol{W}}\boldsymbol{X}.$$
3) Compute the sample covariance matrix
$$\hat{\boldsymbol{R}}_{\boldsymbol{z}}(\tau) = \frac{1}{\phi - \tau}\sum_{t=0}^{\phi-\tau-1} \boldsymbol{z}(t+\tau)\boldsymbol{z}^H(t)$$
for some $0 < \tau < \phi$.
4) Compute the eigendecomposition $\hat{\boldsymbol{R}}_{\boldsymbol{z}}(\tau) = \boldsymbol{V}\boldsymbol{D}\boldsymbol{V}^H$. Denote by $d_1, \dots, d_K$ the eigenvalues and $\boldsymbol{\nu}_1, \dots, \boldsymbol{\nu}_K$ the corresponding eigenvectors of $\boldsymbol{D}$. Find the permutation σ^* on $\{1, \dots, K\}$ that minimizes
$$\left\|\left[\rho_1(\tau), \dots, \rho_K(\tau)\right]^T - \left[d_{\sigma(1)}, \dots, d_{\sigma(K)}\right]^T\right\|^2.$$
The unitary matrix $\boldsymbol{U}$ is estimated as $\hat{\boldsymbol{U}} = \left[\boldsymbol{\nu}_{\sigma^*(1)}, \dots, \boldsymbol{\nu}_{\sigma^*(K)}\right]$, and the uplink channel matrix $\boldsymbol{A}$ is estimated (up to a column-wise phase rotation) as $\hat{\boldsymbol{A}} = \hat{\boldsymbol{W}}^\sharp \hat{\boldsymbol{U}}$.
5) The feedback signals $\boldsymbol{s}_k$, $1 \le k \le K$ are estimated (up to a terminal-specific phase rotation) as
$$\left[\hat{\boldsymbol{s}}_1, \dots, \hat{\boldsymbol{s}}_K\right]^T = \hat{\boldsymbol{U}}^H \hat{\boldsymbol{W}}\boldsymbol{X}.$$
6) An estimate (up to a terminal-specific phase rotation) for the k-th downlink channel $\boldsymbol{b}_k$ is given by
$$\hat{\boldsymbol{b}}_k^T = \beta_k^{-1/2}\hat{\boldsymbol{s}}_k^T(\boldsymbol{\Phi}^T\boldsymbol{C}_k)^\sharp,$$
where $\boldsymbol{C}_k$ is a $\phi \times \phi$ Toeplitz matrix, defined by
$$(\boldsymbol{C}_k)_{i,j} = \begin{cases} c_{k,(j-i)} & \text{, for } 0 \le j-i \le L_k, \\ 0 & \text{, otherwise.}\end{cases}$$

Remark 1. In contrast to Section III, the SBCE algorithm utilizes the sample covariance matrices $\hat{\boldsymbol{R}}_{\boldsymbol{x}}(0)$ and $\hat{\boldsymbol{R}}_{\boldsymbol{z}}(\tau)$, and an estimate for the uplink noise variance σ_{U}^2. Note that the accuracies of these estimates strongly depend on the sample support ϕ.

Remark 2. In [18, III.D], a joint diagonalization of $\hat{\boldsymbol{R}}_{\boldsymbol{z}}(\tau)$ for multiple time lags τ is described in order to increase the robustness of the blind detection algorithm w.r.t. degenerate eigenvalues of $\hat{\boldsymbol{R}}_{\boldsymbol{z}}(\tau)$ for some τ.

V. Performance Evaluation

This section compares the performance of the SBCE and Echo-MIMO algorithms by computer simulations.

A. Simulation Model

We simulate a massive MIMO base station with $M \in \{16, 32, 64, 128, 256, 512, 1024\}$ antennas, which employs the SBCE algorithm for the joint estimation of $K = 4$ up- and downlink channels. The acquired CSI knowledge is exploited for the coherent processing of data signals over the array. We assume that the achieved processing gains are used to reduce the total transmit powers at the base station and the terminals. Thus, the signal-to-noise ratio (SNR) during the training phase is expected to be low, while the data transmissions benefit from the coherent processing gains.

For the UL and DL channel vectors, we assume that both are (flat) Rayleigh fading channels, which are spatially correlated with the exponential model of $\boldsymbol{R} = \mathrm{E}[\boldsymbol{a}_k\boldsymbol{a}_k^H] = \mathrm{E}[\boldsymbol{b}_k\boldsymbol{b}_k^H], \forall k$:

$$\boldsymbol{R} = \begin{bmatrix} 1 & \xi & \cdots & \xi^{M-1} \\ \xi & 1 & & \\ \vdots & & \ddots & \\ \xi^{M-1} & & & 1 \end{bmatrix}.$$

The real number ξ, with $0 \leq \xi < 1$, controls the amount of spatial correlation, which allows us to investigate the impact of the violation of Assumption (A1.a). When $\xi = 0$, we recover i.i.d. Rayleigh fading channels.

Under the above assumptions, the expected DL SNR in (1) at an arbitrary terminal receive antenna can be written as $\gamma_{\mathrm{D}}^{\mathrm{pilot}} = 1/\sigma_{\mathrm{D}}^2$. In the UL, the expected SNR *per feedback signal* at an arbitrary BS receive antenna is $\gamma_{\mathrm{U}}^{\mathrm{pilot}} = 1/\sigma_{\mathrm{U}}^2$ (cf. (3)). Due to different transmit power constraints (e.g.; 23dBm and 43dBm for LTE terminals and base stations, respectively), we assume $\gamma_{\mathrm{D}}^{\mathrm{pilot}} = 100 \cdot \gamma_{\mathrm{U}}^{\mathrm{pilot}}$ in all simulations.

For the correlative filters $c_k(t)$, $1 \leq k \leq K$, we employ truncated IIR filters of order $L_k = 16$ with $c_{k,i} = (p_k)^i$, $0 \leq i \leq L_k$, and $p_k = \alpha_k \exp(j\theta_k)$. The angles θ_k are equidistantly distributed in the interval $[0, 2\pi]$; that is, $\theta_k = k2\pi/K$. The modulus α_k is set to 0.8 for all terminals. One should note that we have distinct $\rho_k(\tau)$, $1 \leq k \leq K$ for $\tau = 1$.

As a reference case, we simulate the Echo-MIMO scheme [8]. This method employs two dedicated training phases for the downlink and uplink channels. The downlink training is almost identical to the procedure described in Sections II-A and II-B except the correlative filtering step in (2), which is omitted. In the second (uplink) training phase, the terminals transmit orthogonal pilot sequences $[\boldsymbol{s}_1, \ldots, \boldsymbol{s}_K]^T = \sqrt{\phi_{\mathrm{p}}}\boldsymbol{\Phi}_{\mathrm{p}}^T$ with $\boldsymbol{\Phi}_{\mathrm{p}} \in \mathbb{C}^{\phi_{\mathrm{p}} \times K}$, $\boldsymbol{\Phi}_{\mathrm{p}}^H \boldsymbol{\Phi}_{\mathrm{p}} = \boldsymbol{I}_K, \phi_{\mathrm{p}} \in \{K, M/8, M/2\}$ and $\mathrm{E}[s_k(t)s_k(t)^*] = 1$, $1 \leq k \leq K$. The received pilot is then

$$\boldsymbol{X}_{\mathrm{p}} = \sqrt{\phi_{\mathrm{p}}}\boldsymbol{A}\boldsymbol{\Phi}_{\mathrm{p}}^T + \boldsymbol{N}_{\mathrm{p}}$$

where $\boldsymbol{N}_{\mathrm{p}}$ follows the same statistics as $\boldsymbol{N}$ in (3). The MMSE estimate for the uplink channels is

$$\hat{\boldsymbol{A}} = \left(\frac{\sqrt{\phi_{\mathrm{p}}}}{\sigma_{\mathrm{U}}^2 + \phi_{\mathrm{p}}}\right) \boldsymbol{X}_{\mathrm{p}} \boldsymbol{\Phi}_{\mathrm{p}}^*.$$

The downlink channels $[\boldsymbol{b}_1, \ldots, \boldsymbol{b}_K] = \boldsymbol{B}$ are then given by the least squares estimate

$$\hat{\boldsymbol{B}}^T = \mathrm{diag}\left(\beta_1, \ldots, \beta_K\right)^{-1/2} \hat{\boldsymbol{A}}^\sharp \boldsymbol{X}\boldsymbol{\Phi}^*.$$

One should note that the entire process requires $2M + \phi_{\mathrm{p}}$ resource samples, in contrast to the $2M$ resource samples needed by the SBCE method.

B. Achievable CSI Accuracy

We first compare the achievable CSI quality of the SBCE and Echo-MIMO (EM) methods. Due to the indeterminacy of a phase rotation in e.g., $\hat{\boldsymbol{a}}_k$, the typical error metric $\mathrm{E}[\|\boldsymbol{a}_k - \hat{\boldsymbol{a}}_k\|^2]^{\frac{1}{2}}$ is not meaningful for the SBCE method. Therefore, we quantify the achievable CSI accuracy in terms of the root-mean-square distance (RMSD) between the two subspaces that are spanned by $\boldsymbol{a}_k$ and $\hat{\boldsymbol{a}}_k$. By using the natural metric on the set of one-dimensional subspaces in $\mathbb{C}^M$ (for details see [22]), the distance between the two subspaces is

$$d(\boldsymbol{a}_k, \hat{\boldsymbol{a}}_k) = \arccos\left(\frac{|\boldsymbol{a}_k^H \hat{\boldsymbol{a}}_k|}{\|\boldsymbol{a}_k\| \, \|\hat{\boldsymbol{a}}_k\|}\right) \quad \text{(rad)},$$

which yields the RMSD

$$\epsilon = \mathrm{E}\left[d(\boldsymbol{a}_k, \hat{\boldsymbol{a}}_k)^2\right]^{\frac{1}{2}} \quad \text{(rad)}.$$

Figure 1 displays the averaged UL and DL RMSDs (obtained by simulating 500 channel realizations) as a function of the number of base station antennas M for $\gamma_{\mathrm{D}} = \gamma_{\mathrm{U}} + 20\mathrm{dB} \in \{10\mathrm{dB}, 20\mathrm{dB}\}$, and for the spatial correlations $\xi \in \{0, 0.8\}$. There are several features in the figures we comment upon. As analyzed in [7], the UL and DL CSI accuracy of the Echo-MIMO scheme improves with an increasing number of UL training symbols ϕ_{p}, and the DL CSI accuracy increases with growing M, which is due to the receive diversity in the UL. For the SBCE method, the number of training symbols that are used for the UL channel estimation is coupled to M, and thus the UL RMSD decreases with growing M, and eventually outperforms the Echo-MIMO method for a sufficiently large M. One would expect that the corresponding DL RMSD of the SBCE method excels the RMSD of the Echo-MIMO method, but this is not the case. Note that the SBCE method relies on the statistical independence of the UL signals $\boldsymbol{s}_1, \ldots, \boldsymbol{s}_K$, and inherently to reinforces this property (i.e., orthogonality) in its estimates for these signals. More precisely, the whitening step in (6) produces uncorrelated observations, and the estimate of the unitary matrix $\hat{\boldsymbol{U}}$ is chosen such that the (cross-) covariance properties are met. Since every UL signal $\boldsymbol{s}_k$ contains the linearly transformed DL channel vector $\boldsymbol{b}_k$, the reinforced geometric properties pass roughly (distorted by $(\boldsymbol{C}_k^H\boldsymbol{C}_k)^{-1}$) to the estimated DL vectors. Though, this effect does not necessarily imply a performance degradation for the coherent signal processing in the DL, as we will illustrate in the next Section. Finally, one should note that the impact of spatial (antenna) correlation, which constitutes a violation of Assumption (A1.a), has some detrimental effect only to the SBCE's UL CSI accuracy, and becomes negligible at low SNRs.

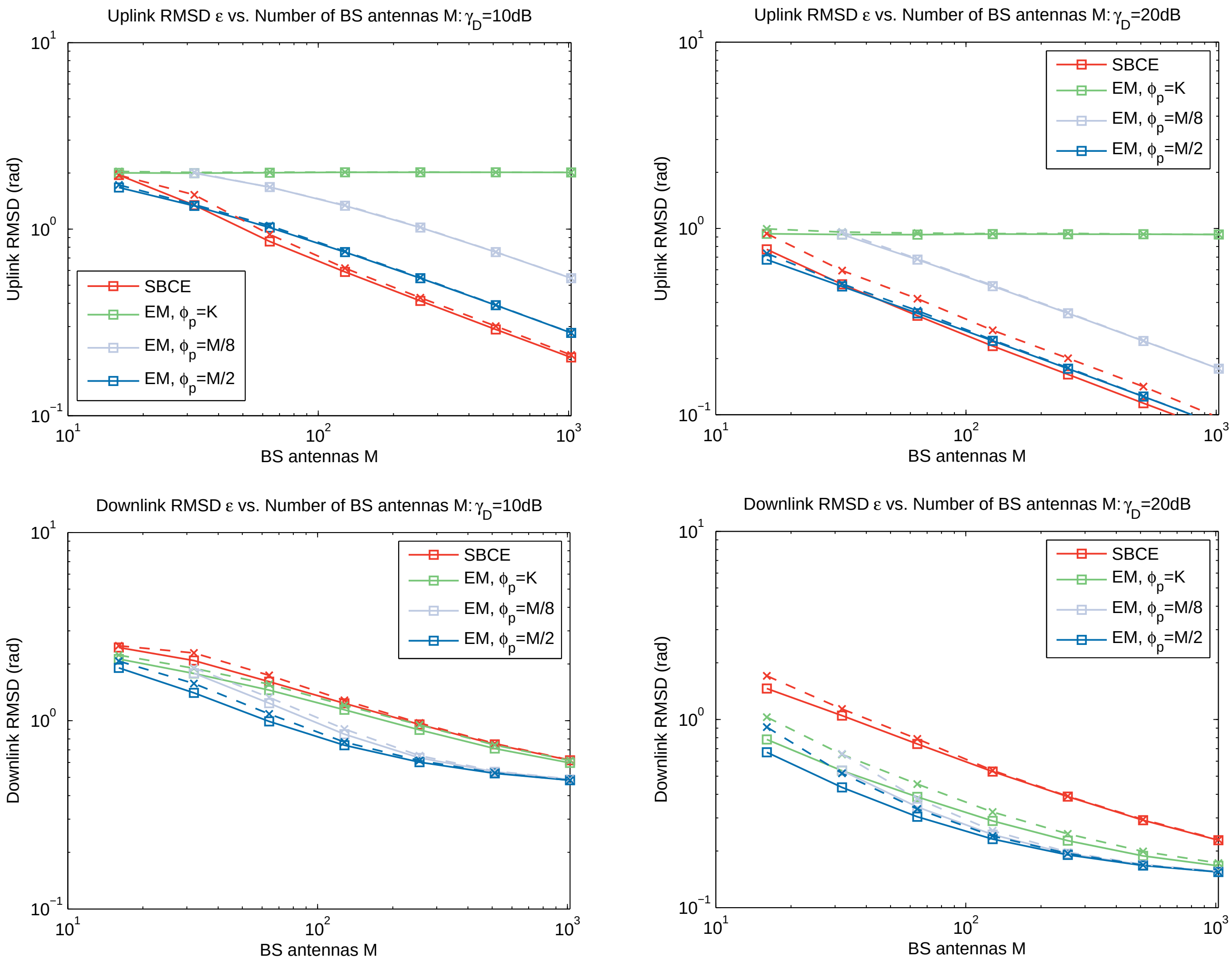

Fig. 1. CSI accuracy in terms of RMSD ϵ (in radians) as a function of the BS antenna number M for $K = 4$, and for $\gamma_{\mathrm{D}} = \gamma_{\mathrm{U}} + 20\mathrm{dB} \in \{10\mathrm{dB}, 20\mathrm{dB}\}$. For the Echo-MIMO (EM) scheme, multiple uplink training lengths $\phi_{\mathrm{p}} \in \{K, M/8, M/2\}$ are simulated. The solid lines depict the i.i.d. Rayleigh fading case (i.e., $\xi = 0$), and the dashed lines illustrate the case of strong spatial correlation with $\xi = 0.8$.

C. Achievable Signal-to-Interference-Plus-Noise Ratio (SINR)

In order to assess the impact of the CSI quality onto the system performance, we evaluate the average UL and DL SINRs under the assumption of zero-forcing (ZF) precoding and equalization at the base station. The ZF precoder $\boldsymbol{w}_k$ for the k-th terminal is given by

$$\boldsymbol{w}_k = \frac{\Pi^{\perp}_{\tilde{\boldsymbol{B}}_k} \hat{\boldsymbol{b}}^*_k}{\left\| \Pi^{\perp}_{\tilde{\boldsymbol{B}}_k} \hat{\boldsymbol{b}}^*_k \right\|}, \tilde{\boldsymbol{B}}_k = \left[\hat{\boldsymbol{b}}^*_1, \ldots, \hat{\boldsymbol{b}}^*_{k-1}, \hat{\boldsymbol{b}}^*_{k+1}, \ldots, \hat{\boldsymbol{b}}^*_K \right]$$

where $\Pi^{\perp}_{\tilde{\boldsymbol{B}}_k} = \boldsymbol{I}_M - \tilde{\boldsymbol{B}}_k (\tilde{\boldsymbol{B}}_k^H \tilde{\boldsymbol{B}}_k)^{-1} \tilde{\boldsymbol{B}}_k^H$ denotes the orthogonal projector onto the null space of $\tilde{\boldsymbol{B}}_k$. The DL data transmission is described by the equation

$$\boldsymbol{r}(t) = \boldsymbol{B}^T \boldsymbol{W} \boldsymbol{q}(t) + \boldsymbol{m}(t)$$

with $\boldsymbol{r}(t) = [r_1(t), \ldots, r_K(t)]$, $\boldsymbol{B} = [\boldsymbol{b}_1, \ldots, \boldsymbol{b}_K]$, $\boldsymbol{W} = [\boldsymbol{w}_1, \ldots, \boldsymbol{w}_K]$, $\boldsymbol{q}(t) = [q_1(t), \ldots, q_K(t)]$, and $\boldsymbol{m}(t) = [m_1(t), \ldots, m_K(t)]$. We assume that $\mathrm{E}\left[\boldsymbol{q}(t)\boldsymbol{q}(t)^H\right] = \boldsymbol{I}_K$ and $\mathrm{E}\left[\boldsymbol{m}(t)\boldsymbol{m}(t)^H\right] = \sigma^2_{\mathrm{D}} \boldsymbol{I}_K$, so that we have the same average transmit power per antenna as for the training phase. The average DL SINR $\gamma_{k,\mathrm{D}}$ at the k-th terminal is then

$$\gamma_{k,\mathrm{D}} = \frac{\left| \boldsymbol{b}_k^T \boldsymbol{w}_k \right|^2}{\sum_{l \neq k} \left| \boldsymbol{b}_k^T \boldsymbol{w}_l \right|^2 + \sigma^2_{\mathrm{D}}}.$$

For the UL, we employ an analogue model with ZF equalization at the base station side, and unit variance transmit signals.

Figure 2 shows the achieved UL and DL SINRs, averaged over 500 channel realizations, as a function of the BS antenna number M. In addition to the CSI obtained by SBCE and Echo-MIMO (EM) method, we plot the average SINRs achieved with perfect channel knowledge, which serve as upper bounds. From the figures, it is easy to verify that the poor DL CSI accuracy of the SBCE method does not translate to bad DL SINRs. For the low SNR case $\gamma_{\mathrm{D}} = 10\mathrm{dB}$, the SBCE method outperforms the Echo-MIMO method with $\phi_{\mathrm{p}} = K$ (i.e., using the minimum number of UL pilots) for

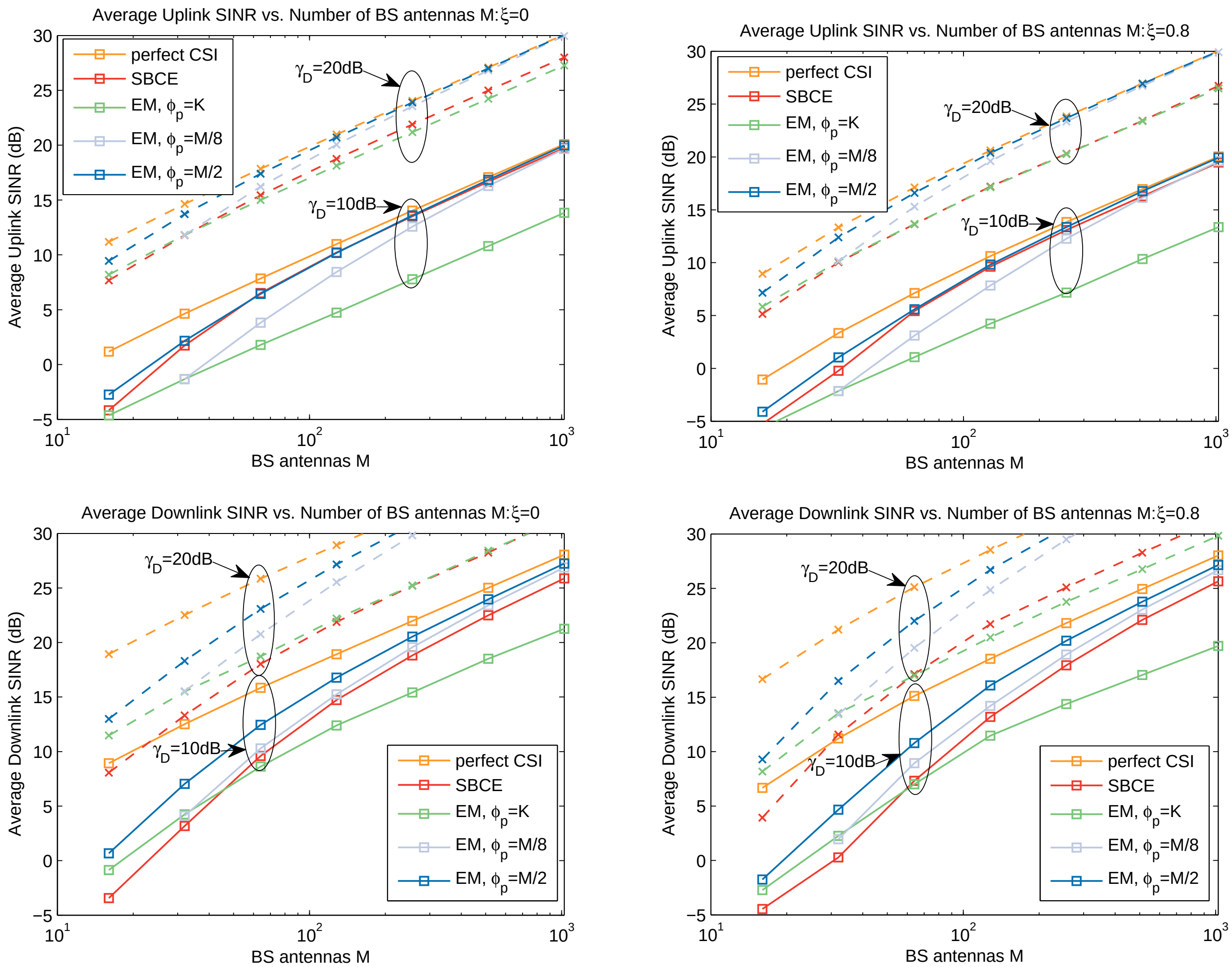

Fig. 2. Average SINR $\bar{\gamma}_{\mathrm{D}}$ as a function of the BS antenna number M for $K = 4$, and for $\gamma_{\mathrm{D}} = \gamma_{\mathrm{U}} + 20\mathrm{dB} \in \{10\mathrm{dB}, 20\mathrm{dB}\}$. For the Echo-MIMO (EM) scheme, multiple uplink training lengths $\phi_{\mathrm{p}} \in \{K, M/8, M/2\}$ are simulated. In contrast to Figure 1, the solid lines depict the results for low SNR $\gamma_{\mathrm{D}} = 10\mathrm{dB}$, and the dashed lines illustrate the results for $\gamma_{\mathrm{D}} = 20\mathrm{dB}$.

all $M > 32$. In contrast, if we assume UL training lengths for the Echo-MIMO scheme that grow with M then the picture is different. Comparing the UL SINRs, the Echo-MIMO method with $\phi_{\mathrm{p}} = M/2$ is always superior to SBCE method, and in terms of DL SINRs, the Echo-MIMO method with $\phi_{\mathrm{p}} = M/8$ always excels the SBCE method. However, both configurations require 50% (resp. 12.5%) more UL resource samples compared to the SBCE method.

VI. Conclusion

We have demonstrated a novel closed-loop channel estimation scheme for FDD Massive MIMO systems, which does not rely on any spatial sparsity assumption. The proposed SBCE method requires only M DL and M UL resources samples as opposed to the (state-of-the-art) Echo-MIMO method with M DL and $M+\phi_{\mathrm{p}}$ UL resources samples. When focusing on the minimum CSI acquisition overhead (i.e., $\phi_{\mathrm{p}} = K$), we have seen for sufficiently large M that the SBCE method provides larger coherent processing gains (in terms of achievable SINRs for the data signals) than the Echo-MIMO scheme, because it exploits the increasing amount of CSI feedback symbols for the UL channel estimation. Of course, by choosing ϕ_{p} sufficiently large, the Echo-MIMO scheme is always able to outperform the SBCE method. We have seen that the therefore required ϕ_{p} needs to grow with M; e.g., $\phi_{\mathrm{p}} = M/2$ in order to provide larger UL and DL performance gains. Regarding the i.i.d. Rayleigh fading assumption on which the SBCE method relies on, we have shown that a violation of this assumption does not cause a signification performance degradation.

There are a number of open problems which are not addressed in this paper: For frequency-flat channels, the correlative filters of the SBCE method can be implemented in the analog domain, so that the CSI does not need to be digitized at the terminal side. For frequency-selective channels, one has to resort to subband-wise processing which typically requires digital processing. A potential alternative would be the application of the blind identification scheme in [23], which applies for multi-tap MIMO channels.

Second, the complexity of the whitening matrix computation as well as its pseudo-inverse computation grows with the number BS antennas M, which demands for efficient processing algorithms.

Regarding the design of the correlative filters, the question arises whether there is a better choice than the truncated IIR filters, whose "poles" are chosen such that they have the maximum angular distance.

And finally, performance comparisons with other CSI feedback schemes (e.g., CSI quantization) potentially provide interesting insights.

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